DESIGN OF COMPACTED CLAY EMBANKMENTS FOR IMPROVED STABILITY AND SETTLEMENT PERFORMANCE

M. J. Goodman
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JOINT HIGHWAY RESEARCH PROJECT

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M. J. Goodman
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Design of Compacted Clay Embankments for Improved Stability and Settlement Performance

To: H.L. Michael, Director
Joint Highway Research Project

From: J.L. Chameau, Research Associate
Joint Highway Research Project

August 30, 1983

Attached is an Interim Report on the HPR Part II study titled "Improving Embankment Design and Performance". The report is entitled "Design of Compacted Clay Embankments for Improved Stability and Settlement Performance". It is authored by M.J. Goodman, J.L. Chameau and C.W. Lovell of our staff.

The report describes the application of the compacted clay investigation presented in previous interim reports to the design and analysis of compacted clay embankments. The alternatives of specifying compaction procedures or compaction results are compared, and a hybrid approach of compaction specification is introduced. Embankment slope design is illustrated for short and long term conditions. Computer programs to compute the magnitude and time-rate of settlement of compacted embankments are developed.

The report is submitted as partial fulfillment of the objectives of the study.

Respectfully submitted,

J.L. Chameau
Research Associate

cc: A.G. Altschaeffl, W. H. Goetz, C.F. Scholer
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Interim Report

DESIGN OF COMPACTED CLAY EMBANKMENTS FOR IMPROVED STABILITY AND SETTLEMENT PERFORMANCE

by

Martin J. Goodman
J. L. Chameau
and
C. W. Lovell

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The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views of policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

Purdue University
West Lafayette, Indiana
August 30, 1983
### DESIGN OF COMPACTED CLAY EMBANKMENTS FOR IMPROVED STABILITY AND SETTLEMENT PERFORMANCE

**Title and Subtitle**

This report is part of the embankment design and performance project conducted by the Joint Highway Research Project which was initiated to improve the ability of highway engineers to design embankments. It has two main objectives. First, it illustrates how the results of the compacted clay investigation can be used in the design and analysis of compacted clay embankments. Second, it completes the analysis package by supplying computer programs for the calculation of embankment settlement.

A hybrid method of specifying compaction which makes the in situ water content of the embankment soil equal to the optimum moisture content is introduced. This approach can help optimize the properties of compacted embankment soils. Embankment side slope design is illustrated for short and long term conditions using laboratory shear strength data. Geometric and probabilistic interpretation of the factor of safety are introduced as alternatives and/or supplements to the conventional strength factor of safety.

A methodology to predict the settlement of embankments is presented. Computer programs to compute the magnitude and time-rate of settlement are included. User's manuals for these programs are provided. Several improvements made to the program STABL are also documented.

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<td>embankment, settlement slope stability, compaction, consolidation, factor of safety, probability</td>
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**Distribution Statement**

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<td>lower bound of a probability density function</td>
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<tr>
<td>(a)</td>
<td>pore pressure parameter relating octahedral shear stress change to excess pore pressure change</td>
</tr>
<tr>
<td>(a_v)</td>
<td>coefficient of vertical compressibility of a soil</td>
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<tr>
<td>(A_{ac})</td>
<td>Skempton pore pressure parameter relating excess pore pressure and deviatoric stress change in a triaxial test</td>
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<tr>
<td>(A_{cr})</td>
<td>area of critical cross-section</td>
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<tr>
<td>(A_f)</td>
<td>Skempton pore pressure parameter at failure</td>
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<td>(A_1)</td>
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<td>(A_5)</td>
<td>term used by STABL to calculate the FS</td>
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<tr>
<td>(b)</td>
<td>upper bound of a probability density function</td>
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<tr>
<td>(B)</td>
<td>pore pressure parameter relating octahedral normal stress change to pore pressure change</td>
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B - number of blows in a compaction test

c - cohesion intercept

c_a - available cohesion

c_h - coefficient of consolidation in the horizontal direction

c_v - coefficient of consolidation in the vertical direction

C_c - compression index

CD - consolidated-drained triaxial test

CDF - cumulative distribution function

C_r - recompression index

CU - consolidated-undrained triaxial test

C_v1 - coefficient of consolidation above a layer interface

C_v2 - coefficient of consolidation below a layer interface

d - dimension used to compute the friction-circle factor of safety

dz - thickness increment

D - depth factor

e - void ratio

_e - total compactive effort

_e_0 - initial void ratio

E - compactive effort

E(x) - expected value of x

f - coefficient of compaction

_f - average value of f
\( f_s \) - probability density function of the load
\( f(x) \) - probability density function of the variable \( x \)
\( F \) - towing force
\( F_R \) - cumulative distribution function of the resistance
\( FS \) - factor of safety
\( FS_c \) - factor of safety on the cohesion intercept
\( FS_{STABL} \) - factor of safety obtained with the Simplified Janbu FS
\( FS_B \) - factor of safety on a slope angle
\( FS_\phi \) - factor of safety on the friction angle
\( F_x(x) \) - cumulative distribution function of the variable \( x \)
\( F_x^{-1} \) - inverse function of \( F_x \)
\( h \) - slice height
\( h_{eq} \) - height of horizontal earthquake force above bottom of a slice
\( H \) - embankment height
\( H \) - clay layer thickness
\( H_{cr} \) - value of slope height at which a slope reaches limit equilibrium
\( H_v \) - horsepower of a compactor
\( i \) - index number indicating position on the \( x \) axis
\( J \) - index number indicating position on the \( z \) axis
\( k \) - time step number
\( k \) - coefficient of compaction

