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Review of Fractal Heat Exchangers

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ABSTRACT

Nature has inspired many scientists and engineers to solve problems through observation and mimicry. One such example is heat transfer enhancement. The enormous natural heat and mass transfer phenomena have led engineers to seek solutions to heat transfer enhancement problems from nature. Fractal geometries are found in respiratory and vascular systems of plants and animals, such as blood vessels, human lungs, leaves, coastlines, etc. Inspired by this, fractal heat exchangers have been developed and were found to have intrinsic advantage of minimized flow resistance and strong heat transfer capability. In current study, a comprehensive literature review was carried out to investigate the thermal and hydraulic performance of fractal heat exchangers, with a focus on fractal channels. Fractal theory, model development, performance comparison with traditional designs, heat transfer and fluid mechanisms and design methods are discussed, separately. Fractal theory was proposed in 1926 and developed in the past decades. Model developed from 1-D to 2-D and 3-D with the assumptions being more complex and realistic, leading to more accurate results. Fractal channels' thermal and hydraulic performances were found to surpass that of traditional parallel channels and serpentine channels due to flow mixing and pressure drop recovery caused by bifurcation. Design methods include traditional scaling laws and a newly proposed topology optimization methodology. At last, the research gaps for future researches are discussed. Main research gaps are insufficient experimental data, lacking of study on optimal design, no research on liquid-to-gas heat exchangers and need of research on topology optimization.

1. INTRODUCTION

Nature has always been the source of inspirations for scientists and engineers to solve problems in various fields. Abundant instructive heat and mass transfer enhancement phenomenon and mechanisms are observed in the nature and partially mimicked and applied in heat and mass transfer enhancements. Heat exchangers design is of significance due to its crucial role in thermal and power systems. Hence, heat exchanger design is one of the main research domains of mimicking the heat and mass transfer phenomenon in the nature. Fractal geometry, widely found in respiratory systems and vascular systems of plants and animals, has been introduced into heat transfer area because of its intrinsic advantage of minimized flow resistance and strong heat transfer capability. In current study, a comprehensive review of fractal heat exchangers is presented. First the fractal theory development is summarized, and then model development is reviewed with a focus on discussion of assumptions. Major findings and mechanisms for the phenomena are analyzed as well as design parameters that would affect the thermal and hydraulic performance. Finally the research gaps are discussed.

2. FRACTAL THEORY

Significant research has been conducted on the theory of fractals (Murray, 1926; Sherman, 1981; Mandelbrot, 1982; West, 1997; Bejan et al., 2008; Bejan and Lorente, 2006, 2007, 2011; Bejan, 1997, 2002, 2003; Xu and Yu, 2006), and the main findings are summarized in Table 1.

Table 1: Fractal theory development

Researchers	Major findings
Murray, 1926	Developed Murray's law: The cube of the radius of a parent branch equals the sum of the cubes of the radii of daughter branches.
Sherman, 1981	Found when Murray's law was obeyed a functional relationship exists between channel diameters and various flow characteristics such as wall shear stress, velocity profile, and pressure gradient.
Mandelbrot, 1982	Described fractal structure from nature: coastlines, leaves and clouds.
West, 1997	Developed scaling laws for a bulk fluid transport problem to minimize the flow work.
Bejan et al., 2008; Bejan and Lorente, 2006, 2007, 2011; Bejan, 1997, 2002, 2003	Developed Constructal Theory: For a finite-size flow system to persist in time (to survive) its configuration must evolve in such a way that it provides an easier access to the currents that flow through it.
Xu and Yu, 2006	Analyzed the transport properties including electrical conductivity, heat conduction, convective heat transfer, laminar flow, and turbulent flow in the networks and also derived the scaling exponents of the transport properties in the networks.

2. MODEL DEVELOPMENT AND MAJOR FINDINGS

2.1 Model development

Fractal theory has been applied in different kinds of heat exchange devices, but mostly in heat sinks for electronic devices, which are reviewed in details. Table 2 is a summary of single phase with a focus on application of heat sink. Different shapes are used for research, such as disk shape as shown in Figure 1(a), rectangular shape as shown in Figure 1(b) and rectangular shape with the branch angle equals to $180^\circ+180^\circ$ as shown in Figure 2. For disk shape, it is usually one layer but for rectangular shape, some researches have tried for two-layer sandwich structure, which is illustrated in Figure 3. Plenty of researches have been done to analytically and numerically investigate the thermal and hydraulic performance of fractal heat sink, but experimental work is insufficient. In Table 2, only number 6, 10, 12 and 13 include experimental work.

