2000

Sound Transmission Through Cylindrical Shell of Hermetic Compressors

J. H. Lee
University of Cincinnati

J. Kim
University of Cincinnati

Follow this and additional works at: https://docs.lib.purdue.edu/icec
SOUND TRANSMISSION THROUGH CYLINDRICAL SHELL OF HERMETIC COMPRESSORS

Joon-Hyun Lee and J. Kim
Structural Dynamics Research Laboratory
Mechanical Engineering Department
University of Cincinnati
Cincinnati, OH 45221-0072

ABSTRACT

Sound transmission through a cylindrical shell is studied in the context of the transmission of the compressor noise. Classical thin shell theory is applied to describe the shell motion. An exact solution of the vibro-acoustic equations is obtained in a form of series solution. This solution is combined with the solution from the one-dimensional wave propagation model that describes the compression wave. The analytical solutions are compared with the measured results, which show quite good agreements. The transmission losses through the compressor shell are compared for various shell geometries and refrigerants. The advantage of having an exact solution procedure is fully utilized for qualitative design parameter studies.

1. INTRODUCTION

A desire to develop a basic design tool for the hermetic compressor shell served as the practical motivation of this study. In the basic design stage, main design parameters of the wall such as the thickness and radius, material have to be determined before details of other parts of the compressor are known. The analysis method developed in this work is intended to serve as a quick, first-cut analysis tool for the compressor shell design. Two major simplifications are made to solve the problem exactly, such that the shell is infinitely long and the incoming wave is a plane wave from the outside space. The second simplification implies that a reciprocal of the real problem in the approximate sense is solved. Two system models that govern different types of wave traveling in the shells are used. The first model is to calculate the sound transmission due to the transverse bending waves in the shells induced by the acoustic waves. The second model is to calculate the sound transmission caused by the compression/rarefaction wave in the shell and the acoustic space in the longitudinal direction across the thin compressor shell. It is found that the TL from the 1D model is lower than the TL from the 2D model in the low frequency range, and vice versa in the high frequency range. Considering the definition of the TL (lower TL means higher transmitted sound), it is easily realized that the lower TL curve should be taken to represent the system response in the entire frequency range. Therefore, the low frequency portion of the combined TL curve is taken from the 1D model and the high frequency portion is from the 2D model. Sound transmissions through thin shell are measured experimentally, with which the theoretical solutions are compared. Thick end caps were used to close both ends of the cylinder to eliminate the effect of the sound radiated from the end plates. TLs are measured in the anechoic chamber with the sound source located inside the cylinder. This experimental system has two major discrepancies in that it has a finite length and it is not subjected to a true incident plane wave. These discrepancies are helpful in some respect in achieving the original purpose of the experiment, checking how well the idealized theoretical model predicts the actual system. The comparison shows that the results agree with each other surprisingly well.

Concerning related literatures, noise transmission through a cylindrical shell was studied by many researchers including Smith [2], White [3], Koval [4-5], Blaise et al [6] and Tang et al [7], for various purposes, notably for design study of aircraft.

2. ANALYTICAL SOLUTION PROCEDURE
2.1 Sound Transmission by Bending Waves in Shells

2.1.1 Formulation of the Governing Equations

Figure 1 illustrates the schematic of the problem studied in this work, in which a plane wave is incident to a cylindrical shell of infinite length with an incidence angle \( \gamma \). The incident wave is a plane wave which is traveling on a plane parallel to the x-z plane. Shown are the rays only on the x-z plane. The shell is defined in terms of the radius \( R_i \), wall thickness of \( h_i \), mass density \( \rho_i \), and Young's modulus \( E_i \). The acoustic media in the inside and the outside of the shell are defined by density and speed of sound: \( \{ \rho_s, c_s \} \) inside and \( \{ \rho_o, c_o \} \) outside. In response to the incident plane wave, a traveling bending wave will be induced in the shell and sound waves will be radiated to the external and internal spaces. All these responses are coupled to one another, therefore the corresponding equations have to be solved simultaneously.

