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*A STUDY OF THE FATIGUE PROPERTIES  
OF AIR-ENTRAINED CONCRETE*

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*Joint  
Highway  
Research  
Project*

*by*

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TECHNICAL PAPER

A STUDY OF THE FATIGUE  
PROPERTIES OF AIR-ENTRAINED CONCRETE

TO: E. B. Woods, Director  
Joint Highway Research Project

October 23, 1958

FROM: S. L. Michael, Assistant Director  
Joint Highway Research Project

File: 5-13-2  
Project: C-38-500

Attached is a technical paper entitled, "A Study of the Fatigue Properties of Air-Entrained Concrete," by Messrs. John deG. Antrim and J. F. McLaughlin. This paper has been prepared for submission to the American Concrete Institute for publication in some forthcoming issue of the Journal.

This paper is a summarization of a research project previously reported to the Board. It is presented to the Board for the record.

Respectfully submitted,


*S. L. Michael*

S. L. Michael, Assistant Director  
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Technical Paper

A STUDY OF THE FATIGUE  
PROPERTIES OF AIR-ENTRAINED CONCRETE

by

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## SYNOPSIS

Fatigue tests were performed on specimens of two types of concrete, each type being designed for the same 28-day compressive strength. The one concrete contained only "accidental" air, while the other contained intentionally entrained air which was maintained at a constant level. Mixes were prepared periodically from each mix design so that there was little variation in the ages of the specimens being tested in fatigue. The fatigue test specimens, which were selected from each mix, were tested in fatigue at several different stress levels. These stress levels were 50, 60, 70, 80, and 90 percent of the ultimate static compressive strength of the respective mixes.

It was found that within the limits of the investigation, the fatigue behavior of air-entrained plain concrete is similar to that of non-air-entrained plain concrete. The results also show, however, that the air-entrained concrete was more uniform than the non-air-entrained concrete with regard to both fatigue and static strength properties.

## INTRODUCTION

As technology has progressed, there has been an increased need for information concerning the behavior of concrete under repeated applications of loads that are less than the ultimate. This type of loading, known as a fatigue loading, does not occur as frequently as the steady or static-type, but it is just as important a factor in design for those cases where it is present.

Experience has shown that it is insufficient for the designer to know the fatigue strength of the concrete at a specified fatigue life, or in the case of certain metals, the value of the fatigue limit; he needs to know the shape of the S-N curve. From the S-N curve he can relate his particular working stresses to a fatigue life or, knowing the number of cycles of stress a structure will receive in its lifetime, he can select the maximum stress which will not cause failure. The particular fatigue limit of concrete which is of greatest concern is that limit for cycles of stress varying from zero to a maximum in one direction only.

Although there has been no information reported on the fatigue behavior of air-entrained concrete, it is known from many studies<sup>1</sup> that air-entrained concrete has certain properties which differ from those of non-air-entrained concrete. Since many properties of concrete are altered by the addition of entrained air, the question naturally arises as to what effect entrained air has on the fatigue properties of concrete. The work reported in this paper had as its objective the establishment and

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<sup>1</sup>"Entrained Air in Concrete," Proceedings, American Concrete Institute, Vol. 42, pp. 602-700, 1946.

comparison of S-N curves for non-air-entrained concrete and for air-entrained concrete.

Most of the investigations concerned with the fatigue behavior of plain concrete were conducted during the period from 1900 to 1938. Not very general conclusions can be drawn regarding the fatigue of concrete since an analysis of the results of these early investigations and also the more recent ones, shows that the methods of investigating the problem varied considerably. Loading arrangement, rate of loading, type and size of specimen, and size of specimen are only a few of the differences existing between investigations. Corby<sup>2</sup> in his review of research on fatigue of concrete up to the present time, has summarized the results of these investigations.

#### TESTING PROGRAM

The tests required for this study were done in two parts. The first part consisted of performing tests on specimens made from a non-air-entrained concrete mix. This mix will be referred to as the PA series. The second part consisted of performing tests on specimens made from an air-entrained concrete mix. This mix will be referred to as the FA series. Each series consisted of eight batches made periodically during the course of the investigation. In general, a batch yielded about twenty-five concrete cylinders, 3 inches in diameter by 6 inches in height.

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<sup>2</sup> Corby, G. E., "Fatigue of Concrete - A Review of Research," Proceedings, American Concrete Institute, Vol. 55, pp. 191-219.

The coarse aggregate was a crushed limestone from central Indiana. It had a maximum size of 1/2-inch. The fine aggregate was a local sand obtained from a river terrace deposit. Type I portland cement from a single clinker batch was used in both mixes and Barox, added at the mixer, was used as the air-entraining agent for the air-entrained concrete.