Table 2: Summary of single phase with a focus on application of heat sink

No.	Researcher	Shape	Layers	Model	Assumptions*	Experiment (N/Y)
1	Pence, 2002	Disk	One	1-D	1, 2, 3a+3b, 4a, 5a.	N
2	Chen and Cheng, 2002	Rectangular	Two	1-D	1, 2, 3a, 4a, 6a.	N
3	Senn and Poulikakos, 2004	Rectangular	Two	3-D	1, 2, 3b, 4b, 5a, 6a.	N
4	Alharbi et al., 2003	Disk	One	3-D	1, 3b, 4b, 5a, 6a.	N
5	Alharbi et al., 2004	Disk	One	3-D	1, 3b, 4b, 5a, 6a.	N
6	Enfield et al., 2004	Disk	One	2-D	1, 3b, 4b, 5a.	Y
7	Wang et al., 2006	Rectangular, angle= $180^\circ+180^\circ$	One	3-D	1, 2, 3b, 4b, 5b, 6a.	N
8	Wang et al., 2007	Disk, with different angles	One	3-D	1, 2, 3b, 4b, 5b, 6a.	N
9	Hong et al., 2007	Rectangular	One	3-D	1, 2, 3b, 4b, 5b, 6c.	N
10	Chen et al., 2010	Rectangular	Two	3-D	1, 2, 3b, 4b, 5a, 6c.	Y
11	Wang et al., 2010	Rectangular	One	3-D	1, 2, 3b, 4b, 5b, 6a.	N
12	Yu et al., 2012	Rectangular, angle= $180^\circ+180^\circ$	One	3-D	1, 2, 3b, 4b, 5b, 6b.	Y
13	Zhang et al., 2013	Rectangular, angle= $180^\circ+180^\circ$	One	3-D	1, 2, 3b, 4b, 5b, 6b.	Y
14	Zhang et al., 2015	Rectangular, angle= $180^\circ+180^\circ$	One	3-D	1, 2, 3b, 4b, 5b, 6b.	N

* Explanation of assumptions:

1: laminar flow and negligible heat loss to environment;

2: negligible gravity;

3a/3b: fully developed flow/developing flow in channel;

4a/4b: negligible/non-negligible effect at bifurcation;

5a/5b: constant/temperature dependent properties;

6a/6b/6c: constant heat flux at channel wall/constant temperature with adiabatic top/constant heat flux at bottom plate and conjugated heat transfer of wall.

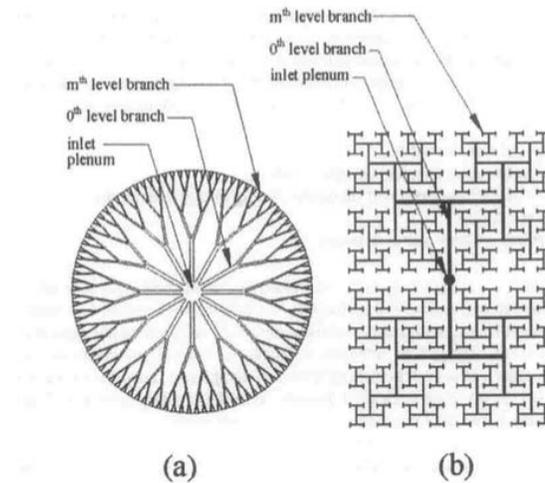


Figure 1: Disk shape (a) and rectangular shape (b) (Pence, 2002)

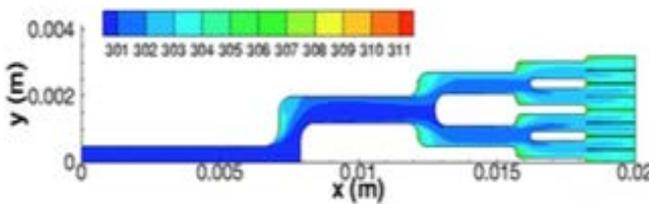


Figure 2: Branch angle equals $180^\circ + 180^\circ$
(Wang et al., 2007)

