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vasillaq.kacani@lmf.at

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Simulation of Reciprocating Compressor Start-Up and Shut down under Loaded and Unloaded Conditions

Vasillaq Kacani
Leobersdorfer Maschinenfabrik GmbH & Co. KG
Vienna
Austria
Vasillaq.kacani@LMF.at

ABSTRACT

During the start-up and shut down phase of reciprocating compressors the loads on all components of driven train system are very high. In this paper a method for the calculating the forces on coupling, e-motor, crank shaft as well other components of the system will be described. The modelling of the electrical induction motor, coupling, crank shaft, damper as well as the compressor resistance torque are extremely important in simulating start-up and shut down of reciprocating compressor. Equally important, are the switching torque of the electrical motor and the instantaneous moment of inertia of the reciprocating compressor crank gear. The transient start-up and shut down process under loaded and unloaded is described using a non-linear differential equation for driven train system: E-Motor – Coupling – Flywheel – Reciprocating compressor - Damper. Shaft torsional moments on the drive train and especially on the coupling, whether elastic or stiff, can then only be calculated using numerical simulation. This paper will describe some of the key elements in modelling, simulating and measurements of drive train start-up and shut down carried out on already operational piston compressor units

1. INTRODUCTION

During the start-up, the inductive motor develops a very high exciting torque, up to six times the rated torque of machine. The dominant frequency is the line frequency. (50Hz, 60Hz). The investigation of the expansion of this torque in the drive train is very important to determine the maximal load on several components of reciprocating compressors. The weakest component in drive train is generally the coupling. The coupling manufacturer provides the main data of the coupling such as stiffness, inertia, maximum torque witch occur during normal transient condition (start-up, passing through resonance) as well as the maximum allowable torque during any abnormal transient condition (short circuits). The goal of this paper is to determine the actual maximal torque on all shafts of the drive train, especially on the coupling and on the compressor shaft during the unloaded start-up of reciprocating compressors. The calculated torques can then be compared with allowable figures. Dependent on the number of spring-masses, the transient phenomena of the start-up is described by a set of nonlinear differential equation. The required torques on the shafts of drive train can be obtained from the numerical solution of differential equations 2), 3). First the torsional stiffness, inertia and damping coefficients must be calculated and the mathematical models of all components are required. Only a clear knowledge of the torsional parameters as well as a proper torsional model of reciprocating compressor components allows a safe dynamic design of drive train.
2. TORSIONAL MODEL OF DRIVE TRAIN COMPONENTS

The accuracy of a simulation depends on the mathematical model of drive train components and the associated parameters. The main data to be specified are the torsional stiffness, mass moment of inertia, damping coefficient as well as the mathematical models.

2.1 Crank shaft

There are different types and design of crank shafts. Depending on the type of the reciprocating compressor there may be one or two throws between the journals. The crank pin usually drives the running gear: connecting rod, X-head, piston rod and piston. V-arrangement compressors have two connecting rods at the same crank pin. W-arrangement compressors have generally three connecting rods at the same crank pin.

**Torsional stiffness**: The first approach calculation of torsional stiffness of the crank shaft can be performed with equations developed by Carter, Ker Wilson, Jackson, Tuplin. The basic dimensions of the journals are needed as inputs, as well as the webs, crank pins, stroke and the shear modulus of shaft material. Generally these formulas are developed for engine crank shafts. The calculated torsional stiffnesses for the crank shafts of reciprocating compressors depends on the equation used. The torsional stiffness value calculated with these equations may differ 100% or more from the actual value. In order to calculate the torsional stiffness, LMF has always used FEA. This can be done in two ways:

a) Calculation of the twist angle difference $\Delta \phi_{\text{Web}}$ between two sections in the journal centre and the crank pin centre of a crank shaft throw under an arbitrary applied torque $T_{\text{Web}}$. One end is symmetrically, rigidly fixed and the torque is applied across the other end of the crank shaft. SURF1546 elements are used for arbitrary applied moment. The ratio between torque and twist angle is then the corresponding stiffness of the crank web. The calculation can then be repeated for the intermediate crank web and for all possible web configuration of the crank shaft.

$$ C_{\text{Web}} = \frac{T_{\text{Web}}}{\Delta \phi_{\text{Web}}} \quad (1) $$

**Figure 1**: Web torsional stiffness

b) Calculation of the rotating inertia for each throw (crank web, crank pin,), and distribution of the shaft inertia parts to the corresponding throw. A FEA modal analysis has to be performed to determine the torsional eigenfrequencies and the corresponding mode shapes (eigenvectors). This data provides the necessarily required information to calculate the torsional stiffness. A solution of an algebraic system of equation must be performed, depending on the number of throw. For one cylinder compressor, the crank shaft is modelled as one spring-mass and the stiffness can be calculated explicitly according to equation (2).

