Calculation on the Inter-Lobe Clearance Distribution of Twin-Screw Compressor by Optimization Method

Wei Xiong
Xi'an University of Architecture & Technology

Follow this and additional works at: https://docs.lib.purdue.edu/icec
CALCULATION OF THE INTER-LOBE CLEARANCE DISTRIBUTION OF TWIN-SCREW COMPRESSOR BY OPTIMIZATION METHOD

Wei XIONG

School of Environmental & Municipal Engineering, Xi’an University of Architecture &Technology, Xi’an 710055, shaanxi, China
Phone: 86-29-82202729
Fax: 86-29-82202729
E-mail: zhangbin@mail.xjtu.edu.cn

ABSTRACT

Small rotor clearance is today a vital prerequisite for an efficient screw compressor. The clearance will increase compressor leakage, and thereby minimize both the volumetric and adiabatic efficiency. But, the inter-lobe clearance is necessary for compressor running. Therefore, a mathematical apparatus to quantify the inter-lobe clearance is precondition of taking a reasonable inter-lobe clearance. An optimization method is presented in this paper in which the inter-lobe clearance is calculated as the minimum distance between the two rotors along the contact line. And the actual position of the rotors in running state was considered, in which the rotors contacted with each other in the drive surface. Therefore the calculation of inter-lobe clearance is transformed to the calculation of the minimum distance distribution between the two rotors along the contact line in contact state. The results show that this algorithm obtains full inter-lobe clearance between the two rotor surfaces with higher accuracy.

1. INTRODUCTION

Screw compressors have the advantage of simple structure, high reliability and easy to operate. So, they are used for the compression of a vast range of gases and vapors including refrigerants. They are widely used in the industrial process industries; they are also used as the compressors in factory based compressed air supplies. Inter-lobe clearance is today a vital factor for an efficient screw compressor. Manufacturers produce clearance between rotors to accommodate tolerance build-up, gas force deflections and the thermal distortions that occur during operation. The clearance will increase compressor leakage, and thereby minimize both the volumetric and adiabatic efficiency. But, the inter-lobe clearance is necessary for compressor running. The reliability of screw compressor depends on the inter-lobe clearance distribution and its variety while the compressor is running. Therefore, a mathematical apparatus to quantify the inter-lobe clearance is the precondition of taking a reasonable inter-lobe clearance. In practice, rotor abrades or rotor seizure often occurs when the compressors are running, this accident damages the compressor greatly and cause great loss to the users. Rotors abrade or rotors seizure occurs because of interference of the two screw rotors while they are running. So, an accurate calculates and analysis of the inter-lobe clearance of the screw rotors in running state will provide the foundation for erasing the accident.

Tang et al. (1994) present two ways of setting clearances of screw compressors and discuss a design technique for obtaining an optimum clearance distribution. Stosic and Smith (2001) present a mathematical apparatus which simplifies the analysis and allows the problem to be presented as a two dimensional problem in the end plane of the rotor, and this apparatus was applied to quantify the static and dynamic effects of the rotor interference in screw compressors. Fong et al. (2001) propose a mathematical procedure to calculate the inter-lobe clearance between two mating screw rotors, this method regards the projection of axial difference of two rotor surfaces in the normal direction as the inter-lobe clearance between two mating screw rotors. But, the sharps of screw compressors are very complicated, there are difference between the projection of axial difference in the normal direction and the inter-lobe clearance.

The contact line of screw compressors is the line of intersection of the two mating rotor surfaces. The contact line divides the cavity of the screw rotor into two portions, the low-pressure portion and high-pressure portion and seals the fluid in the high-pressure portion. In practice, the contact line transforms to seal belt because of the existence of the inter-lobe clearance. Because the points on the contact line of the two rotors are in meshing state at this moment,
the clearance between the two rotors near the contact line is the minimum distance between the two rotors. So, in this paper, the inter-lobe clearance is referred to, what is meant is the minimum distance normal to the two local helical surfaces near the contact line. Therefore, the calculation of inter-lobe clearance is transformed to the minimum distance calculation between the two rotors near the contact line. And, in this paper, a mathematical procedure is proposed to calculate the inter-lobe clearance between the two mating screw rotors in contact state, in which the rotors contacted with each other in the drive surface. So the mathematical procedure can be used to quantify eventual rotor interference in screw compressors due to the bearing clearance and the imperfections in compressor housing manufacturing.