\( k_v \) - vertical earthquake coefficient

\( k_h \) - horizontal earthquake coefficient

\( k_1 \) - permeability above a layer interface

\( k_2 \) - permeability below a layer interface

\( l_c \) - half-width of the cylindrical portion of a sliding mass

\( L \) - towing distance

\( M \) - weighting factor for calculating compactive effort

\( n \) - number of slices

\( n \) - number of strata

\( N_s \) - stability number

\( OCR \) - overconsolidation ratio

\( OCR_o \) - OCR at which excess pore pressure is zero

\( OMC \) - optimum moisture content

\( P_f \) - probability of failure

\( P \) - number of passes of a compactor

\( PDF \) - probability density function

\( P_s \) - compactive prestress

\( P1 \) - perpendicular bisector of the first chord

\( P2 \) - perpendicular bisector of the second chord

\( Q \) - randomly generated number between 0 and 1

\( r \) - correlation coefficient

\( r_o \) - dimension used to calculate stress change beneath an embankment
<table>
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<tr>
<td>$r_2$</td>
<td>dimension used to calculate stress change beneath an embankment</td>
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<tr>
<td>$R$</td>
<td>radius</td>
</tr>
<tr>
<td>$R$</td>
<td>strength</td>
</tr>
<tr>
<td>$R_{\text{min}}$</td>
<td>minimum value of strength</td>
</tr>
<tr>
<td>$S$</td>
<td>load</td>
</tr>
<tr>
<td>$S$</td>
<td>settlement</td>
</tr>
<tr>
<td>$S$</td>
<td>forward speed</td>
</tr>
<tr>
<td>$S_{\text{field}}$</td>
<td>the actual consolidation settlement of a compressible layer</td>
</tr>
<tr>
<td>$S_{\text{lab}}$</td>
<td>calculated value of consolidation settlement of a compressible layer assuming excess pore pressures in the field equal to the pressure developed in laboratory</td>
</tr>
<tr>
<td>$S_{\text{max}}$</td>
<td>maximum value of load</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T$</td>
<td>chord length of segments circumscribing a circle</td>
</tr>
<tr>
<td>$T_L$</td>
<td>time to travel the distance $L$</td>
</tr>
<tr>
<td>$u$</td>
<td>excess pore pressure</td>
</tr>
<tr>
<td>$u_{i,j,k}$</td>
<td>excess pore pressure at the $i^{\text{th}}$ position, the $j^{\text{th}}$ $z$ position and the $k^{\text{th}}$ time step</td>
</tr>
<tr>
<td>$UU$</td>
<td>unconsolidated-undrained triaxial test</td>
</tr>
<tr>
<td>$U%$</td>
<td>percent consolidation</td>
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\[ \begin{align*}
U(z) & \quad \text{the percent volume change due to saturation at the depth, } z \\
V & \quad \text{variable used to compute the friction-circle factor of safety} \\
\phi & \quad \text{parameter used to define the beta distribution} \\
V(x) & \quad \text{mold volume} \\
V(x) & \quad \text{variance of } x \\
w & \quad \text{frequency of vibration of a vibratory compactor} \\
w\% & \quad \text{water content} \\
W & \quad \text{compactor weight} \\
x & \quad \text{the coordinate value on the } x \text{ axis} \\
x & \quad \text{parameter used to define the beta distribution} \\
x_{c1} & \quad \text{ } x \text{ coordinate of the intersection of the first chord and its perpendicular bisector} \\
x_{c2} & \quad \text{ } x \text{ coordinate of the intersection of the second chord and its perpendicular bisector} \\
x_c & \quad \text{ } x \text{ coordinate of the center of the circle} \\
x_1 & \quad \text{ } x \text{ coordinate of the first point on the first chord of a circle} \\
x_2 & \quad \text{ } x \text{ coordinate of the second point on the first chord of a circle} \\
x_3 & \quad \text{ } x \text{ coordinate of the second point on the second chord of a circle} \\
y & \quad \text{the coordinate value on the } y \text{ axis} \\
y & \quad \text{vertical distance to bottom of a slice from the moment center}
\end{align*} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{c1}$</td>
<td>y coordinate of the intersection of the first chord and its perpendicular bisector</td>
</tr>
<tr>
<td>$y_{c2}$</td>
<td>y coordinate of the intersection of the second chord and its perpendicular bisector</td>
</tr>
<tr>
<td>$y_0$</td>
<td>y coordinate of the center of the circle</td>
</tr>
<tr>
<td>$y_1$</td>
<td>y coordinate of the first point on the first chord of a circle</td>
</tr>
<tr>
<td>$y_2$</td>
<td>y coordinate of the second point on the first chord of a circle</td>
</tr>
<tr>
<td>$y_3$</td>
<td>y coordinate of the second point on the second chord of a circle</td>
</tr>
<tr>
<td>$Y$</td>
<td>arbitrary function</td>
</tr>
<tr>
<td>$Z$</td>
<td>net work per vibratory cycle of a vibratory compactor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle the bottom of a slice makes with the horizontal</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>parameter used to define the beta distribution</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>coefficient used in equation 4.19</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>coefficient used in equation 4.19</td>
</tr>
<tr>
<td>$\beta$</td>
<td>sideslope angle</td>
</tr>
<tr>
<td>$\beta_{cr}$</td>
<td>value of sideslope at which a slope reaches limit equilibrium</td>
</tr>
<tr>
<td>$\Delta C_a$</td>
<td>cohesion force acting on the bottom of a slice</td>
</tr>
<tr>
<td>$\Delta e_0$</td>
<td>the difference between the initial void ratio in the soil sample and another soil stratum</td>
</tr>
<tr>
<td>$\Delta N$</td>
<td>normal force acting on the bottom of a slice</td>
</tr>
</tbody>
</table>
\( \Delta Q \) - surcharge load on a slice