The non-air-entrained mix was designed for a strength of 4000 psi and a slump of three inches. The mix had a cement factor of 5.4 sacks per cubic yard of concrete and a water-cement ratio of 0.69 by weight.

The air-entrained mix was designed for a strength of 4000 psi, a slump of three inches, and an air content of seven percent. The mix had a cement factor of 5.0 sacks per cubic yard of concrete and a water-cement ratio of 0.61 by weight.

After 28 days of curing which followed the procedure specified in ASTM Designation: C129-52T, the specimens underwent 3 to 4 days of oven drying at approximately 220°F to dry the concrete and prevent further hydration. Soon after removal from the oven, each specimen was capped on both ends with a sulfur compound and then stored under room conditions until testing.

Shortly after capping, eight specimens from each batch were tested for their ultimate static compressive strength. The average ultimate strength of these eight specimens served as an estimate of the ultimate strength of the batch, and the stress levels at which the fatigue tests were performed were based on this estimate of the batch strength. Following the completion of the fatigue tests on each batch, the remaining specimens in the batch were tested for their ultimate static compressive



strength. The object of these later compression tests was to determine if there was any further strength gain in the concrete during the time taken by the fatigue tests.

The fatigue tests were conducted, when conditions permitted, at five different stress levels. These stress levels were 50, 60, 70, 80, and 90 percent of the estimated ultimate strength of the batch. A minimum stress of approximately 70 psi was maintained regardless of the magnitude of the maximum stress. The process of establishing the desired maximum and minimum loads on the fatigue testing machine required a considerable number of cycles; therefore, the loading was set by first inserting a dummy specimen in the machine. When the loads were established, the machine was stopped and the dummy specimen was replaced by a test specimen. Upon restarting the machine, usually little or no adjustment was necessary to obtain the desired loads. In every case, a virgin specimen was used to obtain the information that is reported in this study. If a test was interrupted for any reason, it was started over again with a new specimen.

The Krouse-Purdue axial-load fatigue machine that was used in this study is shown in Figure 1. It is of the constant deflection type and derives its force from hydraulic pressure acting on a large piston directly connected to the test piece through a piston rod. The load applied by the machine consists of two components: a preload and a pulsating load. The preload is directly proportional to the average differential pressure existing between the two ends of the hydraulic cylinder, and it is controlled by automatically controlling make-up oil pressures. The amount by which the pulsating load varies above and below the preload depends on the throw of a variable-throw crank that can be adjusted before



Fig. 1 - Krouse-Purdus Axial-Load  
Fatigue Machine

or while the machine is operating. The operating speed of the fatigue machine is 1000 cycles per minute and the machine has a maximum loading capacity of  $\pm 50,000$  lbs. The magnitude of the applied loads is measured by means of an electronic system which is actuated by a load cell that is a part of the lower end of the load screw.

#### DISCUSSION OF RESULTS

The results of this study and the dimensions of those results have been divided into two parts. The first part is concerned with the analyses of the physical properties of the plastic concrete and the hardened concrete used in this research work. The second part covers the results of the fatigue testing program.

##### Analysis of Mix Data

The purpose of the study was to compare the fatigue behavior of non-air-entrained concrete and air-entrained concrete that were similar except for air content. Since the two concretes were prepared in batches, the properties of the two mix designs had to be estimated from the corresponding batch properties. These batch properties, slump and air content of the plastic concrete, and strength of the hardened concrete at various ages expressed as means, are tabulated in Tables 1 and 2.

The statistical procedure known as the analysis of variance was used for testing for significant differences among the means. Except for some minor adjustments in the tests because of the nature of the data being tested, the data summarized in Tables 1 and 2 were tested by the same methods. The non-air-entrained mix and the air-entrained mix were

TABLE 1

PHYSICAL CHARACTERISTICS OF MIX DESIGN FN

| Batch Designation | Air Content | Slump  | Average Ultimate Compressive Strength at Age of 7 Days (3 specimens tested) | Average Ultimate Compressive Strength after Oven Drying <sup>1</sup> (8 specimens tested) | Average Ultimate Compressive Strength after Completion of Fatigue Tests <sup>2</sup> (7 to 8 specimens tested) |
|-------------------|-------------|--------|---|---|--|
|                   | percent     | inches | psi.  | psi.  | psi.   |
| FN1               | 0.6         | 3      | None tested <sup>3</sup>  | 4070  | 3970   |
| FN2               | 1.2         | 3½     | None tested <sup>3</sup>  | 3560  | 3520   |
| FN3               | 1.8         | 6      | 2690 <sup>4</sup>   | 3440  | None tested <sup>5</sup>   |
| FN4               | 1.5         | 2½     | 2510  | 4010  | 4150   |
| FN5               | 0.5         | 3      | 2470  | 4150  | 4120   |
| FN6               | 0.1         | 2½     | 2980  | 4270  | 4080   |
| FN7               | 0.8         | 6      | 2760  | 3820  | 3980   |
| FN8               | 1.0         | 6      | 2780  | 4230  | 4180   |

1. Specimen age ranged from 44 to 36 days.
2. Specimen age ranged from 47 to 65 days.
3. Specimens used in the fatigue tests.
4. Specimens tested at the age of 11 days.
5. Specimens tested at a later date.

