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MULTI CRITERIA ANALYSIS OF THE SUPPORTING SYSTEM OF A RECIPROCATING COMPRESSOR

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ABSTRACT

The use of finite element analysis with parametric dynamic models gives a powerful tool for the optimization of a virtual compressor. A multi-criteria approach has been applied to the system to reduce noise, vibration, dynamic stress and costs and improve efficiency. The best project is a compromise of all these factors, because these factors generally give opposite results. This work presents the development of a virtual model for a domestic hermetic compressor and a practical case of balancing optimization, from the numerical analysis to the experimental test.

1. INTRODUCTION

A virtual dynamic model of a hermetic compressor has been created. Even if presently the model has been useful to optimize balancing of the crank mechanism reducing low frequency vibrations (about 50 Hz), the model will be used for many other purposes:
- Analysis of dynamic forces on mechanical components
- Analysis of vibration of the supporting system
- Prediction of power consumption
- Considering flexible shaft also flexional and torsional vibrations can be analyzed
- Analysis of hydrodynamic lubrication
- Optimization of balancing
- Analysis of variable speed compressor
- Start up analysis of the compressor
- Vibro-acoustic analysis
The model will be connected to a thermo-fluid dynamic package (written in Fortran or C) or a Simulink model to calculate the correct PV cycle in specific conditions or the correct speed vs. torque in variable conditions. This is useful to study variable speed compressors. The paper describes the dynamic model, which uses multi-body [1] and flexible elements. So this approach has less simplification than classical analytical approach. Dynamic of a reciprocating hermetic compressor has been described from several authors. Equation of the motion of the slider crank mechanism can be solved to compute the force applied on the supporting system. The analysis of a rigid body applied on the system permits to evaluate acceleration. Unfortunately this approach has several simplifications and it is generally difficult to know the uncertainty of the result. The parametric model has been developed with the use of finite element analysis and dynamic analysis. Some components of the compressor, such as supporting springs and the internal discharge pipe have been considered as flexible bodies [2], so that they have a dynamic transfer function which has been calculated through a spectral analysis performed with Ansys [3]. All other elements has been assumed as rigid, even if it's
possible to analyze a system with all flexible bodies. This is useful to analyze vibration sources in a higher range of frequency.

2. MECHANICAL MODEL

The main assumption is that only discharge pipe and supporting springs are flexible bodies, so that other elements can be considered as rigid. All rigid body elements have ten characteristic parameters:
- A center of mass (three coordinates)
- A mass
- An inertia (six coordinates)

Coupling of substructures permits to analyze flexible bodies, which have a more complex structure because they have the same ten parameters of a rigid body, but they have several modal frequencies (eigenvalues) and modal shapes (eigenvectors). A modal analysis can be performed to find these parameters. The result of the analysis is generally normalized to the maximum displacement, so the real displacement can be evaluated only if the correct excitation set is known. A mechanical dynamic software can evaluate the excitation from the solution of differential equations of the motion of crank mechanism. A great advantage of substructures is that it’s possible to modify independently each component of the assembly. A flexible body has been described as a substructure. A constrained modal analysis of the body has been performed, to evaluate modal frequencies and then power spectral density analysis permits to evaluate participation factor of each mode. It’s possible to evaluate experimentally a damping coefficient for each modal frequency. Normal constrained modes have been transformed into normal free-free modes plus a set of interface eigenvector modes[4]. Rigid body modes have a very low frequency (zero) and are normally disabled. The motion of a flexible body can be described as a combination of shape vectors or mode shapes: the position of master nodes of the flexible body is [5]:

\[ r_i = x + A(s_i + \Theta q) \]

where:
- \( x \) defines the position vector in the global reference system to the local reference frame, \( B \)
- \( A \) is the directions cosines of the orientation between \( B \) and the global origin
- \( s_i \) is the undeformed location of the \( i^{th} \) node in \( B \)
- \( \Theta \) define the contribution of the mode shapes to the \( i^{th} \) node.
- \( q \) is the vector of modal amplitude.

Motion can be represented using Euler angles. Generalized coordinates are:

\[ \xi = \begin{bmatrix} X \\ \psi \\ q \end{bmatrix} \]

where:
- \( X \) is a vector of coordinates of the local body reference frame
- \( \psi \) is the vector of Euler angles of \( B \) relative to the global origin.
- \( q \) is the vector of modal amplitudes of the \( m \) contributing mode shapes.
Equations of the motion of the flexible body can be derived from Lagrange’s equations:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\xi}} \right) - \frac{\partial L}{\partial \xi} + \frac{\partial F}{\partial \dot{\xi}} + \left[ \frac{\partial V}{\partial \xi} \right]^T \lambda - Q = 0
\]

\[
\Psi = 0
\]

where:
- \( L \) is the Lagrangian: \( L=\text{T}-\text{V} \) where \( \text{T} \) and \( \text{V} \) denote kinetic and potential energy.
- \( \Theta \) is the energy dissipation function
- \( \Psi \) are the constrain equations.
- \( \xi \) are the generalized coordinates
- \( \lambda \) are the Lagrange multipliers for the constraints

Kinetic energy can be expressed in generalized coordinates \( \xi \):

\[
T = \frac{1}{2} \tilde{\xi}^T \mathbf{M}(\tilde{\xi}) \tilde{\xi}
\]

\( \mathbf{M}(\tilde{\xi}) \) is the mass matrix.

\[
\mathbf{M} = \begin{bmatrix}
\mathbf{M}_u & \mathbf{M}_{ur} & \mathbf{M}_{um} \\
\mathbf{M}_{ru} & \mathbf{M}_{rr} & \mathbf{M}_{rm} \\
\text{simm} & \mathbf{M}_{mm}
\end{bmatrix}
\]

Where \( t, r, m \) refer to translational, rotational, and modal coordinates.

\( \mathbf{M}_u = m \mathbf{I} \)

where \( m \) is the mass and \( \mathbf{I} \) the identity matrix

\[
\mathbf{M}_{ur} = -\int \rho \mathbf{A}(s_p + \Phi(P) \ q) \mathbf{B} \ dV
\]

the first term is the center of the mass location multiply by the mass. \( \Phi(P) \) is the shape function.

\[
\mathbf{M}_{tm} = \int \rho \mathbf{A} \Phi(P) \ dV \quad \mathbf{M}_{rr} = \int \rho \mathbf{B}^T (s_p + \Phi(P) \ q)^T (s_p + \Phi(P) \ q) \mathbf{B} \ dV
\]

\[
\mathbf{M}_{rm} = -\int \rho \mathbf{B}^T (s_p + \Phi(P) \ q)^T \Phi(P) \ dV \quad \mathbf{M}_{rn} = \int \rho \Phi(P)^T \Phi(P) \ dV
\]

Potential energy contains gravitational energy and elastic energy:

\[
\text{V} = \frac{1}{2} \tilde{\xi}^T \mathbf{K} \tilde{\xi} + \text{V}_g(\tilde{\xi})
\]

where \( \mathbf{K} \) is the generalized stiffness matrix (only modal coordinates \( q \) participate to elastic energy), and \( \text{V}_g \) is the gravitational energy. The damping force is generally dependent on modal velocity \( w \), according to Rayleigh’s dissipation function \( \Theta \).

\[1\ \Phi(P) \ q \text{ is the deformation of a point } \text{P of a flexible body } (\Phi(P) \text{ is the shape function, } s_p \text{ is the undeformed position of the point } \text{P}).\]
$$\Theta = \frac{1}{2} w^T D w$$

where: $D$ is damping coefficient matrix and $w$ is the modal velocity vector.

The asynchronous electric motor has a characteristic speed vs. torque curve, so the speed and the power consumption depend on the load. The angular velocity is about 308 rad/sec, even if this is not constant and brushless motors have also a pulsing torque of double frequency, which has a null mean value. Motor torque characteristic can be determined in regime, if this function is used to study the start up, there is a high degree of uncertainty.

Pressure load on the system has been computed throw a thermo-fluid dynamic software for the specific compressor. So the analysis isn’t fully coupled, but can give a good result in stationary conditions.

The model also considers friction of the sliding parts of the system. Friction model torque has been computed from the theory of hydrodynamic lubrication of bearings. The steady incompressible Reynolds equation has been employed with Sommerfeld boundary conditions.

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) = \frac{w}{2} \frac{\partial h}{\partial x}$$

where:
• $h$ is the film thickness (normally this is $h = h_{\text{min}} + h_{\text{term}} \cos(\phi) + x \sin(\phi)$ where $h_{\text{term}}$ is due to thermal expansion).

• $w$ is the speed

Than hydrodynamic friction force can be evaluated on shear stress.

3. NUMERICAL AND EXPERIMENTAL RESULTS

The first application of the virtual compressor model has been the optimization of balancing. It is possible to optimize the magnitude of a goal function in the frequency domain, for example the fundamental frequency (50Hz) of the goal function. A domestic hermetic compressor has been analyzed varying the counterbalance weight. It is possible to analyze the variation of the effective acceleration in some reference points. Defined as coordinate system on this plane $x$ as the piston motion direction and $y$ direction orthogonal to $x$, and defined as origin the center of the crankshaft, maximum effective acceleration on $x$ and $y$ direction have been computed and measured. This value corresponds to the effective value of the FFT magnitude diagram at the current frequency (about 50 Hz).

Numerical analysis has been performed to find the best counterbalance weight; even if it is always required an experimental validation of the results and the

Fig. 3 Effective acceleration: Numerical and experimental data.

Fig. 4 reference system

\footnote{This compressor uses R134a and it has a cooling capacity of 170 kcal/h}
experimental determination of the goal function. Another validation of the model has been the measure of effective acceleration of the top of the four supporting springs. These accelerations have been measured with a particular setup[6] and they have been calculated numerically. It is possible to notice in tables 1-2 that there is a good correlation.

Table 1 Effective acceleration (measured values)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (m/sec^2)</td>
<td>Phase (deg)</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>Spring 1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>Spring 2</td>
<td>0.7</td>
<td>-80</td>
</tr>
<tr>
<td>Spring 3</td>
<td>0.7</td>
<td>-80</td>
</tr>
</tbody>
</table>

Table 2 Effective acceleration (computed values)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (m/sec^2)</td>
<td>Phase (deg)</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>Spring 1</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>Spring 2</td>
<td>0.6</td>
<td>-70</td>
</tr>
<tr>
<td>Spring 3</td>
<td>0.6</td>
<td>-70</td>
</tr>
</tbody>
</table>

The model can calculate FFT diagram of acceleration and the force transmitted to the shell. These informations can be useful for a vibro-acoustic analysis of the system for the evaluation of emitted noise. It is possible to notice in fig. 5 that there is a good correlation for low frequency vibration (under 200 Hz). For higher frequency it is important to consider other flexible elements. This will be done in future enhancements of the model.

Fig.5 - FFT diagram of acceleration of crankcase
3. CONCLUSIONS

The first use of the virtual compressor model has a good agreement with experimental results, so this approach can give an answer to many questions. Similar results for a single model could be achieved with an experimental approach, but a virtual model remarkably improves design capabilities. Further work is under progress to improve and validate the virtual model of compressor.

4. REFERENCES

[1] Craig, R.R. and Bamton, C.C., Coupling of substructures for Dynamic Analyses, AIAA journal vol.6 no.7 pp. 1313-1319
[2] Adams flex guide Mechanical Dynamics rev. 10.0