Acceleration of the Apex Seals in a Wankel Rotary Compressor Including Manufacturing Process Variation in Location of the Seals

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Abstract: This paper presents equations for the radial and transverse components of the acceleration of the apex seal in the rotor of a Wankel rotary compressor for the "as designed" seal and for the actual seal location as a result of manufacturing process variation. Apex seals are typically designed to be the minimum allowable size to maintain contact with the housing at the maximum lean angle. This allows the weight of the seal to be a minimum which reduces the dynamic forces and the spring force that is necessary to maintain contact between the seal and the housing. The case of an undersized seal is presented in this paper to study the displacement, velocity and acceleration of the seal to maintain the seal in contact with the housing. The resulting forces acting on the seal can then be investigated to minimize seal size, increase efficiencies and ensure seal contact for parts "as made" rather than "as designed."

Nomenclature

O_1	=	center of the smaller pitch circle , O_2 = center of the larger pitch circle
r_1	=	radius of the smaller pitch circle attached to the housing
r_2	=	radius of the larger pitch circle attached to the rotor , r = radius of the generating pin
T	=	number of generating lobes on the larger pitch circle
T − 1	=	number of lobes on the smaller pitch circle
P	=	pole of the two pitch circles, coincident with the instant center
X,Y	=	Cartesian coordinates of the center of the fixed pitch circle
ϕ	=	input angle of the crank, measured counterclockwise with respect to the X-axis
C	=	center of the theoretical generating pin or arc
C_S	=	center of the generating arc on the seal (coincident with C when $\phi = 0^o$ or if the seal does not move relative to the rotor)
$\overline{O_2C}$	=	center line , r_C = radius of the epitrochoidal path of point C (= O_2C)
e	=	trochoid eccentricity , μ = trochoid ratio (= r_C / r_2)
H	=	external point of contact between the generating pin and generated shape
δ	=	displacement of the seal from C to C_S
B_{os}	=	point of intersection of the circular arc with the side of the seal
B_{op}	=	point on theoretical generating pin coincident with B_{os} at the input angle $\phi = 0^o$
β	=	lean angle (the angle between the center line $\overline{O_2C}$ and the line $\overline{PC}$)
β_{max}	=	maximum value of the lean angle , β_o = lean angle when the line $\overline{PH}$ intersects B_{os}
W_{SR}	=	minimum seal width for continous contact with the housing , W_S = actual seal width
X_S, Y_S	=	Cartesian coordinates of B_{os} when $\delta = 0$ (i.e., the seal is not moving relative to rotor)
X_{SH}, Y_{SH}	=	Cartesian coordinates of B_{os} when $\delta \neq 0$ (i.e., the seal is moving relative to rotor and is in contact with the housing)

1. Introduction

Analytical equations for the shape of the housing, the conjugate shape, the generated path curvature, the flow rate, and the compression ratio, are well documented (Hall, 1968; Colbourne, 1975; Beard, et al., 1991; and Beard and Pennock, 1992a). Leemhius and Soedel (1978) performed a velocity and acceleration analysis of the theoretical compressor using vector loops. Beard, et al. (1992b) extended the vector loop analysis and included the minimum radius of curvature of the generated shape but only for the theoretical pin. These papers have proved useful to the compressor community and can be used to design pumps, compressors and engines. However, the papers assume that the actual manufactured shapes are the same as the shapes given by mathematically defined functions. They also assume that the apex seal radius is the same as the "theoretical" generating pin radius and that the proper seal width to contact the housing is the same as the "theoretical" seal width. Process variations prevent the manufacture of a "perfect" seal, therefore, the actual seal must move relative to the rotor to maintain continuous contact with the housing. The equations that appear in the literature for the acceleration of the apex seal are based on the continuous contact of the apex seal with the generated shape. Pennock and Beard (1997) investigated the influence of the start-up torque on the contact forces between the seal and housing and seal and rotor. The analysis, however, was based on the assumption that the width of the apex seal is equal to or exceeds the minimum width of the seal for continuous contact. Utilitzing a seal tip with the same circular arc radius as the generating pin (i.e., a seal width equal to or greater than the actual seal width and centered on the line $\overline{O_2C}$) will ensure that the apex seal follows the housing. This occurs without requiring motion relative to the rotor to maintain contact with the housing. However, if the seal width is less than the minimum seal width for continuous contact or the seal is not centered on $\overline{O_2C}$, then the seal must move relative to the rotor to maintain contact with the housing. If one or both of these conditions occur, either by design or as a result of manufacturing process variation, the dynamic equations (Beard and Pennock, 1997) are not valid for the entire range of motion.

