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ABSTRACT

To simplify the analysis of the three-dimension vibrations of reciprocating compressor crankshaft system under working conditions, a spatial finite element model based on 3-node Timoshenko beam was proposed in this paper. The crankshaft was idealized by a set of jointed structures consisting of simple round rods and simple beam blocks, the main journal bearings were idealized by a set of linear springs and dash-pots, and the flywheel and motor were idealized by a set of masses and moments of inertia. And this model also can be used in dynamic analysis for crankshaft system in subsequent research. In practice, a 6M51 reciprocating compressor crankshaft system was modeled by this method and modal analysis by block Lanczos method which provided by ANSYS package.

1. INTRODUCTION

The design of modern reciprocating compressor is required to make noise and vibration as light as possible. In order to successfully control the noise and vibration, the vibration of reciprocating compressor crankshaft which can cause vibration and noise of the compressor, and some even can destroy crankshaft bearing and crankshaft itself must be estimated and analyzed. So early in design stage, computations of natural frequencies, mode shapes, and critical speeds of crankshaft system are indispensable. Thus, an accurate model for prediction of the vibration of a crankshaft system is essential for reciprocating compressor.

Reciprocating compressor’s crankshaft system vibration is a complex three-dimensional coupled vibration under running conditions, including the torsional, longitudinal and lateral vibrations. In earlier research, the equivalent model and the continuous beam model were applied to analyze the torsional or coupled torsional-axial vibration of crankshaft (Tsuda, 1969, Porter, 1968). With the progress of computer simulation technique and emergence of finite element method, spatial finite element models were introduced to estimate the vibration behavior of crankshaft. The generally used finite element models are in two categories: beam elements and solid elements. It is evident that much better results can be obtained by using the models based on solid elements (Kang and Sheen 1998). However, it is computationally expensive to simulate the motion of the operating crankshaft.

Initial, the finite element beam model was proposed by Ruhl and Booker for rotor-bearing systems (1972). Then Nelson and McVaugh (1976), Zorzi and Nelson (1977) utilized finite beam element models to formulate the dynamic equation for a linear rotor system and determine the stability and steady state responses. In order to study the vibrations of the crankshaft, Okmaura.H (1990 and 1995) proposed a finite element model based on the Euler-Bernoulli beam for crankshaft, and derived the DSM(Dynamics Stiffness Matrix) in closed form from
TMM(Transfer Matrix Method). And in practice, they have calculated a single crank, three-cylinder in-line, four-cylinder in-line and V-6 crankshafts’ bending and axial three-dimensional vibration.

As the Euler-Bemouli beam neglected the shear deformation and rotational inertia effects, a number of Timoshenko beam finite element which considering the shear deformation and rotational inertia effects have been proposed in the literature (Kapur, 1966, Severn, 1970, Davis and Henshell, 1972). And Thomas et al. (1973) proposed a new Timoshenko element beam which has three degrees of freedom at each of two nodes, and this element has good rate of convergence, and for the analysis of simple structures in which shear and rotary inertia are particularly important, this element will give a better rate of convergence. Vebil Yildirim and Erhan Kiral (2000) compared the calculated results of the Euler-Bemouli beam with Timoshenko beam, the results showed that to determine a more satisfactory result in the analysis of free vibration and forced vibration of the thinner component beam, the shear deformation and rotational inertia must be considered. Smaili (1994) proposed a new finite element model involving a new scheme for modeling the stiffness and damping properties of the journal bearings, and the natural frequencies of the crankshaft of a four in-line cylinder engine are determined.

In this paper, a finite element model was proposed by using a 3-node spatial element based on Timoshenko beam theory which provided by ANSYS package for modal analysis of crankshaft. The crankshaft was idealized by a set of simple round and block beams. The flywheel and motor were idealized by a set of masses and moments of inertia. The main journal bearings were idealized by a set of linear spring and dash-pots. And in practice the natural frequencies and mode shapes were calculated by the finite element method of a 6M51 reciprocating compressor crankshaft system. And this model also can be used in dynamic analysis for crankshaft system in subsequent research.

2. MODELING

2.1 Crankshaft
Crankshaft is considered to be a set of rigidly jointed structures consisting of crankjournal crankpins and crankarms. The crankjournal and crankpins of each throw were idealized by rods of diameters \( D_j \) and \( D_p \) and lengths \( L_j \) and \( L_p \), as shown in Figure 1. Here \( D_j \) and \( D_p \) were as equal to their original diameters, while \( L_j \) and \( L_p \) were equal to the length of the original journal together with the thickness of the crankarm \( H \) in accordance with Wlisno and Okmauar (1990). The crankarms was idealized by blocks, which dimensions were equal to the original dimensions so as to keep their centers of gravity, masses, moments and stiffness of the same.

