2000

Direct Identification of Damping Parameters From FRF and Its Application to Compressor Engineering

J. H. Lee  
*University of Cincinnati*

J. Kim  
*University of Cincinnati*

Follow this and additional works at: [https://docs.lib.purdue.edu/icec](https://docs.lib.purdue.edu/icec)

[https://docs.lib.purdue.edu/icec/1474](https://docs.lib.purdue.edu/icec/1474)
DIRECT IDENTIFICATION OF DAMPING PARAMETERS FROM FRF AND ITS APPLICATION TO COMPRESSOR ENGINEERING

Joon-Hyun Lee and J. Kim
Structural Dynamics Research Laboratory
Mechanical Engineering Department
University of Cincinnati
Cincinnati, OH 45221-0072

ABSTRACT

A technique to identify damping parameters directly from the frequency response functions (FRF) in matrix forms has been developed. Furthermore, the technique identifies the viscous damping and structural damping coefficients separately. The technique provides more accurate damping values because it does not rely on the information of the natural modes as in the conventional method. The damping effect is identified as two matrices, which represent the viscous damping and the internal damping effects. Therefore, the best use of the technique is considered to build a hybrid finite element method (FEM) model. For example, the mass and stiffness matrices of a hermetic compressor can be formulated from the FEM program, while the damping matrices, which should include the effects of the internal components such as lubrication oil, are identified by the proposed method. Such a hybrid model will be able to predict the characteristics of complicated systems as a hermetic compressor much more accurately.

1. INTRODUCTION

Damping parameters have been typically of relatively minor concern to test engineers compared to other modal parameters. Often damping characteristics are identified as modal damping ratios, which are combined effects of many different damping mechanisms and lacking spatial information. Even in some case of analytic simulations, the damping effect is considered simply by adding small arbitrarily chosen damping ratios to modal equations. Such a practice does not cause any serious problem for a lightly damped, non-rotating system. However, a heavily damped and/or rotating system including hermetic compressors are different cases. For example, it is well known fact that the internal viscous damping destabilizes a rotating system in the high rotational speed range. In such systems therefore, it is necessary to accurately identify and distinguish different damping mechanisms. The other feature of the method, identifying the damping in matrix forms is an advantage in an important potential application: hybrid modeling of a mechanical system. In the hybrid modeling, the mass and stiffness matrices are formulated by the finite element method (FEM) and the damping matrices are formulated experimentally, which are then combined to obtain the system equation.

Most techniques that have been proposed for damping matrices identification use FRFs indirectly, then damping matrices are formulated using these modal parameters by Pilkey and Inman [2], Lancaster [3], P. Ebersbach and H. Irretier [4] and R. J. Alleman and D. L. Brown [5]. Chen and Tsuei [6] studied the effect of parameter identification on modeling of viscous and structural damping. Authors [1] generalized Tsuei's work and developed the hybrid model. The main theoretical scheme of the method developed in this work is achieved by generalizing and extending the method proposed by Chen and Tsuei.

2. DEVELOPMENT OF THE THEORY
2.1 Theoretical Background

The equation of motion of a dynamic system of \( n \) degrees of freedom (DOF) is represented:

\[
M\ddot{x}(t) + C\dot{x}(t) + (J + D + K)x(t) = f(t)
\]

(1)

where, \( M, C, D \) and \( K \) are \( nxn \) matrices representing the mass, viscous damping, structural damping and stiffness of the system respectively, \( j = \sqrt{-1} \), and \( x(t) \) and \( f(t) \) are \( n \times 1 \) vectors representing the displacements and the applied forces. For a harmonic excitation, \( f(t) = F(\omega)e^{j\omega t} \) and \( x(t) = X(\omega)e^{j\omega t} \), therefore Equation (1) becomes:

\[
[K - M\omega^2]X(\omega) + j\omega C X(\omega) + jDX(\omega) = F(\omega)
\]

(2)

Identifying the normal FRF \( H_N(\omega) \) such as:

\[
H_N(\omega) = \begin{bmatrix} \omega^2 - M \end{bmatrix}
\]

(3)