This paper assumes that the seal radius is the same as the "theoretical" pin radius and only examines a variation in the seal width. Then the paper investigates the condition when the width of the seal is less than the minimum seal width for continuous contact. In order to develop equations for a dynamic force analysis of an apex seal, the equations for the housing shape and the derivatives of the housing shape and the path of the center of the generating pin are required. The paper derives these equations, then presents a method to determine the motion of the seal relative to the rotor for the undersized seal. This motion may be a result of a design choice or from manufacturing process variation. A seal width equal to $2r/\mu$ is required to maintain contact with the housing throughout a rotation of the input crank. A seal width greater than $2r/\mu$ (resulting in an increased mass of the seal) can be used but an increased spring force is required to keep the seal in contact with the housing. Also, a seal width less than $2r/\mu$ can be used, but the theoretical center of the pin would have to move relative to the slot to ensure that the seal will maintain contact with the housing. For a given test case, the paper investigates the displacement of the seal, if the displacement is significant and if the dispacement is beneficial.

2. Kinematics of the Seal Motion

The Cartesian coordinates of point C (i.e., the center of the theoretical generating pin), see Figure 1, can be written as

$$X_C = -e\cos\phi + r_C\cos\frac{\phi}{T} \qquad \text{and} \qquad Y_C = -e\sin\phi + r_C\sin\frac{\phi}{T} \tag{1}$$

The Cartesian coordinates of point P (i.e., the point of contact between the two gears) can be written as

$$X_P = r_1\cos\phi \qquad \text{and} \qquad Y_P = r_1\sin\phi \tag{2}$$

The distance from the point P to point C is given by the relation

$$PC = \sqrt{r_2^2 + r_C^2 - 2\,r_2\,r_C\cos\left(\frac{T-1}{T}\right)\phi} \tag{3}$$

All points on the moving body are generating a path but point H which is the greatest distance from point P is the point of interest (i.e., it is the point which generates the housing). Using similar triangles, the Cartesian coordinates of the generating point can be written as

$$H_X = \left(\frac{PC+r}{PC}\right)(X_C - X_P) \qquad \text{and} \qquad H_Y = \left(\frac{PC+r}{PC}\right)(Y_C - Y_P) \tag{4}$$

The X and Y components of the vector from the center of the smaller pitch circle O_1 to the generated point H can be written as

$$X_H = X_P + H_X \qquad \text{and} \qquad Y_H = Y_P + H_Y \tag{5}$$

which gives the generated shape. Figure 1 also shows the point of contact H as it sweeps along the surface of the generating pin. The location of point H is on the line $\overline{PC}$ (which is collinear with the line $\overline{O_2P}$ when the input angle $\phi = 0^o$) and on the circular arc of the generating pin. Point H sweeps along the circular arc until the lean angle $\beta = \pm\beta_{max}$. The minimum width of the seal for continuous contact with the housing is given by the relation

$$W_{SR} = 2\,r\sin\beta_{max} = \frac{2\,r}{\mu} \tag{6}$$

Figure 2 shows the location of the point of contact on the "theoretical" pin as it follows the shape of the housing and the location of the seal. Point B_{os} is the edge of the actual seal and not the point on the "theoretical" pin. As the contact point on the seal passes a coincident point B_{op} on the theoretical pin (i.e., where B_{op} is coincident with H), the seal moves relative to the rotor until point B_{os} contacts the housing and seals the system. As the theoretical contact point passes B_{op} the seal continues to move until the theoretical contact point reaches the point defined by the angle β_{max} and the arc of the generating pin as shown in Figures 3 and 4.