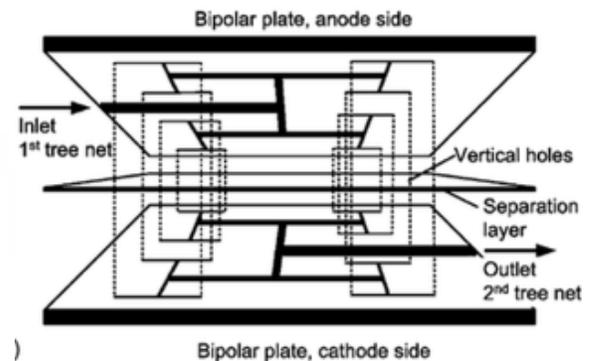


Figure 3: Two-layer sandwich structure
(Senn and Poulikakos, 2004)

Models in literature have been developed over the last decades and summarized in Table 2 and Table 3. The research trend is to eliminate the simplicity of assumptions to make the model closer to the reality, but simple model is still applicable under certain circumstances and have the advantage of simplicity. Here are some discussions about the main assumptions that are made in the model and major findings, respectively.

- Assumption 1: Almost all models assume laminar flow and neglect the heat loss to environment.
- Assumption 2: Most models do not account for the effect of gravity but Guo et al. (2014) pointed out that the inertial force could result in uneven bifurcation, leading to flow mal-distribution. This is especially the case when the channel length isn't long enough for the flow to be fully developed again after the bifurcation.
- Assumption 3: Senn and Poulikakos (2004) pointed out that in Chen and Cheng's (2002) analytical analysis, the assumption of both thermally and hydro-dynamically developed flow was only appropriate when the

hydrodynamic and thermal development lengths were negligible compared with the channel lengths, which could be true if the length to diameter ratio was very high for the flow channels and the Reynolds number was rather small; Otherwise, the assumption was not applicable and the flow should be assumed to be developing flow.

- Assumption 4: In the early stage research period, Pence (2002), Chen and Cheng (2002) assumed that the effect of bifurcation on heat transfer and pressure drop were negligible while Alharbi et al. (2003) observed pressure recovery at bifurcations that resulted from an increase in flow were in fractal channels which made the prediction of pressure drop to be 20% higher if the effect of bifurcation was neglected. Senn and Poulidakos (2004) and Wang et al. (2007) also found pressure drop from bifurcation was substantial and not negligible. Zhang et al. (2011) stated that whether the pressure drop should be taken into consideration was related to the angle of the branches. The effect of bifurcation on the pressure loss could be neglected for disk configuration while it should be taken into consideration for rectangular shape.
- Assumption 5: Alharbi et al. (2004) found the pressure drop for straight channels was under estimated if using constant properties by 17% and the constant properties assumption was not suitable for high heat flux condition because of the large temperature range.
- Assumption 6: Different thermal boundary conditions were made in the open literature, including constant heat flux, constant temperature wall with adiabatic top wall and constant heat flux at bottom plate with conjugated heat transfer of wall. Selection should be made by the operating condition to further approach the reality. Hong et al. (2007) first suggested to use conjugated heat transfer of wall which led to a finding of hotspots at highest branches which wasn't observed in previous research using constant wall flux (Senn and Poulidakos, 2004).

In summary, more complicated assumptions are more accurate, but should be selected while balancing calculation time, simplicity and accuracy.

2.2 Major findings and mechanism discussion

The major findings are summarized in Table 3. The similarities and differences are discussed first, followed by the discussion of design parameters' effects on the performance.

(1) Performance of fractal channel (FC) compared with that of serpentine channel (SC)

It was found that FC had larger heat transfer capability and 50% lower pressure drop (Senn and Poulidakos, 2004; Wang et al., 2006; Chen et al., 2010) and inherent advantage of uniform temperature on the heating surface (Wang et al., 2006; Chen et al., 2010) compared with SC with the same heat transfer area and rectangular area. The mechanism for better heat transfer is that first, the reinitiate of boundary layer at each bifurcation results local heat transfer coefficient spikes at each bifurcation with different magnitude which contribute to the global increase of heat transfer coefficients (Wang et al., 2007); second, there is secondary flow motions initiating at bifurcations, generating longitudinal vortices that result in enhancing thermal mixing and a decrease in required flow rate for heat transfer and this laminar mixing by secondary flow motions also improves the local Nusselt number (Senn and Poulidakos, 2004). However, the transverse vortices may create recirculation at bifurcations that results in hotspot at the inner corners of bifurcation (Senn and Poulidakos, 2004; Zhang et al., 2015). The first reason for lower pressure drop is that the pressure drop recovery phenomenon found at the bifurcation due to the increase in cross section area and it is especially found for the low branch level (Senn and Poulidakos, 2004; Wang et al., 2007), which means that there is a local pressure spikes at each bifurcation and this acts as a buffer to diminish the overall pressure drop. Second, there is not fully developed flow in each branch due to the boundary layer reinitiating at each bifurcation, resulting in lower pressure drop. And these also make the pressure drop to be in a non-linear relationship with mass flow rate, unlike the linear one for parallel heat sink (Hong et al., 2007); thus, it is recommended to apply FC in low flow rate case in practice to make most of the advantage in pressure drop. The reason for better temperature uniformity is due to the increased number of parent channels and branch levels, and Wang et al. (2006) also pointed out that fractal channels could effectively reduce the potential thermal damage by reducing the risk of accidental blockage of channel segments.