2. COORDINATE SYSTEMS AND COORDINATE TRANSFORMATION

As shown in Fig.1, two mating screw rotors rotate with constant gear ratio \( i \) in the opposite directions about parallel axes. There are five coordinate systems in Fig.1. The coordinate system \( o_1x_1y_1z_1 \) fixed to the male rotor, the coordinate system \( o_2x_2y_2z_2 \) fixed to the female rotor, and the coordinate systems, \( O_1X_1Y_1Z_1, O_2X_2Y_2Z_2, \) and \( OXYZ \) fixed to the compressor housing.

The relationship between the five coordinate systems can be expressed by coordinate transformation matrices as follows:

1) The coordinate transformation for the coordinate system \( o_1x_1y_1z_1 \) and \( O_1X_1Y_1Z_1 \).

\[
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1
\end{bmatrix}
= 
\begin{bmatrix}
x_1 & y_1 & z_1 & 1
\end{bmatrix}
\cdot M_{11}
\]

\[
M_{11} = 
\begin{bmatrix}
\cos \varphi_1 & \sin \varphi_1 & 0 & 0 \\
-\sin \varphi_1 & \cos \varphi_1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

2) The coordinate transformation for the coordinate system \( o_2x_2y_2z_2 \) and \( O_2X_2Y_2Z_2 \).

\[
\begin{bmatrix}
X_2 & Y_2 & Z_2 & 1
\end{bmatrix}
= 
\begin{bmatrix}
x_2 & y_2 & z_2 & 1
\end{bmatrix}
\cdot M_{22}
\]

\[
M_{22} = 
\begin{bmatrix}
\cos i\varphi & \sin i\varphi & 0 & 0 \\
-\sin i\varphi & \cos i\varphi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

Figure 1: Coordinate systems
3) The coordinate transformation for the coordinate system \( O_1X_1Y_1Z_1 \) and \( OXYZ \).

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = \begin{bmatrix}
X_1 \\
Y_1 \\
Z_1 \\
1
\end{bmatrix} \cdot M_{10}
\]

\[
M_{10} = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
A & 0 & 0 & 1
\end{bmatrix}
\] (3)

4) The coordinate transformation for the coordinate system \( O_2X_2Y_2Z_2 \) and \( OXYZ \).

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = \begin{bmatrix}
X_2 \\
Y_2 \\
Z_2 \\
1
\end{bmatrix} \cdot M_{20}
\]

\[
M_{20} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (4)

The surfaces of screw rotors can be represented in the coordinate system \( OXYZ \) by the operation of coordinate transformation.

Male rotor:

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = \begin{bmatrix}
x_1 \\
y_1 \\
z_1 \\
1
\end{bmatrix} \cdot M_{k1}
\]

\[
M_{k1} = M_{11} \cdot M_{10}
\] (5)

Female rotor:

\[
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = \begin{bmatrix}
x_2 \\
y_2 \\
z_2 \\
1
\end{bmatrix} \cdot M_{k2}
\]

\[
M_{k2} = M_{22} \cdot M_{20}
\] (6)

3. SCREW SURFACES AND THE NORMAL VECTORS OF SURFACES

The surfaces of the mating rotors are expressed by the following equations:

\[
r_1 = r_1(t_1, \tau_1), \quad r_2 = r_2(t_2, \tau_2)
\]

\[
\begin{cases}
x_1 = x_1(t_1, \tau_1) \\
y_1 = y_1(t_1, \tau_1) \\
z_1 = z_1(t_1, \tau_1)
\end{cases}
\] (7)

\[
\begin{cases}
x_2 = x_2(t_2, \tau_2) \\
y_2 = y_2(t_2, \tau_2) \\
z_2 = z_2(t_2, \tau_2)
\end{cases}
\] (8)

As in Fig.2, the normal vectors of the surfaces are represented in coordinate system \( o_1x_1y_1z_1 \) and \( o_2x_2y_2z_2 \) by

\[
n = \frac{\partial \mathbf{r}}{\partial t} \times \frac{\partial \mathbf{r}}{\partial \tau}
\]
4. CALCULATION ON THE INTER-LOBE CLEARANCE