Equation (2) is written:

\[
[H_N(\omega)]^{-1}X(\omega) + (j\omega C + jD)X(\omega) = F(\omega)
\]

(4)

Pre-multiplying Equation (4) by \( H_N(\omega) \) and if we define a real matrix \( G(\omega) \) as Chen and Tsuei [6] did:

\[
G(\omega) = H_N(\omega)(\omega C + D)
\]

(5)

Then, Equation (4) becomes:

\[
[I + jG(\omega)]X(\omega) = H_N(\omega)F(\omega)
\]

(6)

where, \( I \) is an identity matrix. The displacement vector \( X(\omega) \) in Equation (7) is also related to the input force and the complex FRF \( H^F(\omega) \) as:

\[
X(\omega) = H^C_R(\omega)F(\omega) = \left[H^C_R(\omega) + jH^C_I(\omega)\right]F(\omega)
\]

(8)

where, \( H^C_R(\omega) \) and \( H^C_I(\omega) \) represent the real and imaginary part of \( H^C(\omega) \). The complex FRF \( H^F(\omega) \) is assumed to be available from the measurement, therefore is the known information. Substituting Equation (8) into Equation (7), one obtains:

\[
(I + jG(\omega))(H^C_R(\omega) + jH^C_I(\omega)) = H^N
\]

(9)

From Equation (9), \( G(\omega) \) and the normal FRF \( H^N(\omega) \) are obtained in terms of the known function \( H^F(\omega) \):

\[
G(\omega) = -H^C_I(\omega)H^C_R(\omega)^{-1}
\]

(10)

\[
H^N(\omega) = H^C_R(\omega) + H^C_I(\omega)H^C_R(\omega)^{-1}H^C_I(\omega)
\]

(11)

Because \( G(\omega) \) and \( H^N(\omega) \) are known now, Equation (5) can be written in the following form.

\[
[H^N(\omega)]\begin{bmatrix} C \\ D \end{bmatrix} = G(\omega)
\]

(12)

Equation (12) can be used to solve for the internal and external damping matrices \( C \) and \( D \), which are the objectives of the identification. The procedure can be summarized as follows.

- Obtain the complex FRF matrix \( H^F(\omega) \) from measurement.
- Find the normal FRF matrix \( H^N(\omega) \) from Equation (11)
- Find \( G(\omega) \) from Equation (10)
- Find \( C \) and \( D \) from Equation (12)

2.2 Experimental Damping Identification Procedure
Equation (12) can be applied at as many frequencies as necessary to make the equation over-determined. If \( k \) different frequencies are used, the equation becomes:

\[
\begin{bmatrix}
C_{mn} \\
D_{mn}
\end{bmatrix}_{2n \times n} = \begin{bmatrix}
\omega_1[H_N(\omega_1)]_{n \times n} & [H_N(\omega_1)]_{n \times n} & \ldots & [G(\omega_1)]_{n \times n} \\
\omega_2[H_N(\omega_2)]_{n \times n} & [H_N(\omega_2)]_{n \times n} & \ldots & [G(\omega_2)]_{n \times n} \\
\vdots & \vdots & \ddots & \vdots \\
\omega_k[H_N(\omega_k)]_{n \times n} & [H_N(\omega_k)]_{n \times n} & \ldots & [G(\omega_k)]_{n \times n}
\end{bmatrix}_{2n \times knn}
\]

where, \( + \) means the pseudo-inverse of the matrix. The measured FRF matrix \( H^N(\omega) \) is the necessary and sufficient information to obtain the normal FRF \( H_N(\omega) \) (see Equation (11)), therefore the damping matrices \( [C] \) and \( [D] \) (see Equation (13)). The dimension of the complex FRF matrix \( H^F(\omega) \), therefore the dimension of damping matrices being identified, is determined by DOFs of the experimental model.

The method can be applied to make a FEM / experimental hybrid model, in which the mass and stiffness matrices are obtained from the FEM formulation and the damping matrices are obtained experimentally by the method developed. In such a case, the FEM model usually has a much finer mesh than the experimental model. Moreover, some DOFs of the FEM model, such as the rotational or in-plane DOFs, cannot be measured in the experiment. Thus, usually it will be necessary to expand the measured damping matrices to match with the FEM mass and stiffness matrices. Therefore, it will be convenient to choose the nodal points of the experimental model as a sub-set of those of the FEM model.