Table 3: Summary of major findings in research on fractal heat sinks

Ref.	Major findings
Pence, 2002	Compared with PC with equal surface area, FC has: 1. 60% lower pressure drop for the same total flow rate and 30°C lower wall temperature under identical pumping power conditions. 2. 50% lower density with similar maximum wall temperatures and pressure drop.
Chen and Cheng, 2002	Compared with PC with equal surface area, FC has: 1. Higher total heat transfer rate; 2. Lower total pressure drop; 3. Larger fractal dimension or a larger total number of branching levels will result in a stronger heat transfer capability with a smaller pumping power.
Senn and Poulikakos, 2004	1. Compared with SC with same heat transfer area and same rectangular area, FC has larger heat transfer capability and 50% lower pressure drop; 2. Pressure drop from bifurcation is substantial and not negligible; 3. Lower pressure results from the not fully developed flow in higher branching level. 4. Secondary flow motions initiates at bifurcations; 5. Transverse vortices creates recirculation at bifurcations that results in hot spots at the inner corners of bifurcations; 6. Longitudinal vortices results in enhanced thermal mixing and a decrease in the required flow rate for heat transfer; 7. Laminar mixing by secondary flow motions improves local Nusslet number.
Alharbi et al., 2003	Compared with Pence's 1-D model, the 3-D model: 1. Predicts a 20% lower total pressure drop for fractal channels but similar for straight one, this is due to pressure recovery at bifurcations that results from an increase in flow area; 2. Predicts pressure drop 17% higher for SC when use temperature dependent properties, but similar for FC; 3. Has the reinitiating assumption, which seems to provide plausible trends in pressure distribution.
Alharbi et al., 2004	1. FC has 75% lower temperature variation and a 10% pressure-drop penalty compared with the PC; 2. The assumption of constant properties is not suitable for high heat flux condition.
Enfield et al., 2004	1. Developed a 2-D model for predicting concentration profiles and degree of mixing (DoM); 2. Developed a non-dimension number and a design guideline to determine the optimal number of branch levels to minimize pressure drop and maximize DoM for a fixed initial parent channel width, total path length, and channel depth.
Wang et al., 2006	Compare FC with PC and SC, FC has: 1. The best temperature uniformity; 2. Lower pressure drop than SC but higher pressure drop than PC; 3. Reduced risk of accidental blockage of channel segments; 4. Reduced potential of thermal damage due to the reduced risk of blockage; 5. Increased number of parent channels and branch levels resulted in increased temperature uniformity.
Wang et al., 2007	1. Pressure drop increases as bifurcation angle increases with a decreasing increasing rate and 30° is the optimal angle; 2. Channels with bifurcation angle of 180°+180° has a lower pressure drop compared with PC due to pressure recovery at bifurcation; 3. Increasing angle also increases the risk of appearance of hotspot near the bifurcation; 4. More uniform distribution of the outlet mass flow can be achieved with increased bifurcation angles, but the gradient is reduced with increasing angles.
Hong et al., 2007	1. A modified structure was proposed to address the hotspot issue (by adding serpentine channel structure at the end of highest branches); 2. Hotspot appears at the highest branch (4th) due to assumption of conjugate heat transfer; 3. Effect of bifurcation on pressure drop becomes more obvious for higher flow rate, results in a non-linear relationship between pressure drop and mass flow rate, unlike the linear one for PC; 4. The modified FC is much better than that of PC with respect to pressure drop, thermal resistance and temperature uniformity; and this advantage is much more obvious when the flow rate or the pressure drop is low, which is favored because high pressure drop is not recommended in practice for the design of microsystems.
Chen et al., 2010	1. FC has considerable advantages over SC in both heat transfer and pressure drop; 2. FC has inherent advantage of uniform temperature on the heating surface than SC. 3. The local pressure loss due to confluence flow is found to be larger than that due to diffuence flow.
Wang et al., 2010	1. Leaf-like flow networks has lower pressure drop and higher heat transfer coefficient than symmetric tree-like ones.
Yu et al., 2012	1. FC has a much higher heat transfer coefficient at the cost of a much higher pump power compared with PC with the same heat transfer area. 2. AR (aspect ratio=height/width) of microchannels plays a very important role when considering pressure loss, heat transfer coefficient, and COP ; 3. FC with lowest AR has highest COP, but the one with highest AR has the highest ratio of COP over COP of PC.
Zhang et al., 2013	1. Small aspect ratio is preferred for a smaller pressure drop and a larger heat transfer rate; 2. A high branching level produced a high pressure drop and a large heat transfer rate; 2. The bends with fillets for the fractal-like microchannel reduce the local minor pressure losses, compared with that with the 90° bends, resulting in a lower overall pressure drop.
Zhang et al., 2015	1. Both the flow rate and the AR have large influences on the evolution of the vortices, which promote the fluid mixing and enhance the efficiency of heat transfer; 2. FC with a smaller AR of 0.333 was verified to have lower pressure drop and better heat transfer performance within all the other microchannel networks under investigation in the study; 3. Observed transverse and longitudinal vorticities, secondary flow and recirculation flow motions; 4. Confluence flow has a larger pressure drop than diffluent flow, but not much difference.