$$ C_{\text{Crankshaft}} = 4 \cdot \pi^2 J_{\text{Crankshaft}} \cdot f_{\text{Crankshaft}}^2 \quad (2) $$

**Figure 2**: Crank shaft torsional stiffness
2.2 Running gear

The inertia of the running gear (oscillating masses: connecting rod, cross head, piston and piston rod) varies during each crank shaft revolution (curves in Fig. 3). The equivalent inertia of the running gear can be approximated by adding half of the reciprocating mass multiplied by the crank radius squared to the rotating inertia (Lines in Fig. 3). For simulation of the start-up and the continuous operation of the drive train the instantaneous moment of inertia must be considered. In Fig 3, a dominant amplitude of order two is noticeable for the inertia.

Figure 3: Equivalent mass moment of inertia $J(\phi)$ kgm$^2$ and average inertia (constant) versus crank angle

The equivalent inertia $J(\phi)$ of throw can be found by summing kinetic energy of all moving parts:

$$E_{kin} = \frac{1}{2} J(\phi) \cdot \dot{\phi}^2 = \frac{1}{2} J_R \cdot \dot{\phi}^2 + \frac{1}{2} J_C \ddot{\phi}^2 + \frac{1}{2} m_C (\dot{x}_C + \dot{y}_C)^2 + \frac{1}{2} m_p \dot{x}_p \tag{3}$$

The equivalent variable inertia $J=J(\phi)$ is

$$J(\phi) = J_R + \frac{1}{\phi^2} \left[ J_C \ddot{\phi}^2 + m_C (\dot{x}_C + \dot{y}_C) \right] + \frac{1}{\phi^2} m_p \dot{x}_p \tag{4}$$

The classic average inertia $J=\text{constant}$ is given by

$$J = \text{const.} = \frac{1}{2\pi} \int_0^{2\pi} J(\phi) d\phi \tag{5}$$
Reducing the connecting rod (Fig. 5) in tow masses an oscillating mass \(m_{CO}\) and a rotating mass \(m_{CR}\) the above equation becomes a simplified form in \(\phi\) as follows:

\[
J(\phi) = J_0 + m_{Osc} R^2 \left[ \sin \phi + \frac{\lambda \sin \phi \cos \phi}{\sqrt{1 - (\lambda \sin \phi)}} \right]^2
\]

Where:
\[
\lambda = \frac{R}{L}, \quad J_0 = J_c + m_{CR} \cdot R^2
\]

\[
m_{Osc} = m_p + m_{CO}
\]

**Figure 4:** Crank gear mechanism

The next step is to develop the equation of motion for the crank throw. The equations of motion for the calculation of torsional forced vibration during start-up and continuous operation are described by the derivation of the variable inertia \(J(\phi)\), external torque (gas force, friction force) as well as restoring torques and damping torques of the shafting system.

**Figure 5:** Running gear model

\[
J(\phi) \cdot \ddot{\phi}_i + \frac{1}{2} \frac{dJ(\phi)}{d\phi} \cdot \dot{\phi}_i^2 + M_i(\phi) - M_{i+1}(\phi)
+ M_p(\phi) + M_R(\phi) = 0
\]

Where:

\[
\frac{1}{2} \frac{dJ(\phi)}{d\phi} = m_{Osc} R^2 \left[ \sin \phi + \frac{\lambda \sin \phi \cos \phi}{\sqrt{1 - (\lambda \sin \phi)}} \right]^2
\]

\[
\left[ \cos \phi + \frac{\lambda \cos 2\phi + \lambda^2 \sin^2 \phi}{\sqrt{1 - (\lambda \sin \phi)^2}} \right]
\]
M_R and M_M are the torque, due to the gas force and friction force. For the classical approach a set of linear differential equations is generated considering the average value of throw inertia, equation (5). With variable inertia, equation (4), (6) a system of nonlinear differential equations is generated.

2.3 Motor

Torsional stiffness and rotating inertia: Generally, the Motor manufacturers provide a torsional stiffness and inertia of rotor shaft including all other rotated parts mounted on shaft (fan). The stiffness is then calculated from the end of the shaft (coupling side) to the middle of the rotor shaft. To get more accuracy a FEA is required. For single mass-spring system the 1st NTF and the total inertia of the 3D-model is calculated, Fig. 6. The equation (2) can be then used for the calculation of torsional stiffness.