In the external space, the wave equation becomes [8],

\[
c_1 \nabla^2 (p^I + p^R_i) + \frac{\partial^2 (p^I + p^R_i)}{\partial t^2} = 0
\]

where, \( p^I \) and \( p^R_i \) are the acoustic pressures of the incident and reflected waves, and \( \nabla^2 \) is the Laplacian operator in the cylindrical coordinate system. In the internal cavity, the acoustic pressure of the transmitted wave \( p^T_3 \) satisfies the acoustic wave equation [8]:

\[
c_3 \nabla^2 p^T_3 + \frac{\partial^2 p^T_3}{\partial t^2} = 0
\]

Letting \( \{ u_i^0, v_i^0, w_i^0 \} \) be the displacements of the shell at the neutral surface in the axial, circumferential, and radial directions respectively, Love’s equation [9], classical equations of motion of the thin shell, can be specialized for a cylindrical shell as follows.

\[
L_1 \{ u_i^0, v_i^0, w_i^0 \} = \rho_i h_i \ddot{u}_i^0
\]

\[
L_2 \{ u_i^0, v_i^0, w_i^0 \} = \rho_i h_i \ddot{v}_i^0
\]

\[
L_3 \{ u_i^0, v_i^0, w_i^0 \} + (p^I + p^R_i) - p^T_3 = \rho_i h_i \ddot{w}_i^0
\]

Equation (3), (4) and (5) are equations of motion in the axial, radial and circumferential directions respectively. In the equations, \( \rho_i \) and \( h_i \) are the density of the shell material and the thickness of the shell, respectively. The differential operators \( L_1, L_2 \) and \( L_3 \) are found in reference [9].

On the internal and external shell surfaces, the particle velocities of the acoustic media have to be equal to the normal velocity of the shell, which results in the following equations.

\[
\frac{\partial (p^I + p^R_i)}{\partial r} = -\rho_i \frac{\partial^2 w_i^0}{\partial t^2} \quad \text{at } r = R_i
\]

\[
\frac{\partial p^T_3}{\partial r} = -\rho_3 \frac{\partial^2 w_i^0}{\partial t^2} \quad \text{at } r = R_i
\]

2.1.2 Solution Procedure

The cylindrical coordinate description of the harmonic plane wave \( p^I \) incident from outside to the direction shown in Figure 1 can be expressed as [10]:
where \( p_0 \) is the amplitude of the incident wave, \( j = \sqrt{-1} \), \( n=0,1,2,3, \ldots \), \( J_n \) is the Bessel function of the first kind of order \( n \); \( \omega \) is the angular frequency, \( c_n=1 \) for \( n = 0 \) and \( c_n=2 \) for \( n=1,2,3, \ldots \), \( k_{1z} = k_1 \sin(y_1) \), \( k_{1r} = k_1 \cos(y_1) \), and \( k_1 = \frac{\omega}{c_1} \). It is easily seen that \( k_{1r} = \sqrt{k_1^2 - k_{1z}^2} \).

The harmonic wave description in Equation (8) satisfies the wave equation automatically, and also the boundary conditions in the \( \theta \) and \( z \) directions by the periodicity and traveling wave expression. The boundary conditions in the radial direction will be satisfied after the reflected wave is added to Equation (8) by enforcing Equations (6) and (7). The waves radiated from the shell to the outside and into the cavity, \( p_1^R \) and \( p_3^T \), can be represented as:

\[
p_1^R(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{1n}^R H_n^2(k_{1r}, r) \cos[n \theta] e^{j(\omega t - k_{1z} z)}
\]

\[\text{Equation (9)}\]

\[
p_3^T(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{3n}^R H_n^1(k_{3r}, r) \cos[n \theta] e^{j(\omega t - k_{3z} z)}
\]

\[\text{Equation (10)}\]

where, \( H_n^1 \) and \( H_n^2 \) are Hankel functions of the first and second kind of order \( n \), respectively. The former represents the incoming wave and the second the outgoing wave. Both Equations (9) and (10) satisfy the wave equation and boundary conditions to the circumferential direction (by the periodic form of the solution) and to the axial direction (by the traveling wave form in the \( z \) direction) automatically. In Equation (10), it is known that \( k_{3z} = \sqrt{k_1^2 - k_{1z}^2} \). Because the traveling waves in the acoustic media and in the shell are driven by the incident traveling wave, the wave numbers in the \( z \) direction should match throughout the system, therefore \( k_{3z} = k_{1z} \). Hence, the three components of the shell displacement can be expressed as:

\[
w_1^0(z, \theta, t) = \sum_{n=0}^{\infty} w_{1n}^0 \cos[n \theta] e^{j(\omega t - k_{1z} z)}
\]

\[\text{Equation (11)}\]

\[
u_1^0(z, \theta, t) = \sum_{n=0}^{\infty} v_{1n}^0 \cos[n \theta] e^{j(\omega t - k_{1z} z)}
\]

\[\text{Equation (12)}\]

\[
u_1^0(z, \theta, t) = \sum_{n=0}^{\infty} v_{1n}^0 \sin[n \theta] e^{j(\omega t - k_{1z} z)}
\]

\[\text{Equation (13)}\]

where, \( n=1,2,3, \ldots \) are the circumferential mode numbers. Again, Equations (11) to (12) satisfy boundary conditions in the circumferential and axial directions automatically. Substituting Equations (8) - (13) into the three shell equations (Equations (3), (4) and (5)) results in three equations of motion. Additionally, substituting the same equations into the boundary conditions in the radial directions, Equations (6) and (7), results in two more equations. These five equations involve with six variables: the amplitudes of the outgoing and incoming waves in the exterior cavity, the incident wave in the interior cavity, and three displacements of the shell structure. Therefore, five variables can be represented in terms of the pressure amplitude of the incoming wave from the available five equations. The ratio of the amplitudes of the input and transmitted waves obtained this way allows the transmission loss to be obtained.

**2.1.3 Solution in Terms of the Transmission Loss (TL)**

It is convenient to represent the solution in terms of the transmission loss (TL) for the design purpose. TL can be defined as the ratio of the incoming and transmitted sound powers per unit length of the cylinder.
\[ TL = 10 \log_{10} \left( W_I / \sum_{n=0}^{\infty} W_n^T \right) \]  

(14)

where, \( W_n^T \) the transmitted power flow per unit length of the shell is,

\[ W_n^T = \frac{1}{2} \text{Re} \left\{ p_{3,n}^T \cdot \frac{\partial}{\partial \theta} \left( w_n^0 \right)^* \right\} \times \int_0^{2\pi} \cos^2 n \theta \cdot R_i d \theta \]  

(15)

where \( \text{Re} \{ \cdot \} \) and the superscript * represent the real part and the complex conjugate of the argument, respectively. Substitution of Equations (10) and (11) for \( p_J^3 \) and \( w_n^0 \) into above Equation (15) yields an expression for the components of \( W^T \)

\[ W_n^T = \frac{\pi R_i}{2E_n} \times \text{Re} \left\{ p_{3,n}^T \times H_n^1(k_3, R_i) \left( j \omega w_n^0 \right)^* \right\} \]  

(16)

\( W_I \), the incident power flow per unit length of the shell is,

\[ W_I = \frac{\cos \gamma_1}{\rho_1 c_1} \left( \frac{2}{R_i} \right)^2 \]  

(17)

Finally, the transmission loss can be obtained by substituting Equations (16) and (17) into (14)

\[ TL = -10 \log_{10} \sum_{n=1}^{\infty} \text{Re} \left\{ \frac{\pi R_i}{2E_n} \times \frac{H_n^1(k_3, R_i) \times \left( j \omega w_n^0 \right)^*}{4 \epsilon_n \cos \gamma_1} \right\} \times \rho_1 c_1 \pi \]  

(18)

where, \( \epsilon_n = 1 \) for \( n = 0 \) and \( \epsilon_n = 2 \) for \( n = 1, 2, 3, \ldots \).