(2) Performance of fractal channel (FC) compared with that of parallel channels (PC)

Compared with PC, Pence (2002) found FC had 60% lower pressure drop for the same total flow rate and a 30°C lower wall temperature under identical pumping power conditions, and 50% lower density with similar maximum wall temperatures and pressure drop compared with PC with equal surface area. Chen and Cheng (2002), Hong et al. (2007) also concluded that FC had benefits in both heat transfer and pressure drop over PC. Alharbi et al. (2004) found FC had 75% lower temperature variation and a 10% pressure-drop penalty and pointed out that an optimized design was desired. Wang et al. (2006) found that the pressure drop for FC was marginally higher than that for the PC with same total convective heat transfer and channel volume at the same flow rate. These different findings may be due to the channel shape because in this case the channel height were kept the same while the channel width varied, resulting in different aspect ratio (AR) which was found to be a factor that influenced the performance of FC (Zhang et al., 2015) as discussed later.

(3) Discussions about different design factors

Larger fractal dimension or a larger total number of branching levels result in a stronger heat transfer capability with a smaller pumping power required (Chen and Cheng, 2002) and also result in increased temperature uniformity (Wang et al., 2006). As for bifurcation angle, it was found that pressure drop and distribution uniformity of outlet mass flow increased as bifurcation angle increased with a decreasing rate and Wang et al. (2007) recommended that 30° is the optimal angle. Aspect ratio was also investigated and it is proved that AR had large influence on the evolution of vortices, which therefore increased the fluid mixing and enhanced the efficiency of heat transfer, thus it played a very important role. Lowest AR (0.333) has the highest COP, but highest AR has the highest ratio of COP over COP of straight channels (Yu et al., 2012). Since the fractal structure has inlet and outlet, it is interesting to investigate the confluence flow and diffidence flow. It was found that pressure drop due to confluence flow was larger than that of diffidence flow (Chen et al., 2010; Chen et al., 2015) but the difference is insignificant (Chen et al., 2015).

3. RESEARCH TREND AND GAPS

From the modeling point of view, the model developed from 1-D to 2-D and 3-D. Though most researches focus on steady state modeling, there are already researches using the transient heat conduction to investigate the heat transfer performance of fractal structure (Chen et al., 2015). As discussed before, the assumptions of these models tend to be more complicated and closer to the reality, resulting in more accurate results.

From the experiment point of view, the experiments no longer only focus on the pressure drop and temperature of fractal channels (Enfield et al., 2004; Chen et al., 2005; Luo et al., 2007; Chen et al., 2010; Zhang et al., 2013; Xia et al., 2015). Flow visualization technique had also been applied to further investigate the flow properties (Guo et al., 2014). Also adiabatic flow boiling was investigated (Daniels et al., 2011) and it was found that FC had 50% lower pressure drop and lower exit quality compared with PC with equal surface area, pumping power and flow rate. However, there are conflict results in the literature regarding the pressure penalty, revealing that the fractal concept cannot necessarily guarantee a good design, other factors like branch angle and AR that have large impact should be comprehensively studied.