**Figure 6** 3D-Model of rotor shaft of induction motor for 1.4 MW 375 rpm for a four axes horizontal balanced opposite reciprocating compressor B254.

Switching torque: The air gap torque of induction motor is the main exciter of drive train during start-up. The maximum torque amplitude can reach values up to six times the rated torque of machine. The fundamental frequency is the line frequency (50HZ or 60Hz). The torque is provided by the manufacturer based on basic equations developed in 7. The torque is given by the following equation:

\[
\frac{M}{M_{\text{Rated}}} = M_A + e^{\frac{i}{T_s}} + \frac{M_A}{\sin \beta} \sin(\omega_N t - \beta) e^{\frac{i}{T_s}}
\]

For the 1.4MW motor Fig 6, the coefficients in equation (9) are:

- Rated Torque : \( T_{\text{Rated}} = 35650 \) Nm
- Partial Torque : \( M_A = \frac{T_{\text{Partial}}}{T_{\text{Rated}}} = 0.828 \)
- Time cons. : \( T_0 = 1.22 \text{ s}, \ T_s = 0.0191 \text{ s} \)
- Angle : \( \beta = 0.163 \text{ rad} \)
- Angular Frequency : \( \omega_N = 2 \pi f_{\text{acc}} = 314 \text{ s} \)

Fig. 7 shows the torque, equation (9) over the acceleration time. The dominant amplitude of 1st order with a frequency of 50Hz (line frequency) is closed to six times the rated torque for this application.
Figure 7 Ratio between the motor start torque and the rated torque versus time in s.

Figure 8 Ratio of motor short-circuit torque to the rated torque versus time in s.
2.4 Viscous damper

Viscous dampers are often used in reciprocating engines. Sometimes they are mounted in reciprocating compressors on the crank shaft end to reduce the dynamic torsional torque for resonance operation. A viscous damper consists of a flywheel (inertia $J_{DF}$) that rotates inside a housing (inertia $J_{DH}$) which contains a viscous fluid with stiffness $C_D$ and damping coefficient $k_D$. The mathematical model, equation (10) of viscous dampers implied its mass moment of inertia, damping coefficient and stiffness. The damper manufacturer generally can provide all data required for the damper modelling.

$$J_{DF} \cdot \ddot{\phi}_{DF} - C_D \cdot (\phi_{DH} - \phi_{DF}) - k_D \cdot (\dot{\phi}_{DH} - \dot{\phi}_{DF}) = 0$$

Figure 9 Damper model

3. SIMULATION RESULTS, START-UP

The mechanical system can be reduced Fig. 10 to an equivalent spring-mass system. A drive train with N components leads to a set of $2 \times N$ nonlinear differential equations. If a viscous damper is included, the number of equations is $2 \times N + 2$. The method of solving the nonlinear differential equation in time domain is described in 3), 4).

From the numerical solution, the angles and angular velocities of all masses are obtained. The dynamic torsional shaft torque is calculated from the difference of twist angles between tow masses multiplied by the corresponding torsional stiffness of the shaft. The analysis can be performed for the normal continuous operation of compressor for the start-up torque as well as for ‘non-normal’ operation such as valve failure of compressor or engine misfire.

Below, the use of the simulation method and the results of calculations on a 1.7MW LMF-B254 compressor will be explained. The drive train is composed of following components:

A) A four axis, four throw horizontal compressor. The crank shaft Fig. 10 is reduced to a torsional equivalent vibration system of four spring-masses. The inertias and stiffnesses are: $C_1=18.7 \times 10^6 \text{Nm/rad}$, $C_2=31.3 \times 10^6 \text{Nm/rad}$, $C_3=20.4 \times 10^6 \text{Nm/rad}$, $C_4=31.3 \times 10^6 \text{Nm/rad}$, $J_1=6.52 \text{kgm}^2$, $J_2=6.53 \text{kgm}^2$, $J_3=6.53 \text{kgm}^2$, $J_4=6.51 \text{kgm}^2$

B) Flywheel with inertia $J=1174 \text{kgm}^2$

C) Highly flexible coupling: Stiffness $C=5 \times 10^5 \text{Nm/rad}$, relative damping $=0.8$, max. allowable torque for normal transient conditions $T_{\text{Max}}=150000 \text{Nm}$.