3. THEORETICAL VALIDATION OF THE PROCEDURE

The 3 DOF system shown in Figure 1 is defined by the lumped masses \( m_1, m_2 \) and \( m_3 \) of 10 kg, 14 kg and 12 kg, and the spring constants \( k_1, k_2, \) and \( k_3 \) of 2,000 N/m, 3,000 N/m and 2,500 N/m, the viscous damping coefficients \( c_1, c_2, \) and \( c_3 \) of 2 N.s/m, 3 N.s/m and 2.5 N.s/m, and the structural damping coefficients \( d_1, d_2 \) and \( d_3 \) of 100 N/m, 150 N/m and 200 N/m respectively. The elements of the mass, viscous damping, stiffness and structural damping matrices of the system in Figure 1 are calculated as in Table 1. It is noted that the mass and stiffness matrices are only for reference, and are not necessary in the damping identification procedure. Because the system has 3 DOFs, 9 FRFs are calculated at each frequency, forming a \( 3 \times 3 \) FRF matrix of a function of frequency. Then, it is assumed that the system parameters are unknown, while these FRFs are known from the measurement. The procedure developed in the previous section is applied to find the damping matrices from these FRFs. Table 2 shows the viscous and structural damping matrices identified by the proposed method if the calculated FRFs are used without adding any noise, which are measured FRFs with zero noise. Two cases are shown in the table. Case A is the identification result obtained by modeling only the viscous damping, and case B is the result when both the viscous and structural damping are modeled. Matrices marked as correct matrices in the table are the exact answers: theoretically formulated damping matrices. Case B shows that the original damping matrices are identified exactly, which means that the identification algorithm is working validly in an ideal condition.

In case A, the identified viscous damping matrix is different compared to the original matrix, because in the matrix should include the effect of the structural damping also. Figure 2 compares two re-constructed FRFs corresponding to case A (identified using only the viscous matrix) and case B (identified using both the viscous and structural damping matrices). As it is seen, the two are virtually indistinguishable, which justifies the common practice of using the concept of the equivalent viscous damping to represent the combined effect of all damping mechanisms in lightly damped systems.
4. EXPERIMENTAL WORKS

The damping identification procedure proposed in reference [1] and the hybrid model developed in this work was applied to a uniform beam with its ends clamped shown in Figure 3. The modal test was conducted by the multi reference impact-testing (MRIT) scheme [7] using four acceleration outputs and four impact locations, which results in 16 measured FRFs. Thus, the mass, stiffness and damping matrices are identified as 4 x 4 matrices. Each impact position is 54 mm apart from each other, which defines the mesh size in the experimental model. The physical properties and dimensions of the beam are listed in Table 3.

4.1 Single-Reed Beam Without Modification

The test was done using a single-reed, uniform width beam. Figure 4 shows FRF H11. It shows that the beam has a resonance frequency around 385 Hz, which is very close to the lowest resonance frequency calculated using the Euler beam theory.

To build a hybrid model for the single-reed beam, the system matrices identification procedure developed should be applied to single-reed beam to identify the damping matrices [C], [D] as accurate as possible. When the procedure proposed is applied, the mass and stiffness matrices [M], [K] can be also identified. The 4by4 extracted viscous and structural damping matrix [C] and [D] from the measured FRFs should be expanded in 8by8 damping matrices [C] and [D] by setting all the element of the damping matrices corresponding to the angular displacement components equal to zeros, to match the size of damping matrices to that of mass and stiffness matrices from the finite element method. Matrix (14) and (15) is the identified 4by4 damping matrices [C] and [D].

\[
[C] = 10^{-3} \begin{bmatrix}
0.0726 & -0.4072 & 0.6762 & -1.2475 \\
-0.2001 & 1.0676 & -1.6480 & 2.5028 \\
0.5301 & -1.9084 & 2.7468 & -3.7253 \\
-0.7941 & 2.4228 & -3.3304 & 4.1677
\end{bmatrix}
\]

\[D] = 10^{-6} \begin{bmatrix}
5.0621 & -3.7777 & -2.6326 & -2.5812 \\
-4.4255 & 3.4104 & -2.3707 & 2.1225 \\
-3.8313 & -2.8487 & 1.8165 & -1.2880 \\
-3.592 & 2.4130 & -1.2766 & 0.5925
\end{bmatrix}
\]