Equations for the displacement, velocity and acceleration of the seal from point C to point C_S will now be presented. The displacement, denoted as δ, is along the center line providing the slot is parallel to the center line which is the usual configuration. Therefore, the point of contact between the seal and the housing B_{os} will lie on a line which is parallel to the center line. To obtain an equation for B_{os}, use the equation of the line on which this point lies; i.e.,

$$\hat{Y}_S = m\,\hat{X}_S + b \tag{7}$$

The slope $m = \tan(1/\mu)$ and the Y-intercept, obtained from the vector loop shown in Figure 4, is

$$b = -e \sin\phi + a_1 \sin\left(\frac{\phi}{T} \pm \frac{\pi}{2}\right) + a_2 \sin\frac{\phi}{T} \tag{8a}$$

where $a_1 = r \sin\beta_o$ and

$$a_2 = \frac{e\cos\phi - a_1 \cos\left(\frac{\phi}{T} \pm \frac{\pi}{2}\right)}{\cos\frac{\phi}{T}} \tag{8b}$$

The Cartesian coordinates of point B_{os} before the seal moves relative to the rotor to contact the housing at point H are

$$X_S = -e\cos\phi + r_C \cos\frac{\phi}{T} + r\cos\left(\frac{\phi}{T} \pm \beta_o\right) \quad \text{and} \quad Y_S = -e\sin\phi + r_C \sin\frac{\phi}{T} + r\sin\left(\frac{\phi}{T} \pm \beta_o\right) \tag{9}$$

The point of interest is where the line given by Eq. (7) intersects the housing at H' and the value of the input angle ϕ that is used to generate this point (denoted as ϕ'). Substituting Eqs. (8a) and (8b) into Eq. (7) and solving for ϕ' will give the value for $X_H(\phi')$. The displacement of the seal from point C to point C_S can be obtained from

$$\delta = \left[X_H(\phi') - X_S(\phi)\right] \Big/ \cos\frac{\phi}{T} \tag{10}$$

where $X_H(\phi')$ and $X_S(\phi)$ are given by Eqs. (7) and (9), respectively. Equations for the velocity and acceleration of the seal will now be presented in terms of the first-order and the second-order derivatives of the seal displacement. Differentiating Eqs. (1) with respect to the input angle ϕ, the first-order kinematic coefficients of point C are

$$f_{X_C} = e\sin\phi' - \frac{r_C}{T}\sin\left(\frac{\phi'}{T}\right) \quad \text{and} \quad f_{Y_C} = -e\cos\phi' + \frac{r_C}{T}\cos\frac{\phi'}{T} \tag{11}$$

The Cartesian coordinates of the point on the seal in contact with the housing are

$$X_{SH} = -e\cos\phi + r_C\cos\frac{\phi}{T} + r\cos\left(\frac{\phi}{T} \pm \beta_o\right) + \delta\cos\frac{\phi}{T} \tag{12a}$$

and

$$Y_{SH} = -e\sin\phi + r_C\sin\frac{\phi}{T} + r\sin\left(\frac{\phi}{T} \pm \beta_o\right) + \delta\sin\frac{\phi}{T} \tag{12b}$$

Then differentiating Eqs. (12), with respect to the input angle ϕ, gives

$$f_{X_{SH}} = e\sin\phi - \left(\frac{r_C + \delta}{T}\right)\sin\frac{\phi}{T} - \frac{r}{T}\sin\left(\frac{\phi}{T} \pm \beta_o\right) + \delta'\cos\frac{\phi}{T} \tag{13a}$$

and

$$f_{Y_{SH}} = -e\cos\phi + \left(\frac{r_C + \delta}{T}\right)\cos\frac{\phi}{T} + \frac{r}{T}\cos\left(\frac{\phi}{T} \pm \beta_o\right) + \delta'\sin\frac{\phi}{T} \tag{13b}$$

which are referred to as the first-order kinematic coefficients of the point on the seal in contact with the housing. Note that the slope of the path of point C and point H are the same, therefore

$$\frac{f_{X_C}}{f_{Y_C}} = \frac{f_{X_{SH}}}{f_{Y_{SH}}} \tag{14}$$

Substituting Eqs. (11), (13a) and (13b) into Eq. (14), and rearranging, gives

$$\delta' = r_2\left[\mu\sin\left(\phi - \frac{\phi'}{T}\right) - \sin(\phi - \phi')\right] + (r_2\mu + \delta)\left[\sin\left(\frac{\phi}{T} - \phi'\right) - \mu\sin\left(\frac{\phi - \phi'}{T}\right)\right]$$

$$+r\left[\sin\left(\frac{\phi}{T}-\phi'\pm\beta_o\right)-\mu\sin\left(\frac{\phi-\phi'}{T}\pm\beta_o\right)\right]\Big/ T\left[\cos\left(\frac{\phi}{T}-\phi'\right)-\mu\cos\left(\frac{\phi-\phi'}{T}\right)\right] \quad (15)$$