TABLE 2

## PHYSICAL PROPERTIES OF MIX DESIGN FA

| Batch Designation | Air Content percent | Slump inches | Average Ultimate Compressive Strength    |  | Average Ultimate Compressive Strength after Completion of Fatigue Tests <sup>2</sup><br>(7 to 8 specimens tested) |
|-------------------|---------------------|--------------|--|--|---|
|                   |                     |              | at Age of 7 Days<br>(3 specimens tested) | after Oven Drying <sup>1</sup><br>(8 specimens tested) |   |
| FA1               | 7.6                 | 1 3/4        | 2560                                     | 4020   | None tested <sup>3</sup>  |
| FA2               | 10.5                | 2 3/4        | 2740                                     | 4380   | 4260  |
| FA3               | 7.7                 | 2            | 2860                                     | 4490   | 4410  |
| FA4               | 7.5                 | 1 3/4        | 2900                                     | 4540   | 4430  |
| FA5               | 7.7                 | 2            | 2710                                     | 4300   | 4250  |
| FA6               | 7.6                 | 1 3/4        | None tested <sup>4</sup>                 | 4620   | 4560  |
| FA7               | 9.0                 | 2            | 2670                                     | 4500   | 4160  |
| FA8               | 8.2                 | 2 1/4        | 2490 <sup>5</sup>                        | 4170   | 3910  |
| FA9               | 8.5                 | 2 1/2        | 2640                                     | 4620   | 4510  |

1. Specimen age ranged from 34 to 36 days.

2. Specimen age ranged from 43 to 63 days.

3. Fatigue tests not run, machine out of operation for a month.

4. Scheduling difficulties.

5. Specimens tested at age of 6 days.

treated separately and then a comparison was made of the two mix strengths. All the tests were made at the 5 percent significance level. The results of these statistical tests on data collected from over 200 specimens are best summarized as follows:

1. The mean strengths of the non-air-entrained concrete mix and the air-entrained mix, 4090 and 4430 psi respectively, were not significantly different.
2. The average air content of the non-air-entrained mix was 0.9 percent and the average air content of the air-entrained mix was 8.3 percent. The coefficient of variation of the air content for the non-air-entrained mix was an abnormally high 58 percent as against a coefficient of 12 percent for the air-entrained mix.
3. The average slumps of the non-air-entrained and the air-entrained mixes were 4 inches and 2 inches respectively, with corresponding coefficients of variation of 40 and 15 percent.
4. There was no significant gain in strength of the non-air-entrained concrete during the periods of fatigue testing.
5. There was a significant decrease in the strength of the air-entrained concrete during the periods of fatigue testing. The decrease averaged 140 psi; however, it was assumed that this change did not materially affect the results of the fatigue tests.

Analysis of the Fatigue Data

The analysis of the fatigue data tabulated in Tables 3 and 4 was limited by the scatter that is characteristic of fatigue test results. The actual range of stress cycles endured is unknown for both the 50 and 60 percent stress levels of both the non-air-entrained concrete and the air-entrained concrete because the testing was stopped when the specimens had sustained ten million cycles without failing. There is, of course, a lower boundary for the data representing the 60 percent stress level. It was felt that any curve fitted to all the data for each type of concrete would undoubtedly be a very poor approximation of the true curve; therefore, the curves that were drawn only represent the relationship that exists at the 70 and 80 percent levels.

Referring to the two S-N diagrams in Figures 2 and 3, it can be seen by the position of the points on the right hand portion of both diagrams that it is unlikely that a definite fatigue limit exists in the vicinity of ten million cycles for either type of concrete. It is also apparent that the fatigue test results for the air-entrained concrete are characterized by less scatter than the test results for the non-air-entrained concrete.

Comparison of the FA Mix to the FH Mix. A nonparametric statistical technique was utilized in comparing the fatigue test results from the two mixes, because the type of distribution to which the data belonged was unknown. In this analysis a method called "Runs" was used.<sup>3</sup> This technique, when used to test the data to which it was

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<sup>3</sup> Dixon, W. J., and Massey, F. J., Jr., Introduction to Statistical Analysis, New York: McGraw-Hill Book Co., Inc., 1951.