Figure 5 shows the schematic diagram of FEM model for single-reed beam with uniform cross section. Mass and stiffness matrices [M], [K] for one beam element are not 2by2 matrix but 4by4 matrix, since each end of the beam element has two degree of freedom such as transverse direction displacement (e.g., x) and angular displacement (e.g., θ). Figure 6 shows reconstructed FRFs with mass & stiffness matrices [M], [K] from the finite element model and damping matrices [C], [D] from the measured FRFs by making use of the hybrid model. As can be seen in Figure 6, the reconstructed FRF of the single-reed beam agrees very well to the measured FRF.

3.2 Single-Reed Beam with a Viscous Damper Attached

The experiment was conducted after attaching a small viscous damper to the beam as shown in Figure 7. The damper was attached to a point located 162 mm from left end of the beam. Figure 8 shows the FRF at the driving point measured at this set-up. As can be seen in Figure 8, the clamped beam attached a viscous damper looks like to have more damping effects around the resonance frequencies when compared with the clamped beam without a viscous damper. Similarly, the identified 4by4 damping matrices [C] and [D] can be expressed in Equation (20) and (21).
Mass and stiffness matrices are the same as the single-reed beam, since the viscous damper does not change the system mass, stiffness matrices but will change mostly viscous damping matrix. Figure 9 shows reconstructed FRFs with \([M], [C], [D]\) and \([K]\) matrix from a hybrid model and the measured FRFs from modal testing. As can be seen in Figure 9, the reconstructed FRF of the single-reed beam with a viscous damper attached agrees very well to the measured FRFs.

### 3.3 Double-Reed Beam

Now the damping identification is applied to a beam with two reeds. One identical reed of 3 mm thickness is added to the beam as shown in Figure 10. Because of the friction between the two reeds during the vibration of the beam, it is expected to see higher structural damping in this system. The uniform double beam is shown in Figure 10. Figure 11 shows the measured FRF at the driving point. The extracted damping matrices \([C]\) and \([D]\) can be expressed in the expanded 8 by 8 matrices format (22) and (23).

\[
[C] = 1.0 \times 10^3 \begin{bmatrix}
3.4014 & -5.7481 & 7.2241 & -8.8093 \\
-2.7752 & 4.5065 & -4.9502 & 4.9044 \\
2.7704 & -4.2741 & 4.5241 & -4.4789 \\
-3.0387 & 5.3459 & -6.5707 & 7.8476
\end{bmatrix}
\]

(20)

\[
[D] = 1.0 \times 10^3 \begin{bmatrix}
0.2772 & -1.5135 & 3.4724 & -4.0610 \\
-2.7595 & 4.0655 & -5.0402 & 4.6330 \\
4.1196 & -5.0456 & 5.0029 & -4.2153 \\
-3.9724 & 4.1015 & -3.5095 & 2.9468
\end{bmatrix}
\]

(21)

Again, the result explains the change in the configuration change, increasing the structural damping more relative to the viscous damping. More deviations from the symmetry are believed to be caused by the possible non-linear nature of the double-reed beam problem.

### 6. CONCLUSION

A new method for experimental identification of damping characteristics of a general dynamic system is developed. The method works directly with measured frequency response functions (FRFs) of the system and identifies different types of the damping mechanism such as the internal structural damping and the external viscous damping in separate matrices. Theoretical validation and error study related to the measurement noise are conducted using a simple 3 DOF system. The study shows that the method will provide accurate results if the noise level contained in the measured FRFs is equal or lower than the equivalent damping ratio. The method is applied experimentally to a thin beam with three different configurations. Three distinct configurations used are a single reed, unmodified beam, the single reed beam with a viscous damper attached to, and a double reed beam by adding an identical reed to the single beam. The damping matrices identified reflect the different configurations very well. For example, the magnitudes of the elements of the internal structural damping matrix of the double reed beam increase significantly from those of the unmodified beam, and the magnitudes of the elements of the viscous damping matrix increase in much larger scale compared to those of the structural damping matrix when the
viscous damper is attached to the beam. One of best applications of the method is considered as the hybrid modeling of dynamic systems. In the application, the mass and stiffness matrices are formulated using FEM, and the damping matrices are formulated experimentally by the procedure developed in this work. This approach will be very useful to model hermetic compressors in which there are complicated, hard-to-model energy loss mechanisms.

