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Multivariable Identification and Control of a Calorimeter Used for Performance Evaluation of Refrigerant Compressors

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ABSTRACT

Refrigerant compressors are relatively complex products that require a wide variety of tests to be performed for both product development and quality control. Performance tests are an example of such tests and they are typically performed using test rigs with calorimeters. This paper presents the experimental results of the use of a multivariable dead-time compensating structure to control the conditions at the calorimeter inlet and outlet. Besides the control results themselves, the paper presents the identification of the multivariable model used for control purposes, as well as it discusses tuning and implementation of the proposed controller. The closed-loop experimental results are compared to the ones obtained by using PID controllers and they demonstrate the good performance of the implemented controller. As a result, both the test time and the result uncertainties have been reduced. Besides presenting an important contribution from the point of view of industrial application, this study presents experimental results of dead-time compensators, which are generally analyzed just towards simulation.

1. INTRODUCTION

Before refrigerant compressors can be made available to the market, they are required to be rigorously tested. Some tests are performed just for product development, some of them are performed just for quality control, and others performed for both product development and quality control. Performance test fits into the last category and is widely used by compressor manufacturers. Its objectives are to measure refrigerating capacity, active power consumption, isentropic efficiency, and the coefficient of performance of the compressor. This kind of test is regulated by international standards, such as ISO-917 (ISO, 1989), EN 13771-1 (CEN, 2003), and ANSI/ASHRAE 23 (ASHRAE, 2005), which define topological characteristics of the refrigeration system, measurement uncertainty limits, and control requirements.

Performance data of a compressor are based on the mass flow rate of refrigerant that it is capable of imposing to the circuit, given a test condition. There are different methods for measuring the mass flow rate, but they can be divided into two main groups: direct flow rate measurement methods and indirect measurement methods based on the heat balance inside a calorimeter. For the second kind of test method to be possible, it is necessary to control the inlet and outlet conditions of the calorimeter, what is not always an easy task.
The performance test is performed on special rigs, which operate as a refrigeration circuit with many controlled variables. In addition, it is possible to measure a series of variables that are not generally monitored in refrigeration systems. It is important to reach the test condition with low settling time to minimize the overall test time, which includes the transient and the test time itself in steady state. Although these tests require a heavy infrastructure and generate further costs, many rigs found in industry still use manual control for many variables or the traditional proportional-integral-derivative (PID) controllers. There are almost no references about control strategies applied to performance tests, but papers exploring the use of controllers other than the classical thermostat for refrigeration circuits are beginning to appear and to present good results (Sonntag et al., 2007; Tian et al., 2008; Ekren et al., 2010; Maia et al., 2010).

In the test rig, certain variables, such as the outlet temperature of the calorimeter, exhibit time delays (also known as dead times) and traditional controllers typically do not present good results for dead-time dominant plants. The effects of time delays are easily noticeable in the temperature at the calorimeter outlet. Due to the topology of the circuit inside the calorimeter and the distance between the evaporator outlet and the point where the temperature shall be measured to satisfy standard requirements, a considerable time delay is observed in the temperature response. Flesch and Normey-Rico (2010) have proposed a method to control this variable in closed-loop, considering a single-input and single-output (SISO) approach. A SISO dead-time compensator (DTC) presents better results than a PID controller, but it does not consider the coupling that exists between the process variables of the rig. In this case, a multivariable, also known as multiple-input and multiple-output (MIMO), control approach would be more suitable.

In a general MIMO system, time delays may appear in the input actions, in the measurement paths, and also in the interconnection between internal variables. Therefore, each signal path between the inputs and outputs may show a different delay, increasing the complexity of the control design procedure (Jerome and Ray, 1986). Most of the proposed solutions for dead-time process control are based on a DTC structure, derived from the Smith predictor (SP) proposed for SISO processes by Smith (1957). Several papers have analyzed the extension of the SP to MIMO stable processes, but only two recent papers have studied the control of MIMO square plants with multiple delays and integrating or unstable modes (García and Alberto, 2010; Flesch et al., 2011).

This paper presents a MIMO identification of the dynamic behavior of a $2 \times 2$ system composed by the inlet and outlet temperatures of a calorimeter used for performance evaluation of refrigerant compressors as process variables and by the respective heater electrical powers as manipulated variables. It is shown that this model presents an integrating behavior, what limits the use of classical MIMO DTC structures. The identified system is then used to tune a MIMO-FSP proposed by Flesch et al. (2011) in order to control in closed loop the calorimeter temperatures. The results show how this control structure can improve the time response of the variables when compared to a classical MIMO proportional-integral-derivative (PID) controller.