4.1 Coordinate transformation

The surfaces of screw rotors are expressed in the coordinate systems \( o_1x_1y_1z_1, o_2x_2y_2z_2 \) by equations (7), (8). \( Q(t_1, \tau_1) \) is a point on the surface of the male rotor, \( n_1(t_1, \tau_1) \) is the normal vectors passing the point. \( Q(t_2, \tau_2) \) is a point on the surface of the female rotor. This paper transforms the surfaces of screw rotors and its normal vectors to the coordinate systems \( OXYZ \) by coordinate transformation as below:

\[
Q_1^0 = [X_1^0 \ Y_1^0 \ Z_1^0 \ 1] = [x_1(t_1, \tau_1) \ y_1(t_1, \tau_1) \ z_1(t_1, \tau_1) \ 1] \cdot M_{K1} \tag{10}
\]

\[
n_1^* = n_1 \cdot M_{K1} \tag{11}
\]

\[
Q_2^0 = [X_2^0 \ Y_2^0 \ Z_2^0 \ 1] = [x_2(t_2, \tau_2) \ y_2(t_2, \tau_2) \ z_2(t_2, \tau_2) \ 1] \cdot M_{K2} \tag{12}
\]

4.2 Calculation on the inter-lobes clearance

The Optimization method is used to calculate the inter-lobes clearance between the two rotors in this paper. The inter-lobes clearance is regard as the minimum distance between the two rotors along the contact line. The mathematical model of this problem is as follows:

\[
\min f(x) \\
x = (t_1, \tau_1, t_2, \tau_2) \\
f(x) = (Q_1^0(t_1, \tau_1) - Q_2^0(t_2, \tau_2))^\top \cdot (Q_1^0(t_1, \tau_1) - Q_2^0(t_2, \tau_2)) \tag{13}
\]
The symbol \( Q^h_0, Q^v_0 \) expresses coordinates of the point on the helical surface of the male rotor or female rotor in the coordinate system fixed to the compressor housing which is gained by the formula (10) and formula (12). The parameters which are exhibited as \((t_1, r_1, l_z, t_2)\), express location parameters of the helical surface of the two rotors. The objective function represented by the symbol \( f(x) \), expresses square of the distance between the two rotors.

Because the functional relationship between \( f(x) \) and \((t_1, r_1, l_z, t_2)\) is nonlinear, the direct search method is used to search the minimum value of the distance, Simplex method for instance.

The steps of solving the inter-lobe clearance and its location parameters are as follows. At first, select a point on the contact line on the surface of the male rotor(or female rotor), and set one parameter of its location parameters as constant, and set the rest three parameters as initial value and substitute them in the formula (13). The value of the rest three parameters and corresponding minimum distance are gained by optimization calculation. The minimum distance resolved is just the inter lobe clearance near the point selected. Then select the next point on the contact line in proper sequence, resolve the inter-lobe clearance by the same method. When we adjust the density of the points selected to a proper level, this algorithm obtain full inter-lobe clearance with higher accuracy.

5. CONTACT STATE DETERMINATION OF SCREW ROTORS

5.1 Coordinate transformation matrices of screw rotor rotating about its axis

In the coordinate system \( OXYZ \), \( R \) represents the screw rotor surface in the coordinate system faxed to the compressor housing. \( R' \) represents the screw rotor surface rotated about its shaft with an angle \( \Delta \theta \) in the coordinate system faxed to the compressor housing. Then, the rotor surface after rotating about its shaft with an angle \( \Delta \theta \) can be gained through the following method.

\[
R' = [X', Y', Z'] = R \cdot R_z(\Delta \theta) = [X, Y, Z] \cdot R_z(\Delta \theta)
\]

\[
R_z(\Delta \theta) = \begin{bmatrix}
\cos \Delta \theta & \sin \Delta \theta & 0 & 0 \\
-\sin \Delta \theta & \cos \Delta \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

5.2 Estimate of the contact state between the two mating rotors

A clearance is needed between the rotors to allow for gas loading deflections and thermal distortions of the rotors, bearings and housing, and for manufacturing errors and oil-film thickness. There are in general five methods for producing clearances between rotors: Equidistant profile method, Equidistant helical surface method, Equidistant cutter blade geometry method, Reducing center distance between the cutter and rotor method, Increasing center distance method. In practice running, only the driving face of the male rotor contact with the female rotor and transfer torque. The torque was produced by contact force in the contact point. The position of the two mating rotors can be confirmed after confirming the position of the contact point.