TABLE 03

## FATIGUE TEST RESULTS, NON-AIR-ENTRAINED CONCRETE

| Batch Designation | Specimen Number | Maximum Fatigue Load (lbs.) | Minimum Fatigue Load (lbs.) | Number of Stress Cycles Endured |  |
|-------------------|-----------------|-----------------------------|-----------------------------|---------------------------------|--|
| FN1               | 1               | 14,500 (52) <sup>1</sup>    | 500 (1.8) <sup>1</sup>      | 10,155,000 → <sup>2</sup>       |  |
|                   | 2               | 17,000 (60)                 | 500 (1.8)                   | 10,355,000 →                    |  |
|                   | 3               | 19,500 (69)                 | 700 (2.5)                   | 12,000                          |  |
|                   | 4               | 22,700 (81)                 | 700 (2.5)                   | 282,000                         |  |
|                   | 5               | 25,400 (90)                 | 500 (1.8)                   | 100                             |  |
| FN2               | 1               | 12,300 (50)                 | 500 (2.0)                   | 10,568,000 →                    |  |
|                   | 2               | 14,800 (60)                 | 500 (2.0)                   | 10,106,000 →                    |  |
|                   | 3               | 17,800 (72)                 | 500 (2.0)                   | 31,000                          |  |
|                   | 4               | 19,500 (79)                 | 500 (2.0)                   | 2,200                           |  |
|                   | 5               | 22,200 (90)                 | 1000 (4.0)                  | 900                             |  |
| FN3               | 1               | 12,300 (51)                 | 300 (1.2)                   | 10,330,100 →                    |  |
|                   | 2               | 14,400 (60)                 | 500 (2.1)                   | 3,630,400                       |  |
|                   | 3               | 16,500 (69)                 | 700 (2.9)                   | 25,800                          |  |
|                   | 4               | 18,700 (78)                 | 700 (2.9)                   | 19,800                          |  |
|                   | 5               | 20,500 (85)                 | 500 (2.1)                   | 300                             |  |
| FN4               | 1               | 14,100 (49)                 | 500 (1.3)                   | 10,562,000 →                    |  |
|                   | 2               | 16,500 (58)                 | 500 (1.3)                   | 10,472,000 →                    |  |
|                   | 3               | 19,000 (67)                 | 500 (1.3)                   | 2,349,600                       |  |
|                   | 4               | 22,600 (79)                 | 800 (2.3)                   | 8,900                           |  |
|                   | 5               | Specimen not tested         |                             |                                 |  |
| FN5               | 1               | Specimen not tested         |                             |                                 |  |
|                   | 2               | 17,400 (60)                 | 500 (1.7)                   | 10,740,000 →                    |  |
|                   | 3               | 20,300 (70)                 | 500 (1.7)                   | 99,700                          |  |
|                   | 4               | 22,500 (78)                 | 500 (1.7)                   | 1,500                           |  |
|                   | 5               | 26,100 (90)                 | 700 (2.4)                   | 3,700                           |  |
| FN6               | 1               | Specimen not tested         |                             |                                 |  |
|                   | 2               | 18,000 (62)                 | 500 (1.7)                   | 1,308,400                       |  |
|                   | 3               | 21,000 (72)                 | 500 (1.7)                   | 278,700                         |  |
|                   | 4               | 24,000 (82)                 | 800 (2.7)                   | 1,000                           |  |
|                   | 5               | 26,500 (91)                 | 500 (1.7)                   | 300                             |  |

(continued)



TABLE 03(continued)

| Batch Designation | Specimen Number | Maximum Fatigue Load (lbs.) | Minimum Fatigue Load (lbs.) | Number of Stress Cycles Endured |
|-------------------|-----------------|-----------------------------|-----------------------------|---------------------------------|
| FN7               | 1               | Specimen not tested         |                             |                                 |
|                   | 2               | Specimen not tested         |                             |                                 |
|                   | 3               | 18,800 (69)                 | 500 (1.8)                   | 54,500                          |
|                   | 4               | 21,500 (79)                 | 1000 (3.7)                  | 4,600                           |
|                   | 5               | 24,200 (89)                 | 500 (1.8)                   | 10,900                          |
| FN8               | 1               | Specimen not tested         |                             |                                 |
|                   | 2               | 17,900 (61)                 | 500 (1.7)                   | 1,388,700                       |
|                   | 3               | 20,500 (70)                 | 800 (2.7)                   | 52,200                          |
|                   | 4               | 23,500 (80)                 | 1000 (3.4)                  | 1,700                           |
|                   | 5               | 26,700 (91)                 | 500 (1.7)                   | 500                             |

1. Figure in parentheses is the fatigue load expressed as a percentage of the average ultimate compressive strength of the batch. Average strength based on 15 to 16 specimens except for batch FN3 which had 8 specimens.
2. → indicates that specimen had not failed when test was stopped.