D) Induction motor: Inertia $709 \text{kgm}^2$, stiffness $C=51.62 \times 10^6 \text{Nm/rad}$. The starting torque short-circuit torque date is given above.

E) The running gear data is:
Figure 10. Crank shaft of LMF-B254 compressor, equivalent torsional model and the interface diagram of drive train.
From the numerical simulation the maximal torque on the flexible coupling 111637Nm is obtained. The allowable torque is 150000Nm. The dominant frequency of the coupling torque is NTF of the drive train and not the line frequency (50Hz). Figure 11 shows the crankshaft dynamic torque at the connection to the flywheel and at each throw.

**Figure 11.** Dynamic torque at crank shaft versus starting time in sec, flexible coupling.
Figure 12. Dynamic torque at highly flexible coupling and E-Motor versus starting time in sec.

The next figures show the simulation results for a stiff coupling with fictive torsional stiffness of 2.9x10^7 Nm/rad.
Figure 13. Dynamic torque at crank shaft versus starting time in sec, stiff coupling.

Figure 14. Dynamic torque at stiff coupling and E-Motor versus starting time in sec.
With the stiff coupling solution the torques are much higher and the dominant frequencies are the NTF and the line frequency.

4. SIMULATION RESULTS, SHUTDOWN

The shutdown simulation can be performed in the same way like the start-up, the same mathematical system can be used. The initial conditions for the angles $\phi_i$ and the angular velocities $\omega_i$ of the masses of the torsional system are the result of a previous simulation of the continuous operation of the reciprocating compressor. The instantaneous torque at the shafts and the speed of the compressor components over the slowing time are the output of the simulation. The following figure shows the compressor torque at a specified speed and the speed over the slowing time. The simulation can be performed under loaded condition – the compressor is working at the specified discharge pressure – or unloaded condition, with suction valve unloaders.

Figure 15. Compressor torque at 590rpm and the motor speed over the slowing time.
6. CONCLUSIONS

To ensure a safe design of drive train a start-up shutdown simulation of entire compressor unit is necessary. The torsional parameters of the crank shaft, induction motor as well as the mathematical model of the running gear the compressor torque under unloaded — suction valve unloader - and loaded condition are very important for the accuracy of the calculation. The torsional stiffness value of the crankshaft calculated by using classic methods may differ 100% and more from the actual value. The numerical solution of the nonlinear equations provides the shaft torques. The highly loaded component in drive train is always the coupling.

NOMENCLATURE

- $C_1, C_2, C_3, C_4$: stiffness, torsional model of four throw crank shafts
- $C_D$: stiffness coefficient viscous damper
- $C_{\text{Crank shaft}}$: torsional stiffness, crank shaft
- $C_{\text{Web}}$: torsional stiffness, crank web
- $E_{\text{kin}}$: kinetic energy of running gear
- $i_{\text{Crank shaft}}$: $i^{th}$ NTF of crank shaft
- $J$: average constant inertia of running gear
- $J(\phi)$: equivalent variable inertia of running gear damper
- $J_1, J_2, J_3, J_4$: rotating inertia, torsional model of four throw crank shafts
- $J_c$: inertia, connecting rod
- $J_{\text{Crank shaft}}$: crank shaft inertia
- $J_{\text{DF}}$: inertia, flywheel viscous damper
- $J_{\text{DH}}$: inertia, housing viscous damper
- $J_R$: rotating inertia, throw
- $k_D$: damping coefficient viscous damper
- $L$: length connecting rod
- $m_{co}$: osc. mass, connecting rod
- $m_p$: piston mass
- $m_{osc}$: sum of oscillating masses
- $M_c$: mass, connecting rod
- $M_i$: restoring torque, shaft $i$
- $M_{i+1}$: restoring torque, shaft $i+1$
- $M_p$: gas force torque
- $M_R$: friction force torque
- $M_{\text{Rated}}$: rated torque, induction motor
- $M_A$: partial torque, induction motor
- $R$: crank radius
- $T_0$: time constant, induction motor
- $T_i$: time constant, induction motor
- $T_{\text{Web}}$: twist torque
- $X_c$: CoG, connecting rod
- $Y_c$: CoG connecting rod
- $\phi_{DF}$: angle flywheel, viscous damper
- $\phi_{DH}$: angle damper housing, viscous damper
- $\Delta \phi_{\text{Web}}$: twist angle, between journal centre and crank pin centre
- $\beta$: angle, induction motor
- $\gamma$: angle, connecting rod
- $\lambda$: ratio $R$ to $L$
- $\phi_{\text{N}}$: line angular frequency
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