\( \Delta S_r \) - resisting force acting on the bottom of a slice

\( \Delta t \) - time increment

\( \Delta W \) - slice weight

\( \Delta U_\alpha \) - water force acting beneath a slice

\( \Delta U_\beta \) - water force acting on top of a slice

\( \Delta x \) - increment in the \( x \) direction

\( \Delta z \) - increment in the \( z \) direction

\( \Delta z_1 \) - vertical grid spacing above a layer interface

\( \Delta z_2 \) - vertical grid spacing beneath a layer interface

\( \Delta \sigma \) - stress change

\( \Delta \sigma_v \) - change in vertical stress

\( \Delta \sigma_x \) - change in normal stress acting in the \( x \) direction

\( \Delta \sigma_z \) - change in normal stress acting in the \( z \) direction

\( \Delta \sigma_1 \) - change in major principal stress

\( \Delta \sigma_3 \) - change in minor principal stress

\( \Delta \tau_{xz} \) - change in shear stress acting in the \( z \) direction on the plane perpendicular to the \( x \) axis
$\Delta \theta$ - deflection angle of segments circumscribing a circle

$\gamma$ - unit weight

$\gamma_d$ - dry unit weight

$\gamma_{d\ max}$ - maximum dry unit weight

$\gamma_m$ - moist unit weight

$\gamma_w$ - unit weight of water

$\gamma$ - gamma function

$\lambda$ - nondimensionalized slope stability parameter

$\mu$ - consolidation settlement correction factor

$\mu_c$ - coefficient of variation of the cohesion intercept

$\nu$ - Poisson’s ratio

$\phi$ - friction angle

$\phi_a$ - maximum available friction angle

$\phi_{\text{req}}$ - friction angle required for equilibrium

$\sigma_{\text{oct}}$ - octahedral normal stress

$\sigma'$ - effective preconsolidation pressure

$\sigma_{\text{sample}}$ - the vertical pressure at the depth of the soil sample

$\sigma_{\text{v}}'$ - effective vertical pressure

$\sigma'_{\text{vo}}$ - the effective overburden pressure in a soil stratum

$\sigma_y$ - yield strength

$\sigma_1$ - major principal stress

$\sigma_2$ - intermediate principal stress
$\sigma_3$ - minor principal stress

$\Sigma$ - summation

$\tau_{oct}$ - octahedral shear stress
HIGHLIGHT SUMMARY

This study illustrates how the results of the compacted clay investigation can be best used in the design and stability analysis of compacted clay embankments. It also completes the analysis package by supplying computer programs for the calculation of embankment settlement.

The alternatives of specifying compaction procedures or compaction results are compared and an hybrid approach of specifying compaction that integrates the advantages of these two approaches is introduced. In the hybrid approach of compaction specification, the compactive effort is specified so that the corresponding optimum moisture content is equal to the expected compaction water content.

Embankment side slope designs are illustrated for short and long term conditions using laboratory compacted shear strength data. In these examples the embankment material is assumed to be compacted St. Croix clay and the strength parameters for short and long term conditions are obtained from the reports by Weitzel and Lovell (1979) and Johnson and Lovell (1979), respectively. Several improvements made
to the STABL program during the course of this study are presented:

- The Simplified Bishop factor of safety option was recoded to correct various difficulties discovered by STABL users.

- Recommendations are made to avoid common errors in the use of STABL (surfaces with unacceptable shapes, direction limits on benched slopes, direction of surface generation, etc.).

- A methodology was developed to adjust the Simplified Janbu factor of safety (used in several STABL options) to more familiar definitions of the factor of safety.

Geometric and probabilistic interpretation of the factor of safety are introduced to demonstrate their usefulness as alternatives and/or supplements to the conventional strength factor of safety. The probabilistic approach takes into account the variability of material parameters and provide the reliability of the slope corresponding to the computed factor of safety.

Computer programs are developed to compute the magnitude of settlement that occur within the embankment itself as well as of the consolidation settlement of fine grained soil layers below the embankment. Programs are also provided to estimate the time-rate of consolidation settlement
of the compressible soil layers beneath an embankment. These programs, in conjunction with STABL, form an analysis package for the design of embankments. User's manuals and listings are given for all these computer programs.
I - INTRODUCTION

This report is a part of Purdue's embankment performance project which was initiated to improve the ability of highway engineers to design highway embankments.

This project is composed of two separate objectives. One objective is the investigation of the strength and compressibility parameters of compacted St. Croix clay; a highly plastic residual clay of Southern Indiana. The second objective of this project entails the development of an analysis package to predict the settlement and stability performance of embankments.

**Compacted Clay Investigation**

The study of the parameters defining the behavior of compacted St. Croix clay was performed in two phases. The first phase consisted of tests performed on clay samples that were prepared to simulate standard laboratory compaction specifications. This testing program included the following separate investigations:

1) Compressibility and prestress behavior of St. Croix clay (DiBernardo, 1979).
2) Unconsolidated-undrained (UU) shear strength behavior of St Croix clay (Weitzel, 1979).

3) Consolidated-undrained (CU) shear strength of St. Croix clay (Johnson, 1979).

Parameters defining the behavior of field compacted clay are different than those obtained with laboratory testing because of the difference in compactive effort between field compactors and laboratory compaction tests. Therefore, in the second phase, the tests of the first phase were repeated on samples compacted in the field with two different rollers (Lin, 1981 and Liang, 1982). The swell pressure of both laboratory and field compacted clay was studied by Terdich (1981). These studies provide a unique look at the correlation between the behavior of field and laboratory compacted clays.

