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Tao Liu  
*Lanzhou University*

Zhen Quan Liu  
*Lanzhou University*

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STUDY ON GEOMETRY THEORY OF TRIGONOMETRIC-CURVE MODIFICATION OF SCROLL PROFILE FOR SCROLL COMPRESSOR

Tao LIU 1, Zhenquan LIU 2

1 Scholl of Mechano-Electronic Engineering, Lanzhou University of Technology, Lanzhou, China
Fax: (0931)2757564
Telephone: (0931)2757295
E-mail: liutao0713@sina.com

2 Scholl of Petrochemical Technology, Lanzhou University of Technology, Lanzhou, China

ABSTRACT

The modification of scroll profile for scroll compressor has drawn a lot of attention from researchers worldwide for many years. In order to improve the E.E.R. of scroll compressor, one of the most effective approaches is to change the shape of scroll wrap. Thus various patterns of modification had been presented and put into use in last two decades. Based on conjugate principle, we derived the geometry theory of trigonometric–curve modification. General formulas are presented for calculating the volume of the compression chamber and that of the discharge chamber. Modification parameters that affect the shape of central of scroll wrap, volume of chamber and volume ratio are also analyzed systematically. Under certain condition, the trigonometric–curve modification could convert into circular-arc-curve modification accordingly. These theoretical innovations can contribute considerately to scroll compressor design and improvement of its performance.

1. INTRODUCTION

The scroll compressor has the characteristics of simplicity, reliability, high efficiency and low level of noise and vibration. It therefore is becoming popular and widely used in refrigeration and air conditioners. One of the main problems encountered in the development of scroll compressors is the design of the scroll profile, which plays a key role in determining the performance of a scroll compressor. A favorable scroll profile should be of high efficiency, low vibration, light in weight and easy to manufacture and maintain.

The major components of a scroll compressor include a fixed scroll, an orbiting scroll, an anti-rotation coupling, a crankshaft and a crankcase. The orbiting scroll is turned by 180° and orbits relatively to the fixed scroll. The anti-rotation coupling is used to prevent the moving scroll from self-rotation while it is orbiting. The fluid is thus brought in from suction port and moved continuously inward by the orbiting motion. The volume occupied by the fluid becomes progressively smaller and the fluid is steadily compressed. Finally, the fluid is smoothly discharged from a discharge port. For the simplest case, the profile of a scroll generally is constructed by involute curves. When a typical scroll wrap is machined near to its central part, over-cut is usually encountered because of the intervention between the scroll and the tool. The result is high stress concentration [1], large clearance volume and small volume ratio. To remedy these shortcomings, many researchers have considered modification of the top of scroll wrap. Compared to the non-modified wrap design, the modified one has a more favored performance due to smaller clearance volume, larger volume ratio and smaller size.

Literature survey [2,3,4] has shown that currently circular arc and straight line serve as remedy curves in the widely used PMP modifications across the world. From a geometry point of view, the above two curves are the only possible ones among those various kind of available curves could be used to modify the top wrap of the scroll. An imperfectly meshed pair of scrolls will generally result in poor efficiency, but well modified pair may lead to better properties. The profile of central wrap will have great effects on thermal and mechanical characteristics of the whole scroll set. Therefore, the top scroll profile appears to be very important and needs to be investigated carefully.

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In this present paper, special attention is paid to trigonometric-curve modification at the central portion of scroll and relative geometry theory will be discussed in more detail.

2. TRIGONOMETRIC-CURVE MODIFICATION

2.1 generating conditions

As shown in Figure 1, the basic profile of a scroll can be constructed by two involutes of circle: the inner curve and outer curve respectively. Smooth segments of arcs connecting the outer curves and inner curves of both scrolls are introduced. Segments \( F_1F_0 \) and \( M_1M_0 \), which start from inner curves, could be named connection arcs. Similar, \( F_2F_0 \) and \( M_1M_0 \), which start from outer curves, could be called modification arcs.

To fulfill conjugately meshing motion of the two scrolls, three kind of constrain conditions should be considered. First, however, the following continuity conditions must be met between the inner profile of the scroll and the modification arc as well as between the outer profile and the connection arc:

1) Same coordinate values should be found at connecting points both on involutes and remedy arcs.
2) Slopes at connecting points both on involutes and remedy arcs should keep the same value as well as the same direction.
3) Curvatures and direction of normal lines should be the same at those points.

Second, the two surfaces to be meshed are the inner curve of the first curve and outer curve of the second scroll and vice versa. Therefore, the connection arc of one scroll is conjugately generated by the given modification arc of the other scroll and vice versa.

Finally, a third general condition for conjugation also need to be met [5], that is, at arbitrarily given points, the difference between the modification arc and the connection arc should equal to the rotating radius of main shaft. Thus, the modified profile of central portion of both scrolls could be determined exclusively.