The author confirms the position of the contact point and contact state by the inter-lobe clearance calculation method. Calculating the inter-lobe clearance distribution along the contact line by the method presented as before, if the minimum clearance of the inter-lobe clearance in the driving face of the male rotor is \( d \), the coordinate of the point with the minimum clearance \( d \) is \( Q(x_1, y_1, z_1) \) in the coordinate system fixed to male rotor. The distance between the point and the shaft of male rotor is \( r_\theta = \sqrt{x_1^2 + y_1^2} \), then, rotate the male rotor with an angle of \( \Delta \theta = \frac{d}{r_\theta} \cdot \lambda \) according to formula (14) and formula (15), calculate the minimum clearance again, till the minimum clearance is less than \( \varepsilon \), where \( \lambda \) is amending coefficient, \( \varepsilon \) is destined minimum clearance given before. If the minimum clearance is less than \( \varepsilon \), we consider the two rotors are in contact state.

6. EXAMPLE
Figure 3: The GHH screw rotors

Figure 4: The male rotor and the contact line

Figure 5: The clearance distribution along the contact line by equidistant lobe profile method

Figure 6: The clearance distribution along the contact line by center distance increasing method
This paper calculates an example with GHH rotors. Fig 3 shows a 5/6 rotor pair. The center distance is 45.5 mm and out diameters are 65 mm and 50.94 mm for the male and female rotor respectively. A uniform clearance between the two rotors in the end plane is 0.03 mm.

Figure.4 shows the male rotor and the contact line. Figure.5 shows two graphs of the clearance distribution along the contact line obtained by equidistant lobe profile method. Figure.6 shows two graphs of the clearance distribution along the contact line obtained by center distance increasing method. Figure.5 (a) and Figure.6 (a) show graphs of the clearance distribution along the contact line in the YOZ plane. Figure.5 (b) and Figure.6 (b) show the relationship between inter-lobe clearance and the length of contact line. As it can be seen at Figs 5,6, clearance jump exists along the contact line, the inter-lobe clearance distribution along the contact curve is quite different according to various clearance generating method, and this algorithm obtain full inter-lobe clearance with higher accuracy.

7. CONCLUSIONS

A mathematical apparatus is presented in this paper in which the inter-lobe clearance is calculated as the minimum distance between the two rotors along the contact line. That is, the distribution of inter-lobe clearance is transformed to the calculation of clearance distribution along the contact belt. And the actual position of the rotors in running state was considered, in which the rotors contacted with each other in the drive surface. The results show that the inter-lobe clearance distribution in real contact state has obvious difference with the inter-lobe clearance distribution designed, and clearance jump exists along the contact line, and the inter-lobe clearance distribution along the contact curve is quite different according to various clearance generating method. And the optimization method presented in this paper can calculate the inter-lobe clearance between the two screw rotor surfaces with higher accuracy.

NOMENCLATURE

\( i \) gear ratio
\( M \) coordinate transformation matrix
\( n \) normal vector of rotor surface
\( Q_j \) points in the male rotor \( j = 1 \), and female rotor \( j = 2 \)
\( r_j \) screw surface expressed in the coordinate system fixed to the screw rotor \( j \)
\( R_j \) screw surface of rotor \( j \) expressed in the coordinate system fixed to the compressor housing
\( R_j \) rotating coordinate transformation matrix
\( t_j, \tau_j \) parameter of the helical surface, \( j = 1,2 \)
\( x_j, y_j, z_j \) coordinates of the helical surface in the coordinate system faxed to screw rotor \( j, j = 1,2 \)
\( X_j, Y_j, Z_j \) coordinates of the helical surface \( j \) in the coordinate system faxed to the compressor housing. \( j = 1,2 \)
\( \Delta \theta \) rotate angle about z-axis
\( \phi_1 \) rotate angle of male rotor

Subscripts

1 male rotor
2 female rotor

REFERENCES

Xing, Z. W., 2000, Screw compressor—theory, design and application, Mechanical industry press, Beijing, China: p. 50-63.