![Figure 1: Sketch of the computing modal of crankshaft](image-url)
2.2 Flywheel and Motor
The flywheel and motor were idealized by a set of masses and moments of inertia about three orthogonal axes attached at their original centers of gravity.

2.3 Crankshaft Main Bearings
To take account of oil film pressure of each main bearing (journal bearings), we idealized a set of linear springs and dashpots in the vertical and horizontal directions attached to the crankjournal axis.

2.4 Reciprocating Masses
The reciprocating masses consist of the masses of the piston, connecting rod, piston rod and crosshead. To take account of the mean kinetic energy that would be in due by the reciprocating masses under running conditions, one quarter of the total reciprocating masses were attached at the two crankpin ends.

3. ANALYSIS

ANSYS provide element Beam189 is based on quadratic 3-node Timoshenko beam theory which includes shear-deformation effects. It is an element including shear deformation and rotary inertia effects and suitable for analyzing slender to moderately stubby and thick beam structures. The element is a quadratic three-node beam element in 3-D (as in Figure 2).

\[ \begin{align*}
\{q\} &= [u, v, w, \theta_x, \theta_y, \theta_z, \text{warp}]^T \\
N_i &= \frac{1}{2}\eta (\eta - 1), N_j = \frac{1}{2}\eta (\eta + 1), N_k = 1 - \eta^2
\end{align*} \]

Figure 2: 3-node Timoshenko beam element

With default settings, six degrees of freedom (DOF) occur at each node; these include translations in the x, y, and z directions and rotations about the x, y, and z directions. An optional seventh DOF (warping magnitude) is available, the DOF are defined by

\[ \{q\} = [u, v, w, \theta_x, \theta_y, \theta_z, \text{warp}]^T \]

The shape function is

\[ N_i = \frac{1}{2}\eta (\eta - 1), N_j = \frac{1}{2}\eta (\eta + 1), N_k = 1 - \eta^2 \]

Where \( \eta \) the nature coordinate (-1<\( \eta \)<1). Then the displacement of point p is related to the element nodal displacement \( q_i, q_j \) and \( q_k \) by
\[ q_p = N_i \cdot q_i + N_j \cdot q_j + N_k \cdot q_k \]  \hspace{1cm} (3)

\( q_i, q_j \) and \( q_k \) is the vector of displacement at nodes i, j, and k respectively.

The motion equations of crankshaft are first discretized by the standard finite element technique as

\[ [M] \ddot{q} + [C] \dot{q} + [K] q = [f] \]  \hspace{1cm} (4)

where \( \{q\} \) is the vector of displacements, \( \{f\} \) is the vector of externally applied forces, and \([M],[C],[K]\) are mass, damping and stiffness matrices respectively.

In order to determine natural frequencies or the eigenvalues of the system, omission of the damping and external force terms from the equation of the system leads to

\[ [M] \ddot{q} + [K] q = \{0\} \]  \hspace{1cm} (5)

Assumed the solution can be separated to the amplitude and a time function, we substituted

\[ \{q\} = \{Q\} \sin(\omega_n t + \phi) \]  \hspace{1cm} (6)

into Equation (5), we can derive the following equation:

\[ ([K] - \omega_n^2[M]) \{Q\} = \{0\} \]  \hspace{1cm} (7)

Since Equation (6) is not the standard form for the eigenvalue and eigenvector problems, we can premultiply \( [M]' \) both side of the Equation (6), we can derive the following equation:

\[ ([S] - \lambda[I]) \{Q\} = \{0\} \]  \hspace{1cm} (8)

where \([S]=\[M]'[K]\), called system matrix. In order that \( \{Q\} \) has a value not equal to zero, the following condition must be satisfied:

\[ \det([S] - \lambda[I]) = 0 \]  \hspace{1cm} (9)

Solving this equation for \( \lambda \), we obtain the eigenvalue \( \lambda_n = i\omega_n \) of each mode of vibration of the system. So the vibration problem transform into the eigenvalue problem. In this paper, we utilized block Lanczos method which offered by ANSYS package to determine the eigenvalue of the Equation (9). Block Lanczos method performed with a block of vectors which is used by the block Lanczos method. It is powerful when searching for eigenvector.