Since the values of ϕ and ϕ' are known and the displacement δ is given by Eq. (10) then the first-order derivative of the seal displacement can be obtained from Eq. (15).

Differentiating Eqs. (11) with respect to the input angle, the second-order kinematic coefficients of point C are

$$f'_{X_C}=e\cos\phi'-\frac{r_C}{T^2}\cos\frac{\phi'}{T} \qquad \text{and} \qquad f'_{Y_C}=e\sin\phi'-\frac{r_C}{T^2}\sin\frac{\phi'}{T} \quad (16)$$

Then differentiating Eqs. (13) with respect to the input angle, the second-order kinematic coefficients of the point on the seal in contact with the housing are

$$f'_{X_{SH}}=e\cos\phi-\left(\frac{r_C+\delta}{T^2}\right)\cos\frac{\phi}{T}-\frac{r}{T^2}\cos\left(\frac{\phi}{T}\pm\beta_o\right)-\frac{2\delta'}{T}\sin\frac{\phi}{T}+\delta''\cos\frac{\phi}{T} \quad (17a)$$

and $$f'_{Y_{SH}}=e\sin\phi-\left(\frac{r_C+\delta}{T^2}\right)\sin\frac{\phi}{T}-\frac{r}{T^2}\sin\left(\frac{\phi}{T}\pm\beta_o\right)+\frac{2\delta'}{T}\cos\frac{\phi}{T}+\delta''\sin\frac{\phi}{T} \quad (17b)$$

A numerical differentiation method is used in this paper to approximate the second-order derivative of the seal displacement from C to C_S. The radial and transverse components of the acceleration of the center of the pin (or the moveable seal) can be written, respectively, as

$$A_C^{radial}=(f_X\ddot{\phi}+f'_X\dot{\phi}^2)\cos\frac{\phi}{T}+(f_Y\ddot{\phi}+f'_Y\dot{\phi}^2)\sin\frac{\phi}{T} \quad (18a)$$

and $$A_C^{tran}=-(f_X\ddot{\phi}+f'_X\dot{\phi}^2)\sin\frac{\phi}{T}+(f_Y\ddot{\phi}+f'_Y\dot{\phi}^2)\cos\frac{\phi}{T} \quad (18b)$$

where f_X, f_Y, f'_X and f'_Y are the first and second-order kinematic coefficients of the point of interest (i.e., either C or C_S). The first and second-order kinematic coefficients of point C are given by Eqs. (11) and (16), respectively. The first-order kinematic coefficients of the center of the generating arc on the seal are

$$f_{X_{CS}}=e\sin\phi-\left(\frac{r_C+\delta}{T}\right)\sin\frac{\phi}{T}+\delta'\cos\frac{\phi}{T} \quad \text{and} \quad f_{Y_{CS}}=-e\cos\phi+\left(\frac{r_C+\delta}{T}\right)\cos\frac{\phi}{T}+\delta'\sin\frac{\phi}{T} \quad (19)$$

Differentiating Eqs. (19) with respect to the input angle, the second-order kinematic coefficients of the center of the generating arc on the seal are

$$f'_{X_{CS}}=e\cos\phi-\left(\frac{r_C+\delta}{T^2}\right)\cos\frac{\phi}{T}-\frac{2\delta'}{T}\sin\frac{\phi}{T}+\delta''\cos\frac{\phi}{T} \quad (20a)$$

and $$f'_{Y_{CS}}=e\sin\phi-\left(\frac{r_C+\delta}{T^2}\right)\sin\frac{\phi}{T}+\frac{2\delta'}{T}\cos\frac{\phi}{T}+\delta''\sin\frac{\phi}{T} \quad (20b)$$

Note that Eqs. (19) and (20) are valid when the center of the seal is able to move (i.e., the absolute value of the lean angle $|\beta|\geq\beta_o$).