The rest of the paper is organized as follows: section 2 presents a brief review of the control strategy that is used for experimentation, the MIMO filtered Smith predictor. In section 3 a multivariable model for the inlet and internal temperatures of a calorimeter is obtained. Section 4 presents the controller tuning for the specific case analyzed in this paper, as well as the closed-loop responses obtained experimentally. The paper ends with the conclusions, presented in section 5.

## 2. DESCRIPTION OF THE CONTROL STRUCTURE

This section reviews some aspects of the MIMO filtered Smith predictor (MIMO-FSP), proposed by Flesch et al. (2011). MIMO-FSP is a general MIMO DTC structure which can be used to control stable, integrating, and unstable square MIMO dead-time processes. This section begins with a description of a MIMO plant with multiple time delays and then presents the main aspects of the controller structure, including a simplified performance and robustness analysis.

### 2.1 Description of the plant

Without loss of generality, a square MIMO process can be represented by an $n \times n$ transfer matrix with elements $G_{ij}(z)z^{-d_{ij}}$, each one being the transfer function relating the $j$th input with the $i$th output, where $G_{ij}(z)$ is a delay-
free transfer function and \(d_{ij}\) is the discrete dead time. The effective dead time of each output \(i\) is \(d_i\), computed as the minimal delay of the \(i\)th row as \(d_i = \min_{j=1..m} (d_{ij})\). Thus, defining \(L(z) = \text{diag}\{z^{-d_1}, ..., z^{-d_n}\}\) as the MIMO delay of the plant \(P(z)\) and \(G(z)\) as the model without the common dead times (also called fast model), follows

\[
P(z) = L(z)G(z). \tag{1}
\]

Notice that \(G(z)\) may still contain dead times.

2.2 MIMO-FSP control structure

The MIMO-FSP control structure is shown in Figure 1 (nomenclature is presented as a separated section at the end of the manuscript). From a block diagram reorganization it is possible to find a stable predictor \(S(z) = [G_n(z) - F_r(z)P_n(z)]\), which is necessary to ensure that the control structure is internally stable when the open-loop dynamics of the plant present integrating and/or unstable modes (Flesch et al., 2011). From this structure, in the nominal case, the closed-loop transfer matrices are:

- \(H_y(z) = P_n(z)C(z)[I + G_n(z)C(z)]^{-1}F(z)\) (reference);
- \(H_y(z) = (I - P_n(z)C(z)[I + G_n(z)C(z)]^{-1}F_r(z)) P_n(z)\) (load disturbance);
- \(H_y(z) = I - P_n(z)C(z)[I + G_n(z)C(z)]^{-1}F_r(z)\) (measurement noise).

Moreover, the feedback signal \(y_p(k)\) produced by the predictor is

\[Y_p(z) = F_r(z)Y(z) + S(z)U(z),\]

which, in the nominal case without disturbances, anticipates the system output:

\[Y_p(z) = G(z)P(z)^{-1}Y(z) = L(z)^{-1}Y(z).\]

Thus, \(y_{pi}(t) = y_i(t + d_i)\) for all \(i\).

![Figure 1: MIMO-FSP scheme](image)

The predictor filter \(F_r(z)\) is an \(n \times n\) diagonal filter with diagonal elements

\[
F_r(z) = \frac{\alpha_1 z^{v_1} + ... + \alpha_n z^{v_n}}{(z-a_1)\xi_1 \cdots (z-a_n)\xi_n}, \tag{7}
\]

where \(v_i \geq 1\) is a positive integer, \(a_i\) varies in the range \((0, 1)\), and \(F_r(z)\) is any proper stable filter. As shown in Flesch et al. (2011), for square plants it is possible to design \(F_r(z)\) computing the values of \(v_i\) and \(a_i\) such that any undesired pole of \(G_i(z)\) \(\forall i, j\) is eliminated from the elements of \(S(z)\) (undesired poles are poles with \(|z| \geq 1\) or poles associated with slow dynamics). This allows for a “stable prediction” even in the case of unstable processes. Moreover, if the following constraints are satisfied:
the primary controller, $C(z)$, is designed to stabilize $G_u(z)$ without pole-zero cancelations at the undesired poles of $P_u(z)$;

- $C(z)$ has integral action to allow zero steady-state error for step references;

- $F_r(z)$ does not have zeros at any $z_0$ pole of $P_u(z)$ with $|z_0| \geq 1$,

then:

- the delayed closed-loop system implemented as an equivalent controller is internally stable;

- the undesired stable poles of $P_u(z)$ will not affect the closed-loop disturbance rejection response.