From the application point of view, fractal geometry has been applied in a wide range of heat exchange devices, including heat sink (as shown in Table 2), fuel cells (Senn and Poulikakos, 2004), microreactor (Chen et al., 2011; Chen et al., 2015) distributor (Tondeur and Luo, 2004; Luo et al. 2007; Guo et al., 2014), collector (Guo et al., 2014), tube and shell heat exchanger (Guo et al., 2014), spindle (Xia et al., 2015), Si/Ge nanocomposites (Chen et al., 2015), etc. However most of the research focus on heat sink for electronics and chip cooling due to the inherent advantages of temperature uniformity of fractal structure. For fluid types, researches cover liquid-to-liquid (Tondeur and Luo, 2004; Luo et al., 2007; Guo et al., 2014), solid-to-liquid (as shown in Table 2), solid-to-two phase (Daniels et al. 2011; Daniels et al., 2007; Zhang et al., 2011), solid-to-gas (Chen et al., 2014; Chen et al., 2011; Chen et al., 2015), and solid-to-solid (Chen et al., 2015) heat exchangers; however, no research has been done to liquid-to-gas heat exchangers, which is a research gap and should be investigated.

From the design method point of view, it should be noticed that most design method are based on scaling laws, however, can not necessarily result in optimization design which often lead to the incomprehensive and unfair comparison. Some researchers pointed out that the shape of the bends had an impact on the performance, especially the pressure drop. The bends with fillets for the fractal-like microchannel reduced the local minor pressure losses,

compared with that with the 90° bends, resulting in a lower overall pressure drop (Zhang et al., 2013). Haller et al. (2009) also found that T- and L- junctions with wedges and radii have a lower pressure drop than that with the 90° bends and joints, but suffered a degradation of heat transfer as well. However, this design method integrating fillets and scaling law is imperfect either. Meanwhile, Oevelen and Baelmans (2014) applied topology optimization to the cooling of a constant temperature heat source and optimized the locations where the channels and fins should form, as shown in Figure 4. This topology optimization method leads to branching, tree-like flow network designs. Yaji et al. (2015) utilized topology optimization method for the design of heat sink device as well and achieved a similar fractal design, as shown in Figure 5. Topology optimization should be investigated deeper to achieve optimal design for each application condition.

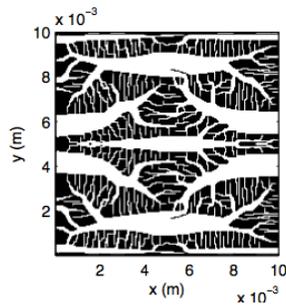


Figure 4: Topology optimized heat sink (Oevelen and Baelmans, 2014)

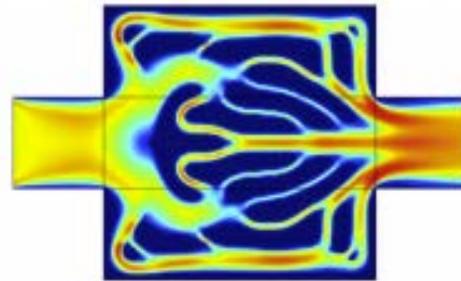


Figure 5: Topology optimized heat sink (Yaji et al., 2015)

4. CONCLUSIONS

Nature has inspired many scientists and engineers to solve problems through observation and mimicking. Fractal geometry mimics the respiratory systems and vascular systems of plants and animals and has been applied to heat exchanger designs. In current study, fractal heat exchangers are reviewed, with a focus on fractal flow channels. Fractal theory has been developed since 1926 and has been applied to many kinds of heat exchangers in the past decades. The model developed from 1-D to 2-D and 3-D with a growing complexity in assumptions. Most researchers found fractal channels to have intrinsic advantage of lower pressure drop, higher heat transfer rate and better thermal uniformity compared with traditional parallel and serpentine channels. Research gaps include, but are not limited to:

- In general, most research focus on analytical or numerical study instead of experiments. More experiments should be done to comprehensively investigate the thermal and hydraulic performance of fractal heat exchangers.
- Most designs are not optimal, and conflictive results for pressure drops are found in literature, due to different design factors selected and lack of optimal design. Thus factors like branch angle and AR should be further studied to optimize designs.
- No research has been conducted to liquid-to-gas heat exchangers in the past due to the manufacturing limitation, which should be investigated now as manufacturing method develops.
- Design methods of fractal heat exchangers are mostly based on scaling laws, more research should be done to investigate more methods, for example, the gradual developing topology optimization method.

NOMENCLATURE

AR	aspect ratio
D	dimension
FC	fractal channel
N	No
PC	parallel channel
SC	serpentine channel
Y	Yes

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