**Analysis Package**

Purdue University has a long standing interest in the development of user-oriented slope stability computer programs. One of the first developments was the SLOPE program package (Carter, 1971) consisting of four separate programs. The subsequent development was the STABL program (Siegel, 1975). This program can evaluate the factor of safety of slopes of almost any description and shape. Boundary surcharge loads and pseudo-static earthquake forces may also be
included. The most significant feature of STABL is its ability to automatically create randomly generated surfaces to help the user search for the minimum value of the factor of safety.

STABL was further developed by Boutrup (1977). Her improvements included:

1) The addition of specialized search routines for simulation of block shaped failure surfaces with active and passive wedges.
2) The inclusion of the Simplified Bishop factor of safety.
3) Changes in the input of pore water pressure that allow the simulation of artesian pressure.

STABL is used on a regular basis by the Indiana Department of Highways for routine evaluation of slope stability.

The most recent of Purdue contributions to the field of slope stability is the development of a factor of safety that takes into account the three-dimensional nature of limit equilibrium surfaces (Chen, 1981). Currently, this method is programmed only for simple slope shapes and failure surfaces.
Report Organization

The purpose of this report is twofold. First, it completes the analysis package by supplying computer programs for the calculation of embankment settlement. Second, it illustrates how the results of the compacted clay investigation may be best used in the design of compacted clay embankments.

Chapter II provides an overview of compaction specification. The alternatives of specifying compaction procedures or compaction results are compared. A hybrid approach of specifying compaction that integrates the advantages of these two approaches is introduced.

Chapter III covers a variety of topics pertinent to the subject of slope stability including:

1) Use of the STABL program for slope stability analysis. The discussion includes recent corrections made to the Simplified Bishop option.

2) Strength parameters of compacted clays to be used when assessing the factor of safety and stability of compacted clay embankments.

3) Discussion of geometric and probabilistic interpretations of the factor of safety.
Chapter IV deals with the settlements caused by the construction of an embankment. User-oriented computer programs to facilitate the computation of the magnitude and time-rate of consolidation settlement are included.

Chapter V presents conclusions of the work that was done and suggestions for further research.
Compaction is the densification of soil by the application of mechanical energy. Densification is achieved by reduction of the size and number of air voids in the soil. As the volume of the voids is reduced, the shear strength of the soil is increased and its permeability is decreased. The increase in shear strength and the reduction in permeability are two factors that make compaction a good technique for constructing highway embankments and dams.

Specifying an adequate level of compaction and range of water content indirectly assures that the soil will have a relatively high shear strength and a low compressibility. This is fortunate because these soil properties are difficult and time consuming to measure on a routine basis.

It is important to be able to quantify the level of compaction to determine if the compaction is adequate. In his pioneering work, Proctor showed that the level of compaction depends on the compactive effort, E, i.e., the amount of mechanical energy imparted into a unit volume of
soil (Proctor, 1933). For a given compactive effort, the dry density, \( \gamma_d \), that can be achieved varies with the water content of the soil (Figure 2.1). There is a value of the water content called the optimum moisture content, OMC, at which the dry density has a maximum value. The soil shear strength is maximized and the soil permeability in service is minimized at or near this water content. The OMC is a function of the compactive effort that varies along a "line of optimums" (Figure 2.2). In general, the dry density will increase and the OMC will decrease as the compactive effort increases.

The concept of compactive effort provides the basis for evaluating the level of compaction. At a specified value of compactive effort, there is a maximum achievable dry density, \( \gamma_d \, \text{max} \), corresponding to the OMC which can be achieved. At water contents other than the OMC, \( \gamma_d \) that is achieved will be less than \( \gamma_d \, \text{max} \). Therefore, it is convenient to define the "percent compaction" as the ratio of \( \gamma_d \) to \( \gamma_d \, \text{max} \).

**Specification of Procedure**

The basic philosophy of specifying the compactive procedure is that if a certain procedure is followed, the compaction is assumed to be satisfactory. Typically, this is achieved by construction of a test pad. Each time the compactor passes over a soil lift in the test pad, the soil
FIGURE 2.1 MOISTURE–DENSITY RELATIONSHIP OF A COMPACTED CLAY
FIGURE 2.2
VARIATION OF THE MOISTURE – DENSITY RELATIONSHIP OF A COMPACTED CLAY VS. COMPACTIVE EFFORT
density is recorded using either a sand cone test or a nuclear density device. The measured density is then plotted versus the number of passes of the compactor. The result is a "density growth curve" (Figure 2.3). The density growth curve indicates the effectiveness of each pass of the compactor. The increase in density diminishes with each pass of the compactor until a pass generates a negligible density increase. Beyond this point, higher densities can only be obtained by using machines that impart more compactive effort. Therefore, although the number of passes should be specified to provide a large fraction (perhaps more than 95%) of the achievable density, the specified number of passes should be less than the value at which each subsequent pass creates only a negligible increase in density.

It is recommended that the density growth curve be developed for various lift thicknesses, various rates of advance of the compactor, and different compactors. This makes it possible to determine the best compactor and the optimum mode of operation for a given job. The performance study done at Purdue (Terdich, 1981) is a good example of the use of the density growth curve. This study showed that a Caterpillar Model 825 tamping compactor is significantly more effective than a RayGo Rascal Model 420C Vibratory compactor to compact St. Croix clay. Advantages of specifying the procedure include:
1) Only limited testing is required to provide quality control of the compaction.

2) The contractor has the assurance that if he operates in accordance with the specified procedure, he will obtain the necessary compaction. This helps reduce the adversarial aspect of the relationship between the contractor and the engineer.