3. Results and Conclusions

The geometrical data that is used for the test case presented in this section are T = 3, $r_2=50$ mm, $r=12.5$ mm, $r_C=100$ mm and assumes that the error in the arc angle is 6°. Substituting the trochoid ratio μ = 2 into Eq. (6) gives a maximum lean angle $\beta_{max}=30^\circ$.

Therefore, the lean angle $\beta_0 = 24^\circ$ or a decreased seal width of 1.165 mm per side. Although this is a larger error in seal width than would be expected in practice, the exaggeration in error should help determine if further research is required. Figure 5 shows the lean angle β as a function of the input angle ϕ. Note that the seal is only moving when the absolute value of the lean angle $|\beta| \geq \beta_0$. The figure also shows that the seal displacement is small even for a 6° error per side in the seal angle. The research showed that the displacement of the seal in comparison to the seal width variation was less than 0.001 %. The next part of the research compares the radial and tangential acceleration of the "theoretical" pin with the actual pin. Figure 6 shows the radial acceleration of point C on the pin and point C on the seal (C_S) during the cycle when the seal is moving relative to the rotor (for clarity only one-half of a cycle is shown). There is a very small difference in the radial acceleration of the two points. For a portion of the cycle, the radial acceleration of C_S is less than the radial acceleration of C. The only significant increase in the radial acceleration of C_S is when the seal initially starts to move. Note that the jerk in the system at the first point after the lean angle $|\beta| \geq \beta_0$ was neglected. The research also showed that there is no significant difference between the tangential acceleration of point C and point C_S throughout the entire cycle and the effects can be neglected.

This paper shows the minimum displacement of the seal that is necessary to maintain contact with the housing for an undersized apex seal. Because the radial acceleration of the undersized seal is similar to the theoretical seal, the spring forces should also be similar. Future research will examine the possible advantages of seal movement to the seal from sticking in the rotor slot. The merits of a "self-cleaning" apex seal or if the fretting and increased seal wear outweigh the "self-cleaning" potential will also be investigated.

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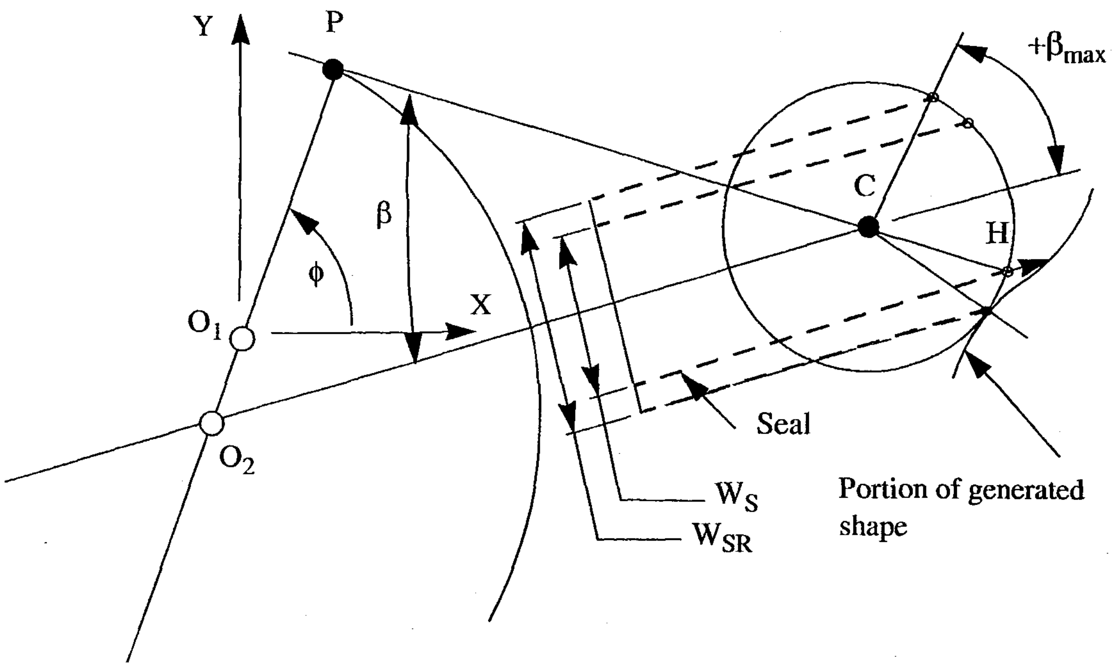


Figure 1. The Basic Generating Shape and the Orientation of the Maximum Sweep Angles on the Surface of the Generating Pin Relative to the Center Line.