In order to guarantee that $S(z)$ does not have any undesired pole, each SISO filter, $F_r(z)$, should be defined so that each element $S_{ij}(z)$ has undesired pole-zero eliminations for all $j = 1, ..., n$. Therefore, the transfer function $[1 - F_r(z)x^{-d_i}]$ must have zeros at all the undesired poles of $G_{ij}(z)$, $j = 1, ..., n$. This is satisfied if $F_r(z)$ meets the condition

$$\frac{d\lambda}{dz\lambda} (1 - F_r(z)x^{-d_i}) \bigg|_{z = z_i} = 0, \quad \lambda = 0,1, ..., m_l - 1, \quad l = 1, ..., k,$$

for every of the $k$ different undesired poles $z_i$ of a row, each one having multiplicity $m_l$. This condition turns the MIMO case into a composition of scalar problems (Flesch et al., 2011).

It is also important to notice that: (i) the design of $C(z)$ can be performed by using any classical MIMO control design approach; (ii) the design of $F_r(z)$ avoids that the disturbance rejection response is governed by the open-loop dynamics of the plant; (iii) the MIMO-FSP structure is able to deal with instabilities at any elements of the transfer matrix (not just in the main diagonal) given that the primary controller is able to stabilize the fast model of the plant.

### 2.3 Closed-loop performance and robustness

In the MIMO-FSP the reference to output closed-loop dynamics can be defined by an appropriate choice of the primary controller $C(z)$ and, if necessary, by using a reference filter $F(z)$. In a second step, the free parameters of $F_r(z)$ are tuned for a compromise between robustness and disturbance rejection response and to obtain $F_r(1) = I$ for step disturbance rejection. As shown in Flesch et al. (2011), the robustness characteristics of the controller are defined by the shape of the scalar functions $F_r(e^{T_5})a_l$, which are defined using the free parameters $a_l$. When $a_l \rightarrow 1$ robustness increases, but at the same time, as these are poles of the filter that affect $H_{yq}(z)$, the disturbance rejection response deteriorates. This trade-off between robustness and performance must be solved for each case when tuning the control structure. Note that, as in the SISO case, when controlling unstable processes, robustness cannot be increased arbitrarily by tuning $F_r(z)$ because a minimal feedback action is necessary to maintain nominal closed-loop stability (Normey-Rico and Camacho, 2007).

As in the SISO case, the filter $F_r(z)$ can be chosen with more poles than zeros to attenuate measurement noise (Santos et al., 2010) and to increase the robustness to modeling errors. A filter $F_r(z)$ with low-pass behavior avoids that high-frequency measurement noise appears directly in the control signal, as can be seen in

$$H_{un}(z) = -C(z)[I + G_u(z)C(z)]^{-1}F_r(z).$$

In addition, a predictor filter $F_p(z)$ in which all elements have a low-pass characteristic helps to increase the robustness at high frequencies, given that the maximum singular value of the filter at high frequencies decreases (Flesch et al., 2011).

### 3. CALORIMETER MODEL IDENTIFICATION

The performance test rig presents many manipulated and process variables. Almost all of the manipulated variables affect more than one process variable, thus characterizing a MIMO process. In this paper a simplified model is obtained, considering just two manipulated variables and two process variables related to the calorimeter. The process variables to be considered are the inlet and the internal temperatures of the calorimeter, and the manipulated variables are the power driven to a silicone-coated resistor wrapped spirally around the calorimeter inlet piping and to the heater inside the calorimeter. A schematic representation of the points of interest is presented in Figure 2.

These two temperatures are very important to the testing process, since they are directly related to the heat balance inside the calorimeter, which is used for measuring the refrigerating capacity of the compressor under test. Thus, for measuring with high accuracy, it is important to maintain both variables as close as possible to their set points.
Step-response tests have been performed near an operating point given by 32°C for a specific model of compressor in order to identify the dynamic behavior of the system. The nominal model obtained from these tests is

\[
\begin{bmatrix}
Y_1 \\
Y_2 
\end{bmatrix} = \begin{bmatrix}
\frac{7.40 \times 10^{-5} e^{-45s}}{s} & \frac{7.41 \times 10^{-5} e^{-90s}}{s} & 0 \\
0 & \frac{0.77 e^{-18s}}{120s + 1} & u_1 \\
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 
\end{bmatrix}
\]

where \(Y_1\) is the internal temperature of the calorimeter, \(Y_2\) is the inlet temperature of the calorimeter, \(u_1\) is the power driven to the heater inside the calorimeter, and \(u_2\) is the duty cycle of a pulse-width modulation circuit which regulates the power driven to the silicone-coated resistor at the calorimeter inlet. It is interesting to notice that the system has a partial coupling, since variations in \(u_1\) do not affect \(Y_2\). Thus, this problem can be solved by using a feed-forward controller together with two SISO FSP or a MIMO-FSP.