The mode shapes can be calculated by substituting the \( \lambda_n \) back into Equation (8) and solving for \( \{Q\} \) in the normalized form.

4. APPLICATION

A 6M51 reciprocating compressor crankshaft is modeled by the proposed 3-node Timoshenko beam, in which the crankshaft is meshed into 136 beams element, the journals is meshed into 16 linear springs and dashpots element, the reciprocating masses, motor and flywheel is meshed into 14 masses element (as in Figure 3, dimensions of member elements, mass and moments of inertia shown in Table 1). These models were analyzed by using the block Lanczos method provided by ANSYS package. The calculated results of the first eight orders natural frequencies and the mode of vibration are listed in Table 2, the expression “L” refers to the lateral vibration and...
“T” refers to torsional vibration. The first eight orders calculated mode shapes of the crankshaft were shown in Figure 4.

Figure 3: Idealized modeling for 6M51 crankshaft system

Table 1: Dimensions of member elements (a) and mass and moments of inertia (b) for 6M51 crankshaft system

<table>
<thead>
<tr>
<th>Element</th>
<th>Parts Number</th>
<th>Type</th>
<th>Real Constant</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal</td>
<td>1, 7, 9, 15, 17, 23</td>
<td></td>
<td></td>
<td>393×Φ320</td>
</tr>
<tr>
<td>Crank pin</td>
<td>3, 5, 11, 13, 19 22</td>
<td>Beam 189</td>
<td>DENS=7850 kg / m³</td>
<td>565×Φ320</td>
</tr>
<tr>
<td>Crank arm</td>
<td>2, 4, 6, 10, 12, 14, 18, 20, 22</td>
<td></td>
<td>EX=2.1e11</td>
<td>750×410×165</td>
</tr>
<tr>
<td>Connection shaft</td>
<td>8, 16</td>
<td></td>
<td>PRXY=0.3</td>
<td>840×Φ398</td>
</tr>
<tr>
<td>Motor shaft</td>
<td>24</td>
<td></td>
<td></td>
<td>1515×Φ320</td>
</tr>
<tr>
<td>Idealized journal</td>
<td>26, 27, 28, 29, 30,31, 32, 33, 34, 35, 36, 37, 38,39, 40, 41</td>
<td>Combin 14</td>
<td>K=1e9 N / m</td>
<td>Null</td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>Parts</th>
<th>Mass (kg)</th>
<th>Moment of inertia (kg·m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ix</td>
<td>Iy</td>
</tr>
<tr>
<td>Reciprocating masses</td>
<td>592</td>
<td>Null</td>
</tr>
<tr>
<td>Flywheel</td>
<td>1460</td>
<td>355</td>
</tr>
<tr>
<td>Motor</td>
<td>19500</td>
<td>4375</td>
</tr>
</tbody>
</table>
Table 2: First eight calculated natural frequencies of 6M51 crankshaft

<table>
<thead>
<tr>
<th>Mode</th>
<th>Calculated Natural Frequency (Hz)</th>
<th>Mode of Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>24.014</td>
<td>L</td>
</tr>
<tr>
<td>2nd</td>
<td>28.182</td>
<td>T</td>
</tr>
<tr>
<td>3rd</td>
<td>47.653</td>
<td>L</td>
</tr>
<tr>
<td>4th</td>
<td>50.940</td>
<td>L</td>
</tr>
<tr>
<td>5th</td>
<td>51.538</td>
<td>L</td>
</tr>
<tr>
<td>6th</td>
<td>54.572</td>
<td>T &amp; L</td>
</tr>
<tr>
<td>7th</td>
<td>58.980</td>
<td>L</td>
</tr>
<tr>
<td>8th</td>
<td>60.319</td>
<td>L</td>
</tr>
</tbody>
</table>

5. Conclusion

A simple model and analysis procedure was proposed in this paper for three-dimensional vibrations of reciprocating compressor crankshaft system, which is very simple and the implementation of simulation is computationally effective. It is competent for the vibration analysis of the crankshaft system of reciprocating compressor. It provides a useful tool for the vibration analysis and the design of crankshaft system. And in
practice, the nature frequencies and mode shapes of a 6M51 reciprocating compressor crankshaft system was calculated by this model and analysis procedure which provide by ANSYS package. The proposed model also can be implemented in dynamic analysis for crankshaft system in subsequent research.

Figure 4 (a-h): First eight orders calculated modes of 6M51 crankshaft

References


