

GPS AND VERTICAL CONTROL  
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February 18, 1994

## INTRODUCTION

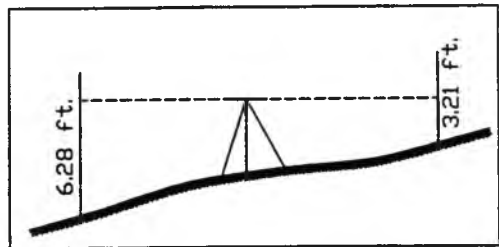
Surveyors and engineers are being asked more and more frequently to gather geopositioning data for use in a Geographical Information System (GIS). And because of the utility of the Global Positioning System (GPS), these positions are often being obtained using this satellite based survey technique.

If applied properly, GPS is an excellent tool with which to gather location data with sufficient accuracy in both the horizontal and vertical components for most GIS's. Often it is not well understood what is required by the user to achieve "sufficient" vertical accuracies. Hopefully the following discussion will shed some light on this topic.

## LEVELING

Leveling, or more specifically differential spirit leveling, is the process of determining height differences between points located at or near the surface of the earth. The common tool of leveling is the level, an instrument which can easily be aligned with the local level surface.

By adding the difference in height between each point and the level, taking care to use the proper sign of the difference, the entire height difference between two points can readily be determined. If one were to begin to level from a point of known height, the height difference could be added to the known starting height to determine an absolute height for the unknown point.



## LEVEL SURFACES/ORTHOMETRIC HEIGHTS

Level surfaces are not necessarily flat or even horizontal. Contrary to intuition or previous experience, they are surfaces on which all points have an equal amount of potential energy. What this means is that a ball placed on a level surface will not roll around. Even though that level surface looks curved or even tilted.

The most well known, and misunderstood, level surface may be mean sea level. If the earth were to stop rotating, the winds

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stopped creating waves, the moon stopped revolving about the earth, and the sun stopped shining, we would be in deep trouble. But besides that, the surface of the oceans would form an idealized level surface. And as Christopher Columbus proved, the oceans are by no means flat.

But when leveling on a local scale, the curvature of the earth need not always be taken into account. If it is, elevation estimates can be computed rather precisely. But if curvature is ignored, systematic errors will be introduced which are dependent on the length of the sights. In any event, level lines formed by the levelling instrument can be assumed, often with minimal error, to be parallel with the local horizon.

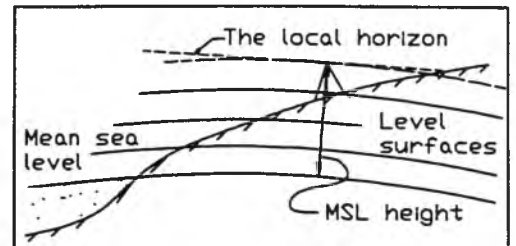
As stated, the operation of leveling determines height differences between two or more points. And these differences typically refer to mean sea level. In Indiana, mean sea level is approximately 600 feet below the surface of the terrain. From a different perspective, Indiana is approximately 600 feet above mean sea level.

The last point about the operation of leveling is that it is a physical measurement. That is, height differences determined using differential spirit leveling are physical quantities referenced to a physical surface, the idealized surface of the oceans. Whereas GPS derived heights, to be discussed shortly, are geometric quantities referenced to a mathematical frame, the ellipsoid.

HEIGHT SYSTEMS

Everyone knows that "water flows down hill". But does it really? The answer to that question depends on one's definition of a hill. And this is where the problem with GPS derived heights begins. It turns out that there is more than one definition.

When one typically thinks of a hill, one may visualize an undulating, inclined surface. Spirit leveling uses this concept of a hill with ups and downs or corresponding highs and lows. A leveling instrument is set up such that it is level or plumb with the local vertical. When level, the instrument sights along a plane tangent to the local equipotential line while at the same time being perpendicular to the local plumb line. Heights are then measured with respect to a datum such as mean sea level along the local vertical.