A graphical comparison between the proposed model and the measured data is presented in Figure 3. It is possible to observe that the proposed model captures the system behavior very well.

**Figure 2:** Schematic representation of the variables of interest

**Figure 3:** Comparison between the identified model and the experimentally measured data in open loop
For a sampling period $T_s = 3$ s, the discrete-time representation of the process is given by

$$ P(z) = \begin{bmatrix} 0.000267z^{-15} & 0.0002223z^{-30} \\ z - 1 & z - 1 \\ 0 & 0.01901z^{-6} \\ z - 0.9753 & z - 0.9753 \end{bmatrix}$$

and its fast model is given by

$$ P(z) = \begin{bmatrix} 0.000267 & 0.0002223z^{-15} \\ z - 1 & z - 1 \\ 0 & 0.01901 \\ z - 0.9753 \end{bmatrix}.$$

### 4. CONTROLLER TUNING AND CLOSED-LOOP RESULTS

A diagonal MIMO proportional-integral (PI) controller with expression

$$ C_{ij}(z) = K_{c_j} + \frac{K_{c_j}T_{ij}z}{T_{ij}^{*}z - 1}, \quad j = \{1,2\}; \quad C_{ij} = 0, \quad i \neq j \quad (10) $$

and tuning parameters $K_{c_1} = 131.4$, $T_{i_1} = 338$, $K_{c_2} = 1.58$, and $T_{i_2} = 117$, with times given in seconds, is used as primary controller. As all the time delays are compensated by the prediction structure, tuning can be performed considering just the delay-free dynamics from the fast model. The presented tuning parameters lead to a critically damped response with settling times of approximately 15 min for the internal temperature and 12 min for the inlet temperature.

As the proposed controller impose dominant zeroes in closed loop, a reference filter

$$ F(z) = \begin{bmatrix} 0.2933z - 0.2845 \\ z - 0.9912 \\ 0 \\ 0.5z - 0.475 \\ z - 0.975 \end{bmatrix}$$

is necessary to avoid a peak in the response. As the non-stable terms only appear in the first row of the transfer matrix, just the first element of the predictor filter needs to be tuned. The pole of the filter was placed at $z = 0.99$ and the zero was determined by using Equation (8). The resulting predictor filter is given by

$$ F_p(z) = \begin{bmatrix} 1.15z - 1.14 \\ z - 0.99 \\ 0 \\ 1 \end{bmatrix}.$$

As the system presents integrating behavior, the implementation needs to be done by using the stable predictor $S(z)$ in order to guarantee the internal stability of the control structure (see Figure 1b). In this case, the stable predictor is given by

$$ S(z) = \begin{bmatrix} 2.67 \times 10^{-3}(z^{15} + 0.02z^{14} + \cdots + 0.02z - 1.28) \\ z^{16} - 0.99z^{15} \\ 0 \\ 2.223 \times 10^{-3}(z^{15} + 0.02z^{14} + \cdots + 0.02z - 1.28) \\ z^{31} - 0.99z^{30} \\ 0.01901(z^6 - 1) \\ z^{7} - 0.9753z^{6} \end{bmatrix}. $$