TABLE 4

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## FATIGUE TEST RESULTS, AIR-ENTRAINED CONCRETE

| Batch Designation | Specimen Number                                  | Maximum Fatigue Load (lbs.) | Minimum Fatigue Load (lbs.) | Number of Stress Cycles Endured |
|-------------------|--|-----------------------------|-----------------------------|---------------------------------|
| FA1               | No tests performed, Fatigue Machine under repair |                             |                             |                                 |
| FA2               | 1  | Specimen not tested         |                             |                                 |
|                   | 2  | 18,350 (61) <sup>1</sup>    | 500 (1.7)                   | 10,048,000 → 2                  |
|                   | 3  | 21,550 (71)                 | 550 (1.8)                   | 702,200                         |
|                   | 4  | 23,900 (79)                 | 500 (1.7)                   | 2,800                           |
|                   | 5  | Loads not verified          |                             |                                 |
| FA3               | 1  | 15,800 (51)                 | 400 (1.3)                   | 10,314,000 →                    |
|                   | 2  | 19,200 (62)                 | 500 (1.6)                   | 6,294,500                       |
|                   | 3  | 22,350 (72)                 | 500 (1.6)                   | 109,400                         |
|                   | 4  | 25,300 (81)                 | 500 (1.6)                   | 600                             |
|                   | 5  | 28,800 (93)                 | 500 (1.6)                   | 300                             |
| FA4               | 1  | Specimen not tested         |                             |                                 |
|                   | 2  | 19,000 (60)                 | 600 (1.9)                   | 8,022,600                       |
|                   | 3  | 22,000 (70)                 | 450 (1.4)                   | 204,800                         |
|                   | 4  | 25,300 (80)                 | 600 (1.9)                   | 3,500                           |
|                   | 5  | Loads not verified          |                             |                                 |
| FA5               | 1  | Specimen not tested         |                             |                                 |
|                   | 2  | 18,000 (60)                 | 600 (2.0)                   | 9,092,900                       |
|                   | 3  | 21,300 (71)                 | 400 (1.3)                   | 1,683,800                       |
|                   | 4  | 24,200 (81)                 | 500 (1.7)                   | 1,500                           |
|                   | 5  | Loads not verified          |                             |                                 |
| FA6               | 1  | Specimen not tested         |                             |                                 |
|                   | 2  | 19,500 (61)                 | 500 (1.6)                   | 10,157,000 →                    |
|                   | 3  | 22,300 (69)                 | 600 (1.9)                   | 226,900                         |
|                   | 4  | 25,900 (81)                 | 500 (1.6)                   | 3,900                           |
|                   | 5  | Specimen not tested         |                             |                                 |
| FA7               | 1  | 15,600 (51)                 | 600 (2.0)                   | 10,105,000 →                    |
|                   | 2  | 18,800 (62)                 | 500 (1.6)                   | 10,256,000 →                    |
|                   | 3  | 22,100 (73)                 | 500 (1.6)                   | 361,800                         |
|                   | 4  | 24,900 (82)                 | 500 (1.6)                   | 600                             |
|                   | 5  | Specimen not tested         |                             |                                 |

(continued)

TABLE 8 (continued)

| Batch Designation | Specimen Number | Maximum Fatigue Load (lbs.) | Minimum Fatigue Load (lbs.) | Number of Stress Cycles Endured |
|-------------------|-----------------|-----------------------------|-----------------------------|---------------------------------|
| FA8               | 1               | 14,400 (51)                 | 700 (2.5)                   | 10,760,000 →                    |
|                   | 2               | 17,400 (62)                 | 650 (2.3)                   | 10,109,000 →                    |
|                   | 3               | 20,500 (72)                 | 500 (1.8)                   | 215,700                         |
|                   | 4               | 23,600 (83)                 | 300 (1.1)                   | 900                             |
|                   | 5               | Specimen not tested         |                             |                                 |
| FA9               | 1               | Specimen not tested         |                             |                                 |
|                   | 2               | 19,400 (61)                 | 500 (1.6)                   | 10,034,000 →                    |
|                   | 3               | 22,700 (71)                 | 400 (1.3)                   | 534,900                         |
|                   | 4               | 26,000 (82)                 | 500 (1.6)                   | 400                             |
|                   | 5               | Specimen not tested         |                             |                                 |

- Figure in parentheses is the fatigue load expressed as a percentage of the average ultimate compressive strength of the batch. Average strength based on 15 to 16 specimens.
- indicates that specimen had not failed when test was stopped.