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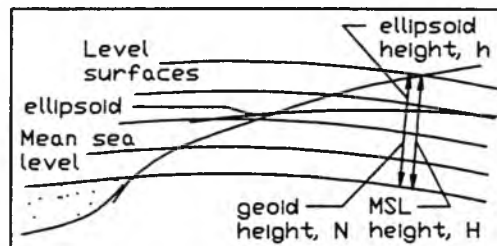
But GPS does not use this local vertical, although GPS network adjustment software often does. GPS references the three dimensional cartesian coordinate system or a reference ellipsoid. Using the cartesian coordinate system, there is no "up". There are the X, Y, and Z axes. The reference ellipsoid uses a height system which is closer to the everyday system of up but with major differences.

The direction of "up" with respect to the ellipsoid (normal to the ellipsoid) may be close to the local vertical (plumb line), but the reference surface is different. Ellipsoid heights are measured along the ellipsoidal normal from the surface of the ellipsoid. In Indiana, the two surfaces, MSL and the reference ellipsoid, are over 30 meters apart and not necessarily parallel. (So down with respect to MSL may actually be up with respect to GPS).

GPS derived elevations refer to heights above the ellipsoid, an abstract three dimensional reference frame. The ellipsoid is used because of the relative ease with which mathematics can be performed on it.

Leveling derived elevations refer to heights above mean sea level, an actual physical, although idealized, surface. This idealized mean sea level, also referred to as the geoid, is used because of its physical significance.

So how many height systems are there? And how does one transform heights referenced in one system to heights referenced in another system. This discussion focuses on three of those height systems, mean sea level heights, ellipsoid heights, and geoid heights. It also discusses how the three heights are related as well as the accuracies associated with the estimate of each.



#### i. Mean Sea Level Heights

The vertical system most evident and important to people in everyday use, and certainly in large construction projects, is related to the earth's gravity field. It is in this system that the words "Water flows down hill" are significant.

The earth's gravity potential field is irregular in nature and rather difficult to model mathematically. Heights determined in this system are called mean sea level heights or orthometric heights and are related to the idealized surface of the oceans, known as the geoid.

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Mean sea level heights are typically obtained using differential spirit leveling. Depending on the care exercised in performing the field work, differences in MSL heights can be obtained with millimeter accuracy.

## ii. Ellipsoid Heights

GPS surveying is used to determine both horizontal and vertical relative positioning with respect to earth centered, earth fixed reference systems. Positioning with respect to these systems is based strictly on geometric observations. These reference systems are mathematical abstractions which have been developed for their ease of computation. Whether they be cartesian or ellipsoidal, the vertical component is not directly applicable for many construction type projects where elevations are referenced to a mean sea level.

Heights obtained from GPS refer to the reference ellipsoid. GPS derived heights are measured between the point in question and the surface of the reference ellipsoid along a normal to the ellipsoid. These heights do not reference mean sea level as heights are commonly referred. Ellipsoid heights can be observed with an accuracy 2-3 cm using relative positioning techniques.

Horizontal positions are also referenced to the ellipsoid as latitude and longitude. These values are readily understood and used by many persons. Ellipsoid heights do not represent a physical quantity and are not useful to most persons. Conversion of ellipsoid heights to mean sea level heights is dependent on geoid heights.

## iii. Geoid Heights

The geoid height,  $N$ , is the height of the geoid, or mean sea level, above or below the reference ellipsoid. As such, it is the one quantity that ties the two vertical reference frames together. Positive geoid heights indicate a geoid which is above the ellipsoid while negative geoid heights indicate a geoid which is below the ellipsoid. Throughout most of the United States, geoid heights are negative.

The geoid is composed of three components: long wavelength (low frequency) global effects, intermediate wavelength regional and local gravity effects, and short wavelength (high frequency) local terrain effects. As it is impacted by the density distribution of the earth, it changes from location to location. Relative geoid height differences can be estimated with an accuracy of approximately 10 cm.