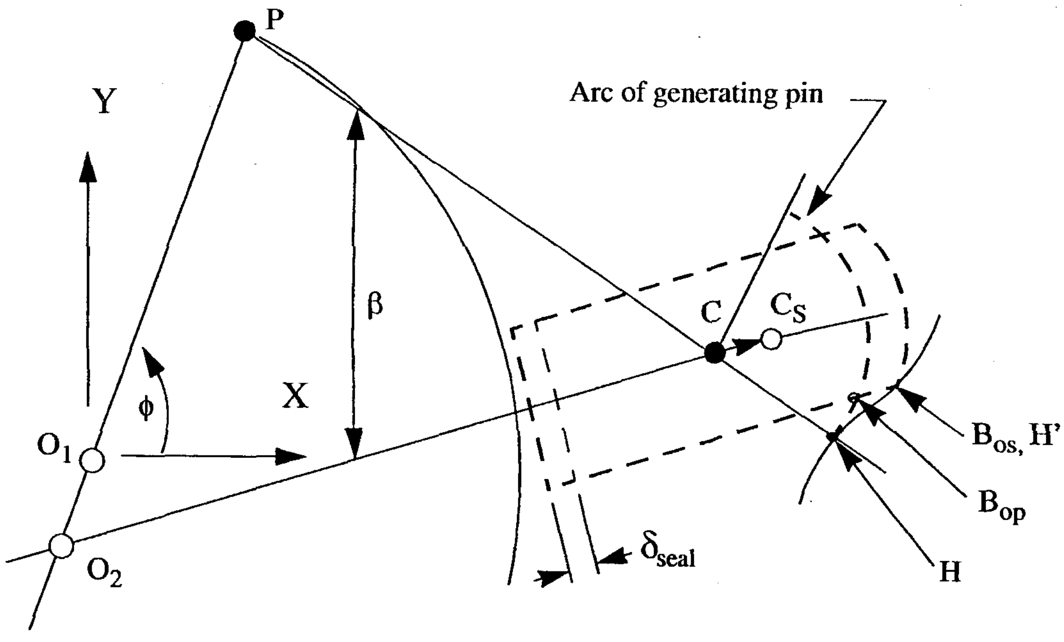


Figure 2. The Movement of the Seal to Compensate for the Manufacturing Process Variation in Seal Width. Point C_S on the Seal (initially Coincident with Point C on the Theoretical Pin) Moves a Distance δ Until Point B_{OS} Contacts Point H' on the Housing.