The closed-loop response of the system is presented in Figure 4, together with the response obtained through simulation. Initially the set point for the inlet temperature was changed from $30^\circ C$ to $32^\circ C$ at $t = 7$ min, and at $t = 22.5$ min the internal temperature set point was changed from $30^\circ C$ to $32^\circ C$. The inlet temperature is changed first because this change excites the system coupling, what does not happen when the outlet temperature is changed. At $t = 67$ min a load disturbance is applied to the system by increasing the compressor inlet pressure by $0.1$ bar (10 kPa). It is easy to notice that the prediction structure is able to satisfy the project requirements, as well as reject load disturbances.
As the desired closed-loop response was tuned to present slow dynamics, in the previously presented case a properly tuned MIMO PID controller is able to achieve very similar closed-loop response if the same reference filters are used. However, if the desired settling times are reduced, the PID controller leads to a very oscillating closed-loop response, what is directly reflected as a higher settling time when compared to the case in which the proposed structure is used. This difference is illustrated in Figure 5, where desired settling times of 4 min and 5 min are imposed to the inlet and internal temperatures respectively. The PID controller was tuned by considering a first-order approximation of the time delay and the MIMO-FSP prediction structure remained the same. The new primary controller for the MIMO-FSP is a PI with tuning parameters $K_{c1} = 370$, $T_{i1} = 148$, $K_{c2} = 3$, and $T_{i2} = 120$, with times expressed in seconds. Initially the set point for the inlet temperature was changed from 32°C to 34°C at $t = 7$ min, and at $t = 22.5$ min the internal temperature set point was changed from 30°C to 32°C. At $t = 44$ min a load disturbance is applied to the system by increasing the compressor inlet pressure by 0.1 bar (10 kPa). In this case, the difference between the responses obtained by using the two control structures is clear, since for the internal temperature the settling time is 5 min when the MIMO-FSP is used and 14 min when the MIMO PID is used.

The price that is paid for a better performance is a more complex control structure. However, a more complex control structure does not mean that it is more complicated to be tuned. The main controller tuning is done considering the fast system (the system without the common delays in each row) and the predictor tuning is done considering the model of the process obtained from a step-response test and Equation (8). On the other hand, when a PID controller is used, its tuning needs to be performed considering the system with the time delays, what is not always trivial. The MIMO-FSP is able to compensate the time delay effects and this is directly reflected as a faster response. As in this example the coupling between the variables is small, the difference between the results obtained by using classical control techniques and the MIMO-FSP is small if the desired dynamics for closed-loop are slow, but it increases significantly as the desired closed-loop dynamics are speeded up.
From the test point of view, the use of a control structure able to compensate the time delays is important to abbreviate the test. If just the settling times for the temperatures are analyzed, the difference between the two control techniques is of about 9 min, but it is important to notice that the power delivered to the calorimeter heater is also a variable that needs to stay into an interval for the test to be considered in steady state. Thus, assuming that all the other test quantities are in steady state, the MIMO-FSP abbreviates the test in 30 min when compared to PID, since in the first case the input power to the heater has a settling time of 10 min and in the case of a PID this time is increased to 40 min.

5. CONCLUSIONS

This paper proposes a first approach for a MIMO DTC control strategy applied to a compressor performance test rig. For the sake of simplicity, a $2 \times 2$ model has been firstly used, considering the inlet and internal temperatures of the calorimeter that encircles the evaporator of the rig. These variables have been chosen because: (i) they are coupled, (ii) they present a significant dead time, and (iii) a proper control of the variables is very important for the test result.

When compared to the results obtained with classical PID controllers, the ones obtained with the proposed strategy are less oscillatory and present lower settling times. The union of both factors is responsible for reducing the test duration, what is directly reflected as productivity increase. Additionally, these factors are indirectly related to the test accuracy, since the better the controlled variables track their references, the better the test accuracy. Thus, besides decreasing the test duration, these gains reflect more trustable information for R&D, catalog data, and product approval.

The tuning of the proposed predictor filter is completely from the tuning of the other elements of the control loop. Thus, the overall control project can be divided into two interchangealbe steps: (i) tuning of the primary controller and (ii) tuning of the predictor filter. The tuning of the primary controller is done as a traditional project for the nominal fast model (without the common dead times) and the tuning of the predictor filter is done to avoid the undesired integrating pole. The primary controllers are two PI and the robustness filter is of first order. Even though the overall control structure may seem complicated, both tuning procedures are simple.
To the best of the authors’ knowledge, this is the first industrial example of use of a MIMO-FSP. Additionally, this is the first experimental example of use of the MIMO-FSP to control an integrating process. Thus, in addition to the contribution to the compressor testing community, this paper is important to show that the proposed control strategy is suitable for industrial processes that are not stable in open loop.

**NOMENCLATURE**

- $t$: time instant
- $K_C$: proportional gain of the controller
- $T_i$: integral time of the controller
- $e_p(k)$: prediction error
- $q(k)$: load disturbance
- $r(k)$: reference
- $n(k)$: output disturbance or measurement noise
- $y(k)$: output
- $\hat{y}(k)$: model output
- $y_p(k)$: output prediction
- $C(z)$: MIMO primary controller
- $F(z)$: MIMO reference filter
- $F_p(z)$: MIMO predictor filter
- $G_n(z)$: nominal fast model
- $L_n(z)$: nominal MIMO delay model
- $S(z)$: MIMO stable predictor

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