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GEOID93, THE GEOID IN USE IN INDIANA

GEOID93 is a computer model used to estimate geoid heights for the continental United States. It was developed by the National Geodetic Survey (NGS) and is available from them for a nominal fee. It estimates geoid heights with a reported accuracy of 10 cm (one standard deviation) over lengths of approximately 100 km.

GEOID93 was constructed using orbital information from artificial satellites, over 1.8 million gravity anomalies gathered over the entire country, and terrain effects. The geoid heights are only computed on a 3' x 3' grid. For random points off the grid, geoid heights are interpolated. Geoid height accuracies may vary from point to point within a project area as the reported accuracies refer to the grid points.

A contour map of the geoid height for the entire State of Indiana has been prepared from GEOID93 (see following page). As can be seen from the map, the geoid is located below the ellipsoid by a distance of approximately 30 to 34 meters (indicated by the negative signs).

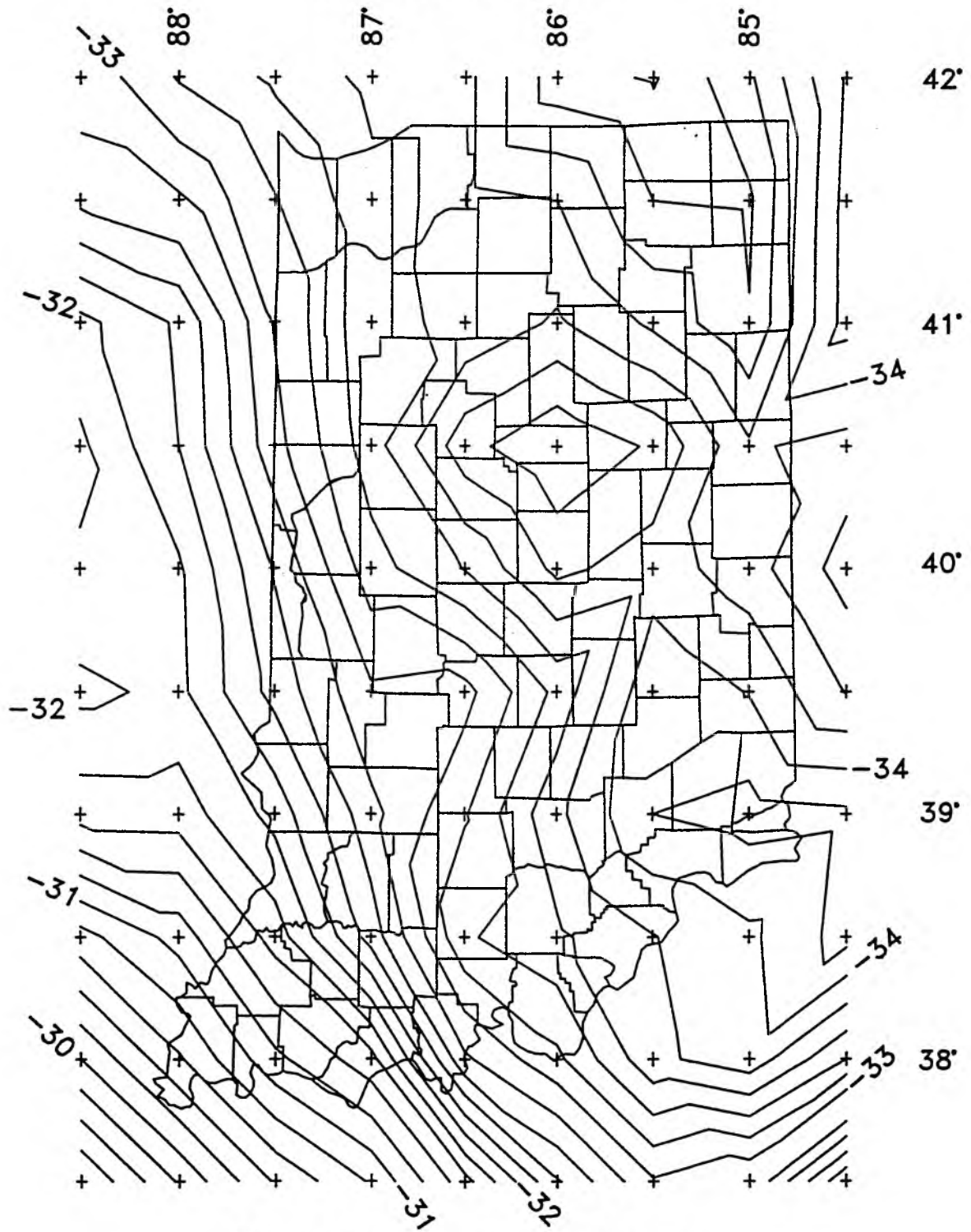
In general, one can see that the geoid slopes down from the southwest to the northeast. The contour interval used in this map is 0.2 m or 20 cm. At 30.48 cm per foot, one contour interval represents a change of 0.66 feet, or 8 inches.