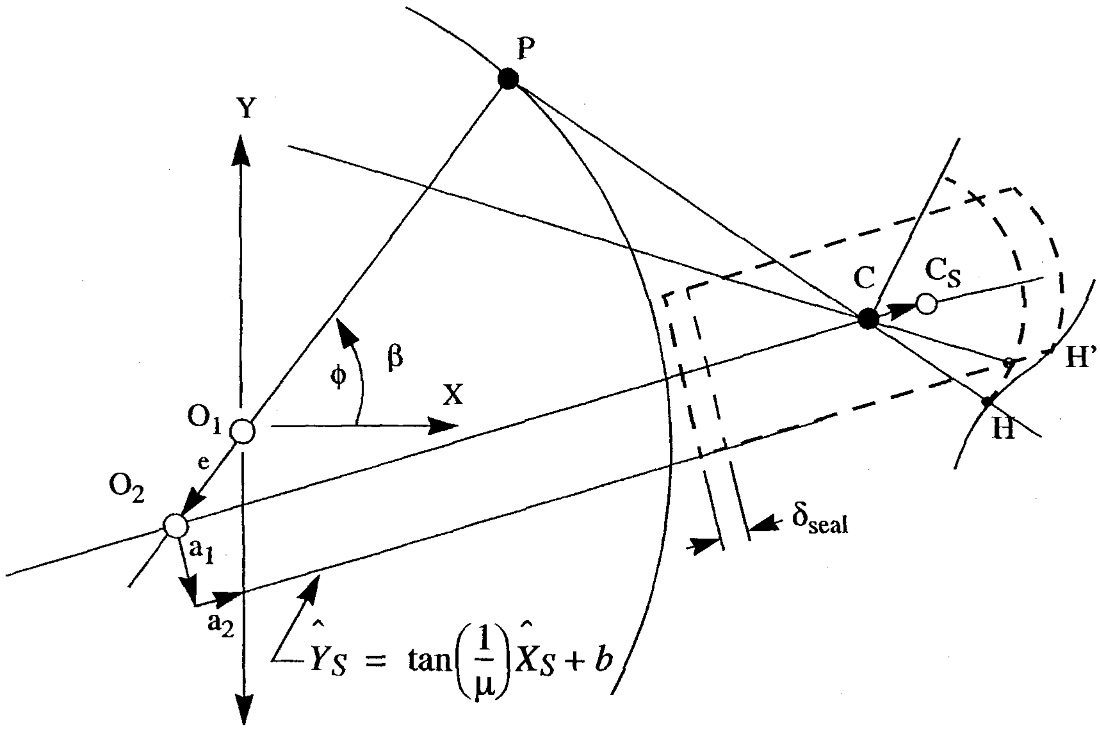


Figure 3. Vectors Used to Locate the Point of Contact H′.