2.2 Equations of equidistant curves

In this present analysis, trigonometric curve is employed as an equidistant curve of connecting arc or modification arc of the scroll. The equation for equidistant curve of connecting arc can be given as

\[
R_1(t) = (R_{10} - \delta_1) \sin^n \left(t + \frac{\pi}{2} - \alpha_1\right)
\]

And the equation for equidistant curve of modification arc could be write as

\[
R_2(t) = (R_{20} - \delta_2) \sin^n \left(t + \frac{\pi}{2} - \alpha_2\right)
\]

\( R_{10} \) and \( R_{20} \) appeared in the above equations are constants derived from conjugation relationship and continuity conditions. Parameters \( \delta_1, \delta_2, \alpha_1, \alpha_2, n_1, n_2 \) that are called modification variables hereafter play an important role in the geometric properties of scroll compressor.

3. CALCULATION OF VOLUME OF COMPRESSION CHAMBER

Based on previous literature, the volume ratio of a scroll compressor can be expressed by dividing the maximum compression chamber by the minimum chamber. The volume of compression chamber will change in accordance with the variation of modification parameters.

From the outmost, sealed chambers enclosed by inner wraps of orbiting scroll and outer wrap of fixed scroll and vice versa are the first, second and discharge chamber in turn. For the meshing motion takes place solely at involutes portion of the wraps, formula for calculating the volume of compression chamber is the same as used in non-modification case. While for the modified portion of scroll wrap, specific equations determining the volume of chamber will be derived. Figure 2 demonstrates an arbitrary compression chamber, the volume of which could be described by the following general equation:

\[
V(\theta) = R_m h (S_{5E} - d \sin \beta)
\]
For the first compression chamber, where $\beta = 0$, the above equation could be rewritten as

$$V_1(\theta) = hR_{or} \left( \int_{\phi_{\max}}^{\theta} a\phi d\phi \right) \quad (0 \leq \theta \leq 2\pi) \quad (4)$$

For the second compression chamber formed by involutes portion, equation is given by

$$V_2(\theta) = hR_{or} \left( \int_{\phi_{\max} - 2\pi}^{\phi_{\max} - 2\pi - \theta} a\phi d\phi \right) \quad (0 \leq \theta \leq \phi_{\text{start}}) \quad (5)$$

While for the modified portion, the volume becomes

$$V_2(\theta) = hR_{or} \left( \int_{\phi_{\max} - 2\pi}^{\phi_{\max} - 2\pi - \theta} a\phi d\phi + d \sin \beta \right) \quad (\phi_{\text{start}} \leq \theta \leq \theta^*) \quad (6)$$

Where

- $a$ = base circle radius
- $h$ = height of scroll wrap
- $\phi$ = involute angle
- $\theta$ = crank angle
- $R_{or}$ = rotating radius
- $S_{SE}$ = arc length of equal distance curve in a compression chamber
- $\phi_{\max}$ = involute end angle of equal distance curve
- $\phi_{\text{start}}$ = involute start angle of equal distance curve
- $d$ = distance between two sealing points of a compression chamber
- $\beta$ = relative angle between $d$ and $R_{or}$
- $\theta^*$ = discharge angle

### 4. VOLUME CALCULATION OF DISCHARGE CHAMBER

Much effort has been made in determining the volume of the discharge chamber in previous literature. The equations for volume calculation are found to be more complicated. In this study, equidistant curve of the inner or outer profile of scrolls is introduced into volume calculation, thus easier formulas could be obtained. Under circumstance that the discharge volume is constructed only by involute curves, volume of discharge chamber is determined based on geometric relation and is given by

$$V_3(\theta) = V_{EM2ABM,F} = 2hR_{or} \left[ \int_{\phi_{\text{start}}}^{\phi_{\max} - \theta} a\phi d\phi + \left( S_1 + \sin(\pi - \theta^*) \right) \right] \quad (0 \leq \theta \leq \phi_{\text{start}}) \quad (7)$$

When discharge chamber is formed by both involutes and modification curves, as demonstrated in figure 3, the volume of which will change to

$$V_3(\theta) = V_{CABD} = hR_{or} \left[ S_1(\theta) + S_2(\theta) + l \cdot \sin(\pi - \theta^*) \right] \quad (\phi_{\text{start}} \leq \theta \leq \theta^*) \quad (8)$$

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Finally, when crank angle reaches \( \theta \), the previous two compression chambers merge into the maximum discharge chamber, as shown in figure 4, and hereafter the volume of it could be expressed as

\[
V_1'(\theta) = V_{EM,ABM,F}
\]

\[
= 2hR_{\theta} \int_{\theta_{min}}^{\theta_{max}} (\psi - \phi) d\phi + \frac{S_1 + S_2}{2} - \frac{a + l}{2} (\theta' \leq \theta \leq 360^\circ) \tag{9}
\]

Where
- \( S_1 \) = arc length of equidistance curve of whole connecting curve in a discharge chamber
- \( S_2 \) = arc length of equidistance curve of the modification curve in a discharge chamber
- \( S_1(\theta) \) = arc length of equidistance curve of partial connecting curve in a discharge chamber
- \( S_2(\theta) \) = arc length of equidistance curve of partial modification curve in a discharge chamber
- \( l_{O,OD} \) = distance of curvature centers of the two sealing points in a discharge chamber
- \( l \) = length of equidistance curve of straight line in a discharge chamber