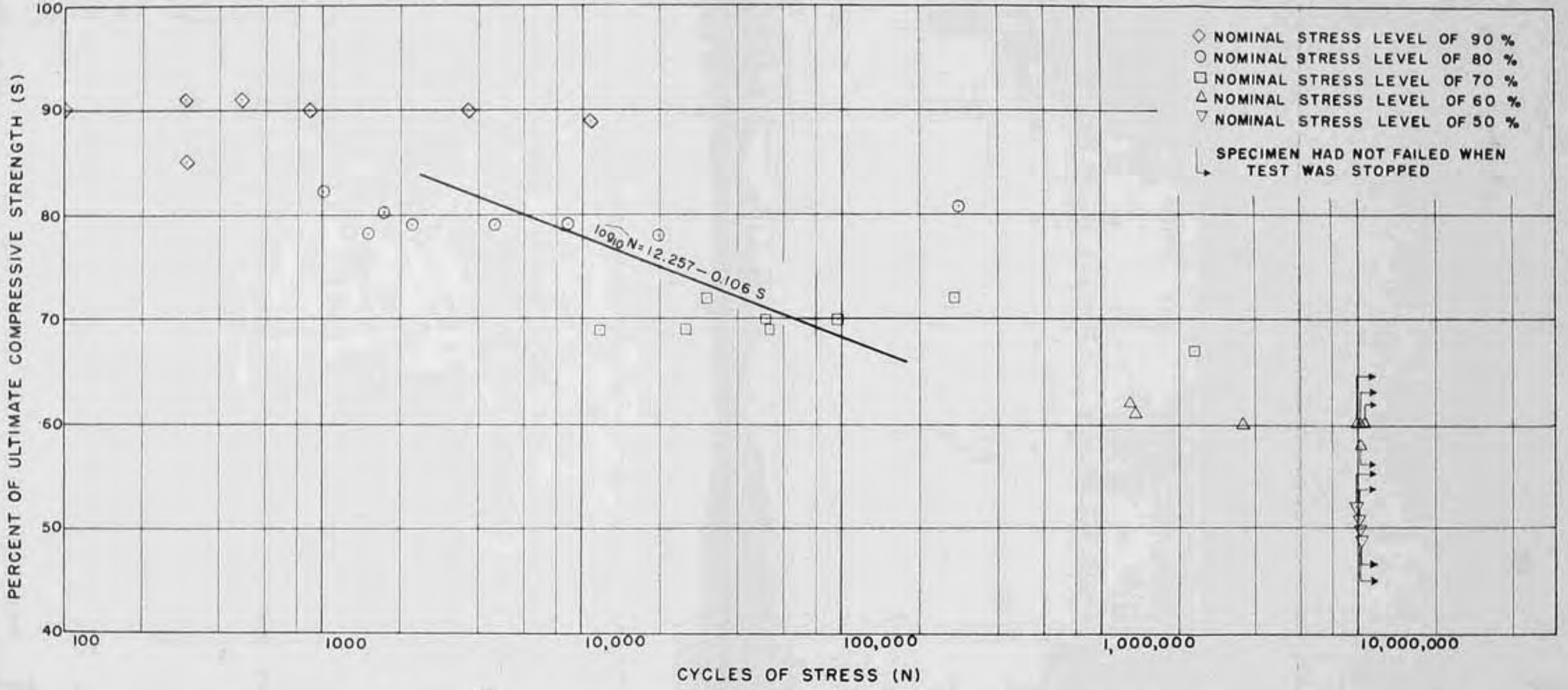


FIGURE 2. S-N DIAGRAM FOR NON-AIR-ENTRAINED CONCRETE

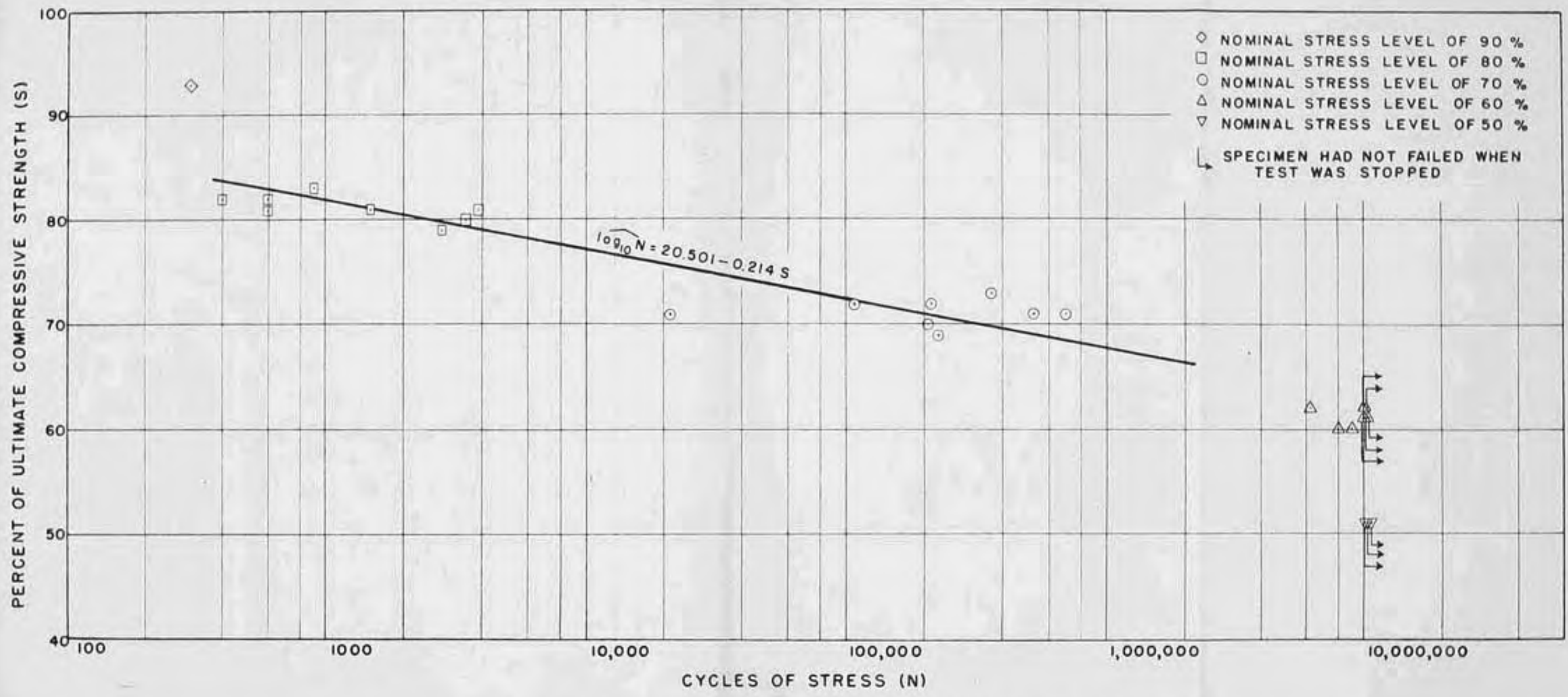


FIGURE 3 S-N DIAGRAM FOR AIR-ENTRAINED CONCRETE

applicable, indicates that there is reliable evidence at the 5 percent significance level that at the respective stress levels the data are from the same population.

Linear Regression Analysis. A consequence of plotting the cycles to failure to a logarithmic scale was a contraction or drawing together of the  $N$  values. The data in this transformed condition appear to have a normal distribution with respect to cycles to failure. It is on this assumption that the following analyses were performed.

Only the data for the 70 and 80 percent stress levels were considered in the analyses because it is only at these levels that an equal number of specimens from each mix design had been tested. Fitting a curve to the data of each mix design was accomplished by the method of least squares.

The least squares linear regression equation resulting from an analysis of the data for the non-air-entrained concrete is:

$$\widehat{\log_{10} N} = 12.257 - 0.106 S \dots\dots\dots 1$$

where  $N$  is cycles to failure and  $S$  is percent of the ultimate stress. This equation can only be used to estimate  $N$  when the value of  $S$  is between 66 and 84 percent since this is the range of the  $S$  values used in the analysis.

The least squares linear regression equation resulting from an analysis of the data for the air-entrained concrete was calculated to be:

$$\widehat{\log_{10} N} = 20.901 - 0.214 S \dots\dots\dots 2$$

The same limits on  $S$  that apply to the equation for the non-air-entrained concrete, apply to this equation when estimating  $N$ .

Along with the linear regression equations, the correlation coefficients for the two sets of data were calculated. The degree of association among the data for the air-entrained concrete is very high, the correlation coefficient being  $-0.94$ . The degree of association for the non-air-entrained concrete is less than that for the air-entrained data, the correlation coefficient in this case being  $-0.62$ . A significance test for differences between correlation coefficients performed at the 5 percent significance level indicates that there is reliable evidence that the two correlation coefficients are significantly different. This test only compares the linear association of the data and does not compare the curves determined by the regression analysis. However, it does show that the fatigue data for the air-entrained concrete has considerably greater uniformity than the fatigue data for the non-air-entrained concrete.

A test was also performed to find whether or not the slopes of the two equations were significantly different. The test indicates that at the 5 percent significance level there is no reliable evidence that the slopes are significantly different. This indicated equality of the slopes suggests very strongly that the relationship existing between the stress level and the cycles to failure for the non-air-entrained concrete is the same as the relationship for the air-entrained concrete.