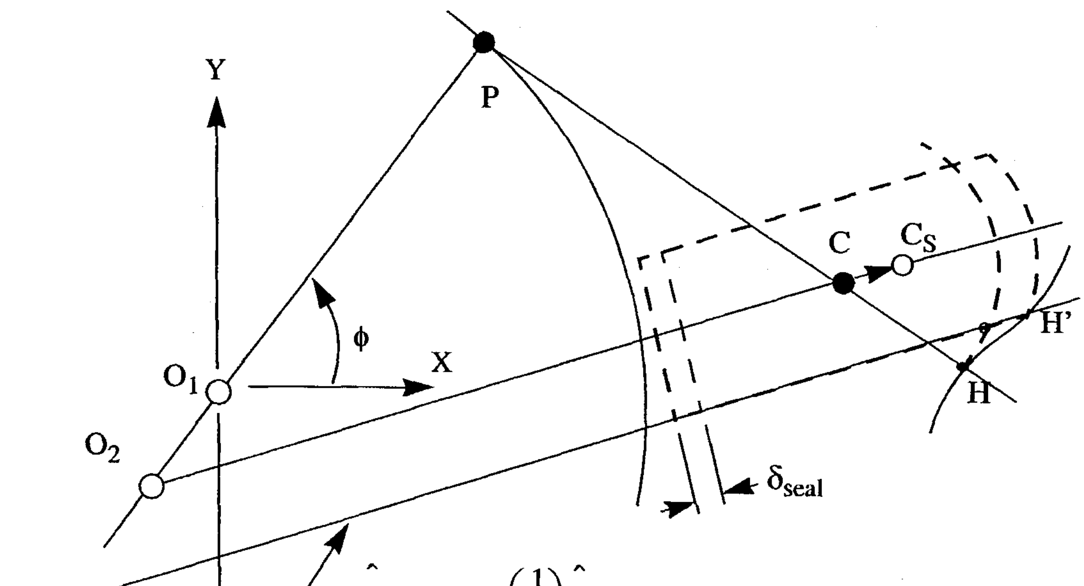
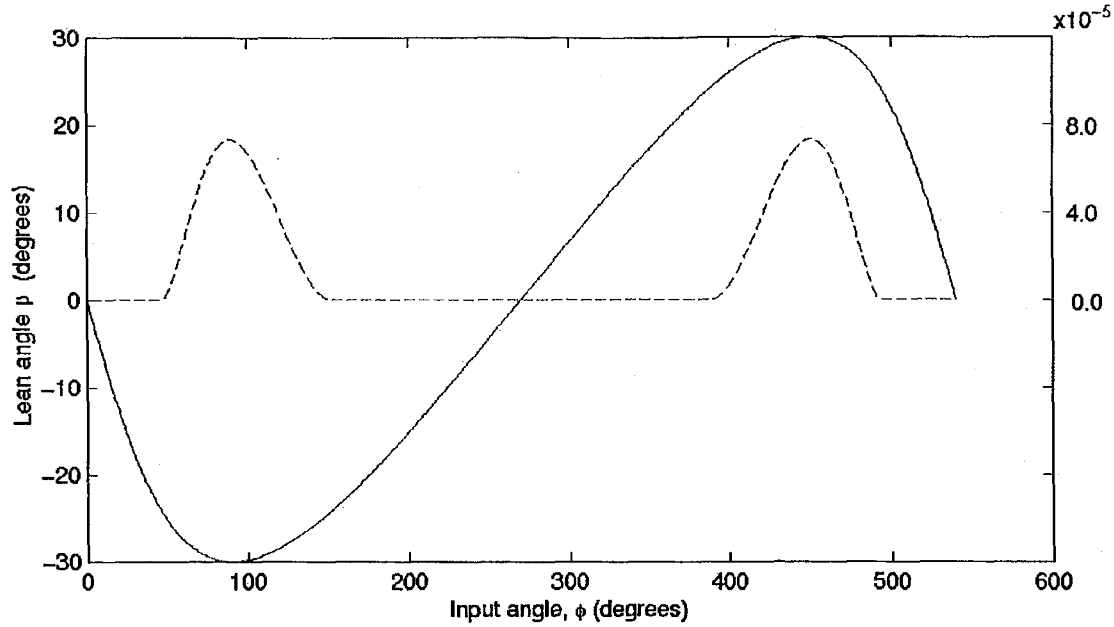
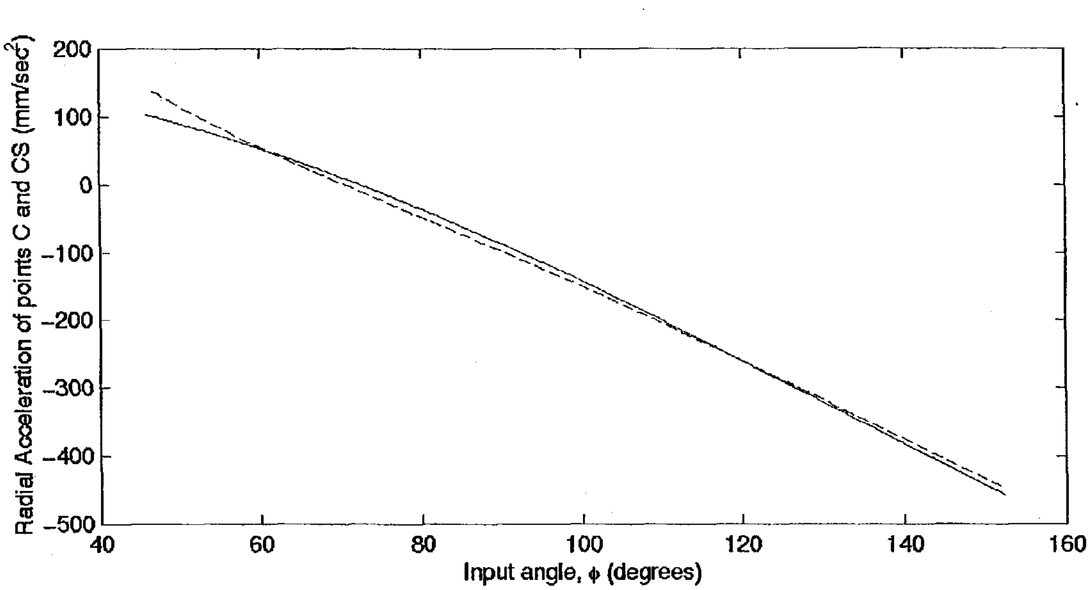


Figure 4. The Intersection of the Seal Action Line and the Housing Used to Determine the Value ϕ' that is Necessary to Generate H′.



------- seal displacement
______ lean angle

Figure 5. Lean Angle and Seal Displacement Against the Input Angle



-------- Point C_S
______ Point C

Figure 6. The Radial Acceleration of Points C and C_S Against the Input Angle.