One can now also see that the change in geoid height is related to distance surveyed. The longer the baseline, the greater the difference in geoid heights. So it is easy to see that by ignoring geoid heights, errors of considerable proportion can be introduced into GPS derived elevations or mean sea level heights. Over an average county, the geoid height changes by approximately 40 cm (two contour intervals), or 16 inches, a number which should be considered when computing GPS derived heights.

Differences in geoid heights may be neglected for the determination of orthometric heights in some instances. A single "project" geoid height may be used where the project area is very limited, where the geoid and ellipsoid are approximately parallel, or where the project accuracy requirements can be met without the need for geoid height differences.

But before ignoring geoid height differences, consider your specific project. Does the project have a large spatial extent? Do geoid heights change much over the project area based on GEOID93? And can you be reasonably sure there are no other factors which impact geoid height differences, such as systematic rotations between the geoid and the GPS reference system, the ellipsoid.

STATE OF INDIANA  
GEOID UNDULATIONS (meters)  
GEOID 93 MODEL



Contour Interval = 0.2 m

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LEVELING BY GPS

The use of GPS to level, or obtain mean sea level heights, actually involves the marriage of the two disparate vertical reference frames described above. As mentioned, elevations derived directly from GPS are not the same as elevations derived from leveling.

The relationship between GPS derived heights and spirit leveling derived heights is expressed as,

$$H = h - N$$

where H represents MSL heights, h represents GPS or ellipsoidal heights, and N represents geoid height. Ellipsoidal height minus geoid height equals MSL height or orthometric height.

Given any two of these two height elements, the third is easily computed. If one had the ellipsoidal height of a particular point, obtained using GPS, and the geoid height at the same point, obtained some other way, one could easily compute the MSL height or elevation of that same point without ever using a level. Likewise, given the ellipsoidal height and the orthometric height, one could easily compute the geoid height. So the key to using GPS to level is to convert ellipsoidal heights into MSL heights.

The problem of estimating MSL heights from GPS derived heights is to accurately estimate the geoid height and to apply it once estimated. Some methods of estimating MSL heights ignore the estimate of N by assuming it to be the same throughout the project area. By using height differences rather than absolute heights, one can readily see what happens when the geoid height, N, is assumed to be constant. The relationship shown above, rewritten using height differences rather than absolute heights, is

$$(H_2 - H_1) = (h_2 - h_1) - (N_2 - N_1).$$

Written in terms of differences, this equation can be rewritten as,

$$\Delta H = \Delta h - \Delta N.$$

When N is assumed to be constant throughout a project area, the term  $\Delta N$  disappears. However, if the geoid height is not constant within that same area, errors in MSL height estimates will occur. How much error will occur depends on where one is located, the length of the baseline used to perform relative positioning, and the orientation of the baseline.

When adjusting GPS networks, arguably the easiest, or at least most simplistic, method of estimating MSL heights is to hold fixed

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the MSL elevation of any one survey station. For this method, the network adjustment software computes the elevations of all unknown survey stations while applying very little in the way of corrections to account for the differences between GPS and local control. This method is easiest because only one mean sea level elevation is needed for an entire project.

The only corrections to be applied to the GPS observations are a scale factor and a rotation angle. These two trend or bias parameters are determined by the network adjustment software in addition to the differential corrections to the coordinates of the network survey points during the overconstrained adjustment. The scale factor is used to scale the satellite derived network to local control while the rotation angle is used to orientate, in the local horizon system, the GPS network with local control.

There are two different ways to estimate MSL heights using only one fixed elevation. One way is to use no geoid heights. The other way is to use geoid heights for all network points. Each method works. The question is "How well do they both work?"

The difficulty with using either of these approaches is that correct estimates of MSL heights are dependent on the correctness of the one known elevation. Accurate estimates of MSL heights are also dependent on the unmodeled behavior of the geoid, and some of the errors generated by the GPS itself. All three error components can be significant, particularly the first two.

Consequently, the resulting elevations will most likely be very rough estimates indeed. Nevertheless, achievable accuracies need be determined. This information would be a benefit to those currently using the one fixed elevation method (with and without geoid heights) as well as to those projects in which accuracies achievable by this method are actually acceptable.

Other methods exist whereby the GPS derived network is transformed to best fit local control, including the use of multiple existing bench marks. This technique uses differential rotation angles and scale factors to fit the satellite derived network to the local network which may be distorted somewhat. This method estimates the tip and tilt between the ellipsoid and the geoid and applies that information to points with no known elevation.

The tip and tilt of the geoid, with respect to the ellipsoid, can be determined on a local level by combining GPS observations with MSL heights determined using differential leveling. An absolute minimum of three points located around the project area with both GPS derived ellipsoid heights and spirit leveling derived MSL heights will provide three local geoid heights. From these



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three geoid heights, the tip and tilt of the geoid with respect to the ellipsoid can be computed assuming the geoid to be a plane within the project area.

Using this method, in combination with geoid heights for all points, will produce better estimates of mean sea level heights. The achievable accuracies depend on the accuracy of geoid height estimates and the degree to which the relationship between the ellipsoid and local vertical control can be modeled.

SUMMARY

In summary, GPS produces ellipsoidal heights rather than mean sea level heights most people are accustomed to using. In order to use these height estimates where conventional mean sea level heights are required, geoid heights must be taken into account.

Accurate determination of the geoid heights, and their application, are the key to the estimation of mean sea level heights using GPS. The use of the GEOID93 model may allow for rather high differential mean sea level height accuracies.

Considering ellipsoidal heights can be determined with an accuracy of approximately 2-3 cm, differential mean sea level heights, using GPS, may be determined with a comparable accuracy, i.e., approaching 2-3 centimeters for short baselines (< 10 km) in areas where the geoid is well-behaved (e.g., Indiana).

GPS leveling may be very practical over small areas or in areas with a well behaved and modeled geoid. A scattering of control points with known mean sea level heights, however, should be available within the project area to improved the accuracy of height estimates. Today's techniques may not be able to provide the highest of accuracies but they should yield accuracies sufficient for many engineering projects and GIS's.

In general, such accuracies may not be attainable, however, where the geoid heights are quite varied or gravity data is sparse. Until the accuracy of the geoid model is refined further, GPS leveling will not be able to provide accuracies approaching high order National Vertical Control accuracies.

Should GPS leveling become an everyday reality, its impact on data acquisition for GIS's should be significant. No longer will survey crews need to trek over hill and dale to acquire MSL heights. Those same heights can be obtained at the same time horizontal positions are being obtained, with very little additional effort.