7. REFERENCES


![Figure 1. 3 DOF lumped parameter system](image1)

![Figure 2. Comparisons of reconstructed FRFs of 3 DOF lumped mass system](image2)

![Figure 3. Experimental set-up of the single-reed beam](image3)
Figure 4. Measured FRF of the single-reed beam

Figure 5. FEM model for the single-reed beam

Figure 6. Comparison of the measured and the reconstructed FRF by the hybrid model of the single-reed beam

Figure 7. Experimental set-up of the single-reed beam attached to a viscous damper

Figure 8. Measured FRF of the single-reed beam attached to a viscous damper

Figure 9. Comparison of the measured and the reconstructed FRF by the hybrid model of the single-reed beam with a viscous damper attached
Figure 10. Experimental set-up of the double-reed beam

Figure 11. Measured FRF of the double-reed beam

Table 1. Matrices of the 3 DOF lumped parameter system

<table>
<thead>
<tr>
<th>Mass Matrix (kg)</th>
<th>Viscous Damping Matrix (N.s/m)</th>
<th>Stiffness Matrix (N/m)</th>
<th>Structural Damping Matrix (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[M]</td>
<td>[C]</td>
<td>[K]</td>
<td>[D]</td>
</tr>
<tr>
<td>10 0 0</td>
<td>5 -3 0</td>
<td>5000 -3000 0</td>
<td>250 -150 0</td>
</tr>
<tr>
<td>0 14 0</td>
<td>-3 5.5 -2.5</td>
<td>-3000 5500 -2500</td>
<td>-150 350 -200</td>
</tr>
<tr>
<td>0 0 12</td>
<td>0 -2.5 2.5</td>
<td>0 -2500 2500</td>
<td>0 -200 200</td>
</tr>
</tbody>
</table>

Table 2. Estimated damping matrices from FRFs with 0% noise

<table>
<thead>
<tr>
<th>Estimation Method</th>
<th>Viscous Damping [C]</th>
<th>Structural Damping [D]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Data</td>
<td>5 -3 0</td>
<td>250 -150 0</td>
</tr>
<tr>
<td>Case A</td>
<td>-3 5.5 -2.5</td>
<td>-150 350 -200</td>
</tr>
<tr>
<td>Viscous Damping</td>
<td>0 -2.5 2.5</td>
<td>0 -200 200</td>
</tr>
<tr>
<td>Model</td>
<td>15.6 -6.5 -1.1</td>
<td>- -</td>
</tr>
<tr>
<td>Case B</td>
<td>-7.0 20.2 -9.4</td>
<td>- -</td>
</tr>
<tr>
<td>Viscous &amp; Structural Damping Model</td>
<td>-0.7 -11.4 14.6</td>
<td>- -</td>
</tr>
<tr>
<td>Case B</td>
<td>5 -3 0</td>
<td>250 -150 0</td>
</tr>
<tr>
<td>Viscous &amp; Structural Damping Model</td>
<td>-3 5.5 2.5</td>
<td>-150 350 -200</td>
</tr>
<tr>
<td></td>
<td>0 -2.5 2.5</td>
<td>0 -200 200</td>
</tr>
</tbody>
</table>

Table 3. Physical properties and dimensions of the beam tested

<table>
<thead>
<tr>
<th>Material</th>
<th>Steel</th>
<th>Length (mm)</th>
<th>270.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus (Pa)</td>
<td>1.9×10^{11}</td>
<td>Width (mm)</td>
<td>19.0</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7,750</td>
<td>Height (mm)</td>
<td>6.0</td>
</tr>
</tbody>
</table>