By the choice of different parameter values, the profile design can usually result in various modification cases, and the above derivations can be simplified as well. For example, if \( \delta_1 + \delta_2 = 0 \) is chosen, and \( \delta_1 = 0 \) is further adopted, then \( S_1 = S_2 \) as well as \( l = 0 \) are obtained. Under this condition, the central profile of scroll is consisted solely of arcs. Otherwise, if \( \delta_1 = -\delta_2 \), then \( S_1 = S_2 \) and \( l \neq 0 \) could be derived, in which the modified profile of both scrolls are constructed by both symmetric arcs and straight lines. Under condition of \( \delta_1 + \delta_2 \neq 0 \), \( \delta_1 = \delta_2 \) will result in \( S_1 \neq S_2 \) and \( l = 0 \), in which the central profile of scroll is consisted of asymmetric arcs only. When \( \delta_1 \neq \delta_2 \), the results are \( S_1 \neq S_2 \) and \( l \neq 0 \), which means the central profiles of both scrolls are constructed by both asymmetric arcs and straight lines.

5. EFFECTS OF MODIFICATION PARAMETERS ON GEOMETRY PROPERTY

Volume of the first compression chamber, which is constructed solely by involute curves, has nothing to do with modification parameters. Comparatively, volumes of second chamber and discharge chamber appear to be very sensitive to the change of parameter values. Several numerical examples are performed and some results will be shown herein. In carrying out the computer simulation, the following basic parameters are given: \( a = 3mm \), \( t = 5.4mm \), \( N = 2.5 \), \( h = 40mm \), \( \theta^\prime = 269^\circ \).

To study the influence of \( \alpha \) on the scroll shape design, figure 5 is constructed, in which all of the other parametric values are the same as before. As indicated in the figure, the central portion of the scroll wrap becomes thicker as \( \alpha \) becomes larger. In this present analysis, when \( \alpha \) takes value of 90° and 45° respectively, compared to \( \alpha = 135° \), \( \theta^\prime \) correspondingly increased by 39° and 73°, while augmentation of 14.6% and 31.7% are found in volume ratio. The variable \( \alpha \) is therefore a very important parameter in the optimal design of scroll wraps.

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With all other parametric values except that of $\delta$ fixed as before, figure 6 shows the influence of $\delta$ on the central part of the scroll wrap. As can be seen from the figure, a change of the value of $\delta$ results in a change of wrap thickness. The top of scroll wrap becomes thinner as $\delta$ becomes smaller. Figure 10 and figure 12 shows the influence of $\delta$ on the second compression chamber. When $\delta_1 = -\delta_2$, the two compression chambers are not identical, the increased amount of volume of chamber $V_{21}$ is the same as the decreased amount of chamber $V_{22}$. It can be seen in figure 13 that the change of $\delta$ has no influence on the volume of discharge chamber. It indicates that the average volume ratio is kept the same whatever the value of $\delta$ is. Under condition of $\delta_1 = \delta_2$, the larger the value of $\delta$, the smaller the discharge angle $\theta^*$, the larger the maximum volume of discharge chamber, as shown in figure 11, and the smaller the volume ratio. For example, when $\delta$ takes value of 1 mm and 2 mm respectively, compared to $\delta = 0$, the $\theta^*$ decreased by 41° and 53°, while the volume ratio is 18.8% and 23.1% less, correspondingly.

Figure 7 shows the influence of $n$ on the top scroll profile. Again, in obtaining this figure, parametric values except that of $n$ are kept the same as those originally selected. As can be seen, the central part of scroll becomes thicker as $n$ becomes larger, thus lead to a stronger scroll wrap. The influence of $n$ on the second compression chamber and discharge chamber are shown in figure 14 and figure 15, respectively. The simulation results obtained from the change of $n$ are compared in the figures. As demonstrated, when $n$ takes the value of 0.5, compared to $n = 0$, the augmentation in $\theta^*$ is found to be 19°, and the volume ratio increased by 9.9%. However, when $n = 3$, compared to $n = 0$, a reduction of 24° in $\theta^*$ and 11.4% less of volume ratio are found.
6. CONCLUSIONS

Because profile affects the scroll shape and mechanical analysis of a scroll compressor, trigonometric-curve modification of top profile has been carefully studied and presented in this paper. Both geometric derivation and numerical simulation have been carried out. Based on the above work, the following conclusions can be drawn:

- The wrap thickness at the central portion of scroll can be change by the control of modification parameters. A thicker scroll wrap in general results in a stronger scroll wrap and a smaller volume ratio.
- Geometry properties such as volume of compression chamber and that of discharge chamber are also affected by modification parameters.
• Under the condition of n=0, trigonometric-curve will simplify and change into circular-arc-curve. From both mechanical and manufacturing viewpoints, this case is being widely used in many kinds of scroll fluid machines.
• With the development of NC machines, it’s possible that other trigonometric-curve modification cases could be applied in practical use for special design considerations.

The modification parameters as presented will play an important role in optimizing the design and improving performance of the whole scroll set. The derivations of this study can also be used in the profile design of scroll pumps, package air conditioners and car conditioners.

REFERENCES