Prediction Intervals. Although the estimating equations for both mix designs have been calculated, it must be realized that if a number of tests were to be performed at a given  $S$  value, the observed  $N$ 's would cluster about the calculated or estimated  $N$ . It is, therefore, advisable to determine the limits of the predicted value of an individual  $N$  for a given  $S$  value.

Ninety-five percent prediction intervals were calculated for each of the two  $S-N$  curves. As shown in Table 5, these intervals are extremely wide, especially in the case of the non-air-entrained concrete. It can be seen that for a given  $S$  value the predicted  $N$  for the air-entrained concrete lies within the limits for the predicted  $N$  for the non-air-entrained concrete. The fact that there are overlapping prediction intervals along with essentially equal slopes is good reason to assume that there is no difference between the fatigue behavior of the two types of concrete.

#### Other Observations

It was noticed that if an air-entrained specimen was exchanged for a non-air-entrained specimen in the fatigue machine without any adjustment in the load control mechanism, the loads applied to the air-entrained specimen were different from those that had been established for the non-air-entrained specimen. This change consisted of an increase in the minimum load and a corresponding decrease in the maximum load. The change is explained by the ability of the air-entrained specimen to compress farther under a similar load. The desired loads were easily obtained by increasing the crank throw, which in turn increased the travel of the loading piston and, thereby,



TABLE 5

NINETY-FIVE PERCENT PREDICTION INTERVALS FOR NON-AIR-ENTRAINED AND AIR-ENTRAINED CONCRETE

| Stress Level<br>(percent) | Prediction Intervals       |                         |                         |                         |
|---------------------------|----------------------------|-------------------------|-------------------------|-------------------------|
|                           | Non-Air-Entrained Concrete |                         | Air-Entrained Concrete  |                         |
|                           | Lower Limit<br>(cycles)    | Upper Limit<br>(cycles) | Lower Limit<br>(cycles) | Upper Limit<br>(cycles) |
| 80                        | $9.67 \times 10^{-5}$      | $3.26 \times 10^{13}$   | 8.94                    | $5.17 \times 10^7$      |
| 75                        | $1.41 \times 10^{-5}$      | $5.51 \times 10^{13}$   | 1.53                    | $5.24 \times 10^8$      |
| 70                        | $7.00 \times 10^{-4}$      | $3.14 \times 10^{14}$   | 12.18                   | $9.08 \times 10^9$      |

compensated for the decrease in height of the air-entrained specimen.

Another phenomenon noticed was an indicated change in the applied load while a test was in progress. This was noticed only during the testing of air-entrained specimens and it was not detected for all specimens. In those cases where the load change was detected, it was only after the specimens had been subjected to a million or more cycles. A strange feature was that a number of the specimens endured ten million cycles without any load changes occurring. Those changes in the loadings consisted of an increase in the minimum load and an increase or a decrease in the maximum load. The changes, however, were of different magnitudes ranging from a few hundred pounds to over a thousand pounds.

#### Summary of Fatigue Test Results

The analyses of the results of 65 specimens tested in fatigue indicate the following:

1. The portion of the S-N curve for the non-air-entrained concrete between the stress levels of 66 and 84 percent  $f_{ac}$ , as determined by a linear regression analysis, the relationship:

$$\widehat{\log_{10} N} = 12.257 - 0.106 S$$

The portion of the S-N curve for the air-entrained concrete between the stress levels of 54 and 84 percent  $f_{ac}$ , as determined by a linear regression analysis, the relationship:

$$\widehat{\log_{10} N} = 20.501 - 0.214 S$$

It was found, however, that the slopes of these lines are not significantly different and, in addition, that one prediction interval lies within the other. On this basis it may be said that the relationship of stress level to cycles to failure is not significantly different for the two types of concrete when a fluctuating compressive stress condition is maintained and the minimum stress is held constant at slightly in excess of zero stress.

2. The fatigue test data for the air-entrained concrete had considerably less variation than the data for the non-air-entrained concrete.

3. The fatigue test data accumulated in this study suggest that the S-N curve is continually sloping downwards; that is, there is no indication of a definite fatigue limit for either the non-air-entrained or the air-entrained concrete.

#### CONCLUSIONS

The following conclusions are based on a laboratory investigation of fatigue of concrete in which 3- by 6-inch cylinders were tested by a fluctuating compressive stress in which the maximum was varied and the minimum stress was maintained at a constant value near zero. Subject to these qualifications, the following conclusions appear to be justified:

1. The fatigue behavior of non-air-entrained plain concrete and air-entrained plain concrete are not significantly different.

2. It appears that there is considerably less variation present among fatigue test data for the air-entrained plain concrete than there is for the non-air-entrained plain concrete.

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