The TL curves of shell calculated in this manner are shown in Figure 2. As shown in Figure 1, the geometry of the shell used in the calculation are 50 mm radius and 3 mm thickness for the shell. The material of the shells is steel, therefore the Young’s modulus \( E = 1.9 \times 10^{11} \) Pa, and the Poisson’s ratio \( \mu = 0.3 \). The temperature of the interior cavity is taken as 40°C (6 kg/cm² (Abs.)), which is a typical operating condition of the low-pressure type compressor. The incident angle of 45° is used and R134a as a refrigerant is adopted. Parameters used in the calculation to obtain Figure 2 are listed in Table 2 for the compressor shell. The condition in Table 2 is referred as the compressor condition.

### 2.2 Sound Transmission by Compression Waves in Shells

The compression-rarefaction type waves in shells induced by the acoustic wave incoming in the perpendicular direction are considered. In this case, the problem is idealized as a one-dimensional wave propagation problem as shown in Figure 3 by assuming that the shells are locally reacting, behaving like a fluid. The four-pole method can be used conveniently for this type of analysis [11]. The four-pole equation between the system’s input and output variables are:

\[
\begin{bmatrix}
    p_A \\
    v_A
\end{bmatrix} = 
\begin{bmatrix}
    \cos k_A l_A & j r_A \sin k_A l_A \\
    -j \sin k_A l_A & \cos k_A l_A
\end{bmatrix} \begin{bmatrix}
    1 \\
    1
\end{bmatrix} \begin{bmatrix}
    p_B \\
    r_B
\end{bmatrix}
\]  

(19)

From Equation (19), the incident pressure \( p_i \) and the reflective pressure \( p_r \) can be separated, and the reflection coefficient \( R \) can be obtained:

\[ R = \frac{p_r}{p_i} = \frac{\cos k_A l_A + j r_A \sin k_A l_A - r_A \cos k_A l_A - r_A \cos k_A l_A}{r_B \cos k_A l_A + j r_A \sin k_A l_A + r_A \cos k_A l_A + r_A \cos k_A l_A} \]  

(20)

Because the cross-sectional area of the input and output side of the system are the same in this case, the power transmission coefficient becomes:
Finally the transmission loss can be estimated from the power transmission coefficient:

\[ TL(dB) = 10 \log_{10} \frac{1}{T_r} \]  

The TL curves calculated by this 1-D model are shown in Figure 4 for the same system at the same condition used to obtain Figure 4.

2.3 Combined Solutions

The TL curves in Figures 2 and 4 can be combined to represent the system behavior in the entire frequency range. Considering that lower TL means higher transmitted noise, the rule to combine two TL curves should be picking up the lower TL at each frequency. Figure 5 shows the TL curves of the shell obtained by combining the curves in Figures 2 and 4. These combined TLs are used for comparisons of the analytical result and the experimental result.

3. EXPERIMENTAL COMPARISON

Figure 6 shows the experimental set-up to measure TLs of the shells experimentally. A pair of microphones shown in the figure is to measure the sound intensity on the external surface. One more pair of microphones is located at the same position on the internal surface of the shell to measure the sound intensity on the internal surface. The geometry of the shells used in the experiment is that, with reference to Figure 1, 0.1m radius and 1 mm thickness for the shell. The shells were made of the steel, which has the Young's modulus \( E = 1.9 \times 10^{11} \) Pa, and the Poisson’s ratio \( \mu = 0.3 \). The temperature was measured to be 20°C through the entire system at the measurement. Parameters corresponding to this condition are listed in Table 1 for the shell. The condition defined by Table 1 is referred as the test condition. The measurement was conducted using an internal sound source. The internal source was a 3-inch speaker. A white noise with the maximum frequency of 6,400 Hz was used to drive the sound sources. The frequency resolution of the measurement was 16 Hz. Figures 7 shows the measured and calculated TLs of the shell. The calculation was made using the test condition (see Table 2 for parameters). The incident angle of 70° was used in the calculation, which was estimated based on the relative location between the sound source and the intensity probes. As the figures indicate, the measured TL curves show quite good agreements to the calculated TL curves.

4. PARAMETER STUDIES

Important design parameters are studied analytically to see their effects on the TL of the shell for different refrigerants and shell geometries. Refrigerants to be used, the simulations are made at the compressor condition listed in Table 2. The material of the shells is steel, therefore the Young’s modulus \( E = 1.9 \times 10^{11} \) Pa, and the Poisson’s ratio \( \mu = 0.3 \).

4.1 Effects of Thickness

TLs calculated for three different thickness of the shell (\( R=50 \) mm) are plotted in Figure 8 when R134a was used as a refrigerant. For other refrigerants, which are R12, R22, R410A, and R407C, TLs can be easily calculated. TLs with three thicknesses of 2.0 mm, 3.0 mm and 4.0 mm are considered. As it is seen in Figure 8, changing the thickness has a broadband effect on TL over the entire range of the frequency. In general, TL increases 6 dB as the thickness doubles, which is well anticipated. In a practical situation, the shell will have to be thick only as necessary because of the weight constraint. The type of analysis in this work will be very useful in such a situation. For example, if the noise level in the internal cavity and the
target noise level of the compressor are known, a proper thickness can be adopted easily by utilizing the analysis method reported here.

4.2 Effects of Radius

TLs calculated for three different radius of the shell (t=3 mm) are shown in Figure 9 when R410A was utilized as a refrigerant. TLs with three radiusses of 25 mm, 50 mm and 75 mm are considered. As shown in Figure 9, the smaller the radius makes the TL higher, which is caused by the curvature effect of the shell on its stiffness. Combined with the large surface area, the shell of a large size hermetic compressor will become an effective radiator, therefore the structure of such a compressor will have to be designed with more care.

5. CONCLUSION

An exact solution procedure is developed to study the sound transmission through a cylindrical compressor shell. The solutions are obtained from two models, the first of which describes the sound transmission caused by the bending waves traveling in the shell and the second describes the sound transmission by the compression waves traveling across the layer of the compressor shell. For the bending wave solution, two acoustic wave equations and one shell vibration equations are solved simultaneously, which provides a series solution whose convergence is confirmed To solve the compression wave solution, the four pole method is used. Both solutions are represented in terms of the transmission loss (TL), which are then combined into a single TL curve. The idea of combining the solutions is used for the first time in this work. The analytical solutions obtained as such are compared with the measured TLs, which shows that the analytical results agree with the measurement quite well. The effects of important compressor design parameters such as the thickness and the radius with respect to two alternatives (HFC) refrigerants (e.g., R134a and, R410A) are studied using the analytical solutions.

6. REFERENCES

Figure 1. Schematic description of the problem: 2-D model

Figure 2. TLs calculated from 2-D model at compressor condition

Figure 3. Schematic description of the problem: 1-D model

Figure 4. TLs calculated from 1-D model at compressor

Figure 5. Combined TLs from Figures 4 and 6
Figure 6. Experimental set-up of TL measurement

Figure 7. Calculated TL compared with measured TL using inside source, shell (R=0.1 m, h=1.0mm)

Figure 8. TLs of the shell (R=50 mm) with respect to thickness for R134a

Figure 9. TLs of the shell (t=3 mm) with respect to radius for R410A

Table 1. Parameters to calculate TLs of the shell at the test condition

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Outside Air</th>
<th>Shell</th>
<th>Inside Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>20</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>ρ₁</td>
<td>ρ₂, 1.21</td>
<td>ρ₃, 7,750</td>
</tr>
<tr>
<td>Speed of Sound (m/s)</td>
<td>c₁, 343</td>
<td>c₂, 6,100</td>
<td>c₃, 343</td>
</tr>
</tbody>
</table>

Table 2. Refrigerants and parameters to calculate TLs of the shell at the compressor condition

<table>
<thead>
<tr>
<th>Refrigerants</th>
<th>Density (kg/m³)</th>
<th>Sound Speed (m/s)</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>R134a</td>
<td>25.82</td>
<td>153.7</td>
<td>Alternative refrigerant for R12</td>
</tr>
<tr>
<td>R410A</td>
<td>17.48</td>
<td>195.0</td>
<td>Alternative refrigerant for R22</td>
</tr>
</tbody>
</table>

(The compressor is surrounded with ambient air (20°C, 1 kgf/cm²))