**Specification of Results**

On jobs where the compacted soil is expected to be quite variable, specifying the procedure may not be practical because the level of compaction will change in an unknown fashion from soil to soil, even if identical procedures are used. When this occurs, the results of the compactive operation must be specified.

Specification of results entails two stages. The first stage involves the development of a moisture-density relationship from compaction tests run on soil samples compacted in the laboratory. The laboratory compaction should produce similar moisture-density results as those produced by the compactive effort that is expected in the field. The relationship between the moisture and the density is called the "control curve". The second stage involves the comparison of field measurements of the dry density with the maximum dry density on the control curve. Generally, if the percent compaction is between 95% and 100%, the compaction is judged
to be satisfactory. If the percent compaction is consistently greater than 100%, the laboratory test is probably not imparting as much compactive effort as the compactor. Conversely, if the percent compaction is consistently less than 95%, either the lab compaction is imparting more compactive effort than the field compactor is able to deliver, or more passes of the field compactor are required.

To adjust the laboratory compaction technique to approximate the effects of the compactive effort in the field, it is helpful to quantify the value of the compactive effort of a desired compactor (Selig, 1971). Selig proposed that all compactors be represented by a single drum towed by a separate machine. In this representation, the total compactive effort of the roller for smooth wheel, pneumatic tire, and tamping compactors is

\[ e = F \cdot L \cdot P \]  

(2.1)

where

- \( e \) = total compactive effort (ft·lb)
- \( F \) = towing force (lb)
- \( L \) = distance towed (ft)
- \( P \) = the number of passes over the distance \( L \)

This implies that all of the compactive effort of these compactors is provided by the towing unit through the drawbar.
In contrast, the compactive effort of vibratory rollers is imparted by the vibratory energy of the roller drum. The effect of the towing unit on the drawbar is assumed to be negligible. Therefore, the total compactive effort of a vibratory roller is

\[ e = Z \cdot \omega \cdot T_L \]  

where

- \( Z \) = net work per cycle (ft'lb/cycle)
- \( \omega \) = frequency of vibration (cycles/min)
- \( T_L \) = time to travel the distance \( L \) (min)

Although equations 2.1 and 2.2 both assume that all of the mechanical energy of the roller is transformed into compactive effort, only a fraction of the mechanical energy is actually transformed. The ratio of the compactive effort to the mechanical energy is called the "efficiency." The compactive effort per unit volume may be expressed with quasi-analytic expressions which are specific to a compactor. These expressions are given in Table 2.1. All these expressions contain a coefficient of rolling friction, \( f \). Like compactive effort, this coefficient has almost never been determined experimentally (Selig, 1971). To overcome this, Selig estimated the compactive effort of several types of compactors by comparing the densities that they could achieve with the densities obtained in Standard and Modified Proctor compaction tests. Substituting these values into the expressions in Table 2.1, he evaluated the values of \( f \).
Table 2.1 Expressions for the Compactive Effort of Various Types of Compactors (after Selig, 1971)

<table>
<thead>
<tr>
<th>Roller Type</th>
<th>Compactive Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth Wheel</td>
<td>( E = \frac{fWP}{Bt} )</td>
</tr>
<tr>
<td>Pneumatic</td>
<td>( E = \frac{fWP}{ht} ) or ( E = \frac{fWP}{Bt} ) if ( d &lt; 2b )</td>
</tr>
<tr>
<td>Tamping</td>
<td>( E = \frac{fWP(d + 2)}{k_o + cNA} ) or ( E = \frac{fWP}{Bt} )</td>
</tr>
<tr>
<td>Vibratory</td>
<td>( E = \frac{375H_v P}{5Bt} ) or ( E = \frac{fWP}{Bt} )</td>
</tr>
</tbody>
</table>

where \( f = \frac{375H_v}{WS} \)
Table 2.1 (continued)

A = contact area of tamping foot (ft\(^2\))
B = roller width (ft)
b = tire width (ft)
c = foot area correction factor > 1.0
D = diameter of roller drum (ft)
E = compaction effort per unit volume (ft\(^3\) lb/ft\(^3\))
f = coefficient of compaction
H\(_V\) = horsepower of vibrator engine
h\(^V\) = nb for d ≥ 2b
    = b + (n-1)d for d < 2b
k = overlap correction factor ≤ 1.0
l\(^O\) = tamping foot length (ft)
N = number of tamping feet
n = number of tires
P = number of passes
S = forward speed (miles/hour)
t = compacted lift thickness (ft)
W = total weight of compactor (lb)
Therefore, Selig's f coefficient is not actually a coefficient of rolling resistance, but a general purpose correction factor called the coefficient of compaction, which reflects the number of compactor passes, the soil lift thickness, and the soil type. The range of values of f and the recommended average design values are given in Table 2.2.

The relations for compactive effort in Table 2.1 assume that the compactive effort delivered to the soil is linearly proportional to the number of passes of the compactor. In practice, this is not the case. As the number of passes of the compactor increases, the densification per pass diminishes. This implies that as the number of passes increases, a lesser amount of the work done by the compactor is transformed into compactive effort. Selig (1971) showed that this phenomenon could be incorporated in his model with the following expression for the f parameter:

\[ f_i = k t / P \]  \hspace{1cm} (2.3)

where

- \( f_i \) = coefficient of compaction after the \( i^{th} \) pass
- \( k \) = compaction constant
- \( t \) = compacted layers thickness
- \( P \) = number of passes

It follows that the average value of \( f \) over \( P \) passes is
Table 2.2 Values of $f$ and $k$ for use in Table 2.1 and Equation 2.6 (after Selig, 1971)

<table>
<thead>
<tr>
<th>Roller Type</th>
<th>Soil Type</th>
<th>$f$</th>
<th>$f_{avg}$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheepsfoot</td>
<td>2</td>
<td>0.20–0.50</td>
<td>0.35</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.05–0.15</td>
<td>0.10</td>
<td>1.6</td>
</tr>
<tr>
<td>Pneumatic</td>
<td>1</td>
<td>0.05–0.25</td>
<td>0.15</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.10–0.30</td>
<td>0.25</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.05–0.25</td>
<td>0.15</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.05–0.30</td>
<td>0.15</td>
<td>1.4</td>
</tr>
<tr>
<td>Smooth Wheel</td>
<td>1</td>
<td>0.20–0.50</td>
<td>0.35</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.15–0.25</td>
<td>0.15</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.20–0.50</td>
<td>0.25</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.10–0.40</td>
<td>0.30</td>
<td>1.3</td>
</tr>
<tr>
<td>Vibratory</td>
<td>1</td>
<td>0.20–0.40</td>
<td>0.30</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.15–0.40</td>
<td>0.25</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.30–0.60</td>
<td>0.40</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.50–1.00</td>
<td>0.80</td>
<td>2.7</td>
</tr>
<tr>
<td>Segmented Pad</td>
<td>2</td>
<td>0.10–0.30</td>
<td>0.20</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.10–0.25</td>
<td>0.15</td>
<td>1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>sand</td>
</tr>
<tr>
<td>2</td>
<td>silty clay</td>
</tr>
<tr>
<td>3</td>
<td>gravel-sand-clay</td>
</tr>
<tr>
<td>4</td>
<td>crushed stone</td>
</tr>
</tbody>
</table>
Substituting \( \bar{f} \) into the basic expression for compactive effort (Table 2.1) yields:

\[
E = k \cdot W \cdot \frac{M}{E}
\]  
(2.6)

Suggested values of \( k \) for use in equation 2.6 are given in Table 2.2. Unlike the equations in Table 2.1, equation 2.6 does not reflect the effect of variables specific to individual compactors such as tire spacing, tamping foot length, vibrator horsepower and operation speed. This simplification was necessary due to insufficient data. Even so, the compactive effort should be estimated with equation 2.6 because, unlike the equations in Table 2.2, it simulates the diminution in compaction per pass. The two formulations of the compactive effort equations are compared in the following two examples for the compactors which were used in the Purdue embankment performance study (Terdich, 1981).

Example 2.1

It is desired to use a Caterpillar Model 625 segmented
pad tamping roller to compact one foot lifts of St. Croix clay. The specifications for this roller are provided in Table 2.3. Determine the variation of compactive effort delivered to the soil with the number of passes of the roller.

The total weight of the roller is approximately 60,000 pounds. The compaction is performed with four drums each with a 44.5 inch width. Using the expression for compactive effort of tamping rollers given in Table 2.1, the average compactive effort is:

\[ E = \frac{(f)(60,000)(P)}{(44.5/12)(1)} = 16,180 \text{ fP} \]

The coefficient \( f \) may be assumed to be approximately 0.2 (using the value for silty clay in Table 2.2). It follows that:

\[ E = 3236 \text{ fP} \quad (\text{ft} \cdot \text{lb/ft}^3) \]

Alternately, the compactive effort may be evaluated using equation 2.6. The \( k \) coefficient may be approximated by the value of \( k \) for a segmented pad compactor operating on silty clay, i.e., \( k = 0.9 \). Substituting into equation 2.6 yields

\[ E = \frac{(0.9)(60,000)(M)}{(44.5/12)} = 14562 \text{ M} \]

The results of this calculation are given in Table 2.4. The results of the two expressions for compactive effort are
Table 2.3 Specifications of Compactors Used in Purdue's Embankment Performance Project (after Terdich, 1981)

**Caterpillar Model 825**

<table>
<thead>
<tr>
<th>Dimensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, with dozer</td>
<td>23 ft, 4 in.</td>
</tr>
<tr>
<td>Width, w/o clearers</td>
<td></td>
</tr>
<tr>
<td>w/o dozer</td>
<td>11 ft, 11 in.</td>
</tr>
<tr>
<td>w/o dozer</td>
<td>12 ft, 6 in.</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>13 ft, 7 1/2 in.</td>
</tr>
<tr>
<td></td>
<td>140 in.</td>
</tr>
<tr>
<td>Weight (shipping)</td>
<td></td>
</tr>
<tr>
<td>w/o dozer</td>
<td>59,000 lb.</td>
</tr>
<tr>
<td>with dozer</td>
<td>63,000 lb.</td>
</tr>
<tr>
<td>Number of Drums</td>
<td>4</td>
</tr>
<tr>
<td>Number of Pads/Drums</td>
<td>65</td>
</tr>
<tr>
<td>Each drum width</td>
<td>44 1/2 in.</td>
</tr>
<tr>
<td>Max. ballast</td>
<td>244 U.S. Gal</td>
</tr>
<tr>
<td>Bulldozer Dimensions</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>14 ft.</td>
</tr>
<tr>
<td>Height</td>
<td>40 1/2 in.</td>
</tr>
<tr>
<td>Maximum Speeds</td>
<td></td>
</tr>
<tr>
<td>Gear</td>
<td>MPH</td>
</tr>
<tr>
<td>1</td>
<td>3.1</td>
</tr>
<tr>
<td>2</td>
<td>7.0</td>
</tr>
<tr>
<td>3</td>
<td>17.0</td>
</tr>
</tbody>
</table>
Table 2.3 (continued)

### RayGo Rascal Model 420C

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>18 ft, 9 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, with blade</td>
<td>9 ft</td>
</tr>
<tr>
<td>Width</td>
<td>9 ft</td>
</tr>
<tr>
<td>Wheelbase</td>
<td></td>
</tr>
</tbody>
</table>

| Weight              | 25,160 lb.   |
| Vibration Drive     | Hydraulic, Direct Drive |
| Frequency           | 1100 to 1500 rpm |
| Dynamic Force       | 32,000 lb     |

| No. of Drums        | 1            |
| No. of Pads/Drums   | 140          |
| Each drum width     | 84 in.       |
| Maximum Speeds      |              |
| Gear                | MPH          |
| 1                   | 4            |
| 2                   | 6            |
| 3                   | 8            |
### Table 2.4 Calculations - Examples 2.1 and 2.2

**Compactive Effort**  
(\text{ft}'lb/\text{ft}^3)

<table>
<thead>
<tr>
<th>Passes</th>
<th>( M )</th>
<th>( E = 14562 \ M )</th>
<th>( E = 7907 \ M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.50</td>
<td>21843</td>
<td>11861</td>
</tr>
<tr>
<td>4</td>
<td>2.08</td>
<td>30289</td>
<td>16447</td>
</tr>
<tr>
<td>2</td>
<td>1.50</td>
<td>21843</td>
<td>11861</td>
</tr>
<tr>
<td>4</td>
<td>2.08</td>
<td>30289</td>
<td>16447</td>
</tr>
<tr>
<td>6</td>
<td>2.45</td>
<td>35676</td>
<td>19373</td>
</tr>
<tr>
<td>8</td>
<td>2.72</td>
<td>39608</td>
<td>21508</td>
</tr>
<tr>
<td>10</td>
<td>2.93</td>
<td>42666</td>
<td>23169</td>
</tr>
<tr>
<td>12</td>
<td>3.10</td>
<td>45142</td>
<td>24513</td>
</tr>
<tr>
<td>14</td>
<td>3.25</td>
<td>47326</td>
<td>25699</td>
</tr>
<tr>
<td>16</td>
<td>3.38</td>
<td>49219</td>
<td>26727</td>
</tr>
</tbody>
</table>
compared in Figure 2.4. The equation for compactive effort of Table 2.2 underestimates the compactive effort up to 15 passes because it does not simulate the diminution in compaction per pass. Note, that the values of compactive effort for this roller are generally intermediate to the total values of the Standard and Modified Proctor compaction tests.

Example 2.2

Repeat Example 2.1 for a RayGo Rascal Model 420C vibratory roller. Specifications are provided in Table 2.3. The width of the drums is 84 inches. The total weight of the compactor is 25,160 pounds. Assume that the roller proceeds at 1.5 miles per hour. The $f$ coefficient is estimated to have the value given in Table 2.2 for a vibratory roller on silty clay, i.e., $f = 0.25$. The horsepower of a vibrator is (Table 2.1):

$$H_v = 0.0027 f W S$$  \hspace{1cm} (2.7)

Therefore, the horsepower of this particular vibrator is:

$$H_v = (0.0027)(0.25)(21,600)(1.5) = 21.87 \text{ hp}$$

The expression for the compactive effort of a vibratory roller from Table 2.1 is:
FIGURE 2.4  ESTIMATED COMPACTIVE EFFORT OF A CATERPILLER 825 COMPACTOR

Modified Effort

E = 14562 M

Standard Effort

E = 3236 P

Compactive Effort (kft. lb/ft³)
E = \frac{(375)(21.87)(P)}{(1.5)(84/12)(1)} = 781 P

Alternately, the compactive effort may be obtained from equation 2.6. Note that equation 2.6 employs the static weight instead of the dynamic force. This discrepancy is accounted for in the k coefficient (k = 2.2, see Table 2.2). Substituting into equation 2.6 yields:

E = \frac{(2.2)(25.160)(M)}{(84/12)} = 7907 M

The results of this calculation are presented in Table 2.4 and Figure 2.5. Although equation 2.6 does not include speed of the roller, it does account for the reduction in compaction per pass as compaction proceeds. Therefore, as was the case with the tamping roller, the compactive effort of the vibratory roller is better predicted by equation 2.6 than the expression in Table 2.1. This particular compactor will yield the value of total Standard compactive effort after approximately two passes (See Figure 2.5). A comparison of Figures 2.4 and 2.5 indicates that the Caterpillar tamper is expected to be more effective than the Rascal Vibratory roller for compacting clay.

Assuming a selection of compactor and its use, based upon experience, a laboratory test may be selected, assuming that the total energy in the field and laboratory are equal. The equation for calculating the compactive effort in a compaction test is:
Figure 2.5: Estimated Compactive Effort of a Raygo Model 420 C Compactor
\[
E = \frac{H \cdot W \cdot B \cdot L}{V}
\]  
(2.8)

where

\( E \) = compactive effort \\
\( H \) = height of fall of the hammer \\
\( W \) = weight of the hammer \\
\( B \) = number of blows per soil layer \\
\( L \) = number of soil layers \\
\( V \) = volume of the mold

If equipment is already available to perform an impact compaction test, the volume of the mold, the weight of the hammer, and the height of fall are predetermined. If the number of layers is set as in the Standard and Modified compaction tests, the only unspecified variable is the number of blows per layer. This may be adjusted as desired to approximate the rollers compactive effort. Rearranging equation 2.8, the required number of blows is:

\[
B = \frac{V \cdot E}{H \cdot W \cdot L}
\]  
(2.8a)

**Example 2.3**

It is desired to prepare a soil sample to the level of
compaction expected from the Rascal compactor after four passes (Example 2.2). The sample will be compacted with the equipment used for performing standard compaction tests:

\[ V = \frac{1}{30} \text{ ft}^3 \]
\[ L = 3 \text{ layers} \]
\[ W = 5.5 \text{ lb.} \]
\[ H = 1.0 \text{ ft.} \]

From Table 2.4, the energy corresponding to four passes is 16447 ft·lb/ft\(^3\). Therefore the number of blows/layer should be

\[ B = \frac{(1/30)(16447)}{(1)(5.5)(3)} \approx 33 \text{ blows/layer} \]

This represents approximately 25% more compactive effort than standard compaction.

**The Hybrid Approach**

Figure 2.2 shows that the OMC varies along the line of optimums, and as the compactive effort increases, the OMC decreases. When a compactor imparts a compactive effort associated with an OMC equal to the water content in the field, the compaction process is efficient. Therefore, the specified compactive effort should correspond to an OMC equal to the expected compaction water content. This combination of specification of results and specification of procedure involving a field compactive effort is called the hybrid approach.
Example 2.4

The insitu water content of St. Croix clay that is to be used to construct an embankment is 20%. The variation of the compactive effort vs. the OMC for laboratory compaction is shown in Figure 2.6. The compaction water content is equal to the OMC for a compactive effort of approximately 25,000 ft·lb/ft\(^3\). Assuming that the efficiency of the compaction in the field is approximately equivalent to laboratory compaction, three passes of Caterpillar Model 825 should be sufficient to impart this compactive effort to the soil (Figure 2.4). Once the choice of compactor has been made, a density growth curve should be developed on a test section to check the prediction.
<table>
<thead>
<tr>
<th>Method</th>
<th>OMC %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified</td>
<td>16</td>
</tr>
<tr>
<td>Standard</td>
<td>22</td>
</tr>
<tr>
<td>Low Energy</td>
<td>24</td>
</tr>
</tbody>
</table>

**FIGURE 2.6** VARIATION OF OPTIMUM MOISTURE CONTENT VS. COMPACTIVE EFFORT FOR ST. CROIX CLAY (AFTER DIBERNARDO, 1979)
III - SLOPE STABILITY CONSIDERATIONS FOR EMBANKMENT DESIGN

The Concept of the Factor of Safety

The first criterion to be satisfied in the design of a compacted embankment is the stability of the embankment side slopes. Typically, the relative stability of a slope is assessed with a factor of safety obtained by limit equilibrium analysis. The factor of safety is defined as the ratio of available strength to applied shear stress along a surface of unit thickness beneath the free surface of the slope. Each slope has a family of such surfaces. The surface with the minimum factor of safety is referred to as the critical surface. If the factor of safety on the critical surface is greater than one, the slope is considered stable. Conversely, if the factor of safety on the critical surface is less than one, the slope is considered unstable. Since the value of the factor of safety that controls a design is the minimum value, the minimum factor of safety will be referred to as the factor of safety throughout this chapter, unless specified otherwise.

Factors that complicate the relationship between the
critical surface and the expected failure surface include:

(1) The dependence of the position of the critical surface on the factor of safety.

(2) The deviations of the soil shear strength behavior from the mathematical models used to quantify it.

(3) Errors inherent in the way slope stability analysis methods calculate the normal stresses along the trial surface.

(4) Additional resistance due to end effects in actual three-dimensional surfaces.

In structural mechanics, the factor of safety is computed by comparing the resistance of a structure to a load that is applied to the critical surface. For example, if an axial load is applied to the prismatic elasto-plastic bar shown in Figure 3.1, the critical surface will be the cross-section with the smallest area. Therefore, the factor of safety will be

\[ FS = \frac{\sigma_y A_{cr}}{P} \]  

(3.1)

where

- \( \sigma_y \) = yield strength of the bar
- \( A_{cr} \) = area of the smallest cross-section of the bar
- \( P \) = the applied load

The factor of safety in this case is a measure of the proximity to failure of the cross-section on which the bar is
FIGURE 3.1 CRITICAL SURFACE OF AN AXIALLY LOADED TAPERED PRISMATIC BAR
expected to fail if the load is increased or the yield strength lowered. This is not the case for the factor of safety used in slope stability. Consider the embankment shown in Figure 3.2a, whose geometry is defined by the slope height, \( H \), and the slope angle, \( \beta \). The shear strength is assumed to be a constant everywhere in and under the embankment. If the value of the soil density increases, the factor of safety will decrease and the critical surface will move progressively deeper under the embankment provided that the soil strength does not increase because of the density increase (Figure 3.2b). Each value of the factor of safety corresponds to a different critical surface. This implies that the slope is expected to fail along a different surface than the critical surface.

Typically, a shear strength envelope is obtained by using peak values of the deviatoric stress from a triaxial test run to simulate insitu conditions. However, the soil in the slope may only be able to sustain a reduced deviatoric stress because of strain softening. Since the states of stress and strain vary greatly from position to position within an embankment, it is unlikely that the maximum values of the strength envelope can be developed simultaneously along any trial surface on which the factor of safety is to be evaluated. Therefore, using a strength envelope based on peak values of the deviatoric stress will usually result in an overestimate of the factor of safety. Although the
FIGURE 3.2a)  SIMPLE SLOPE

FIGURE 3.2b)  CRITICAL SURFACES ON A SIMPLE SLOPE

$\gamma_2 > \gamma_1$
unconsolidated-undrained (UU) shear strength envelope of unsaturated compacted clay is nonlinear if the water content is less than the OMC (Weitzel, 1979), it may be represented by a straight-line Mohr-Coulomb envelope (Figure 3.3). This simplification underestimates the strength along portions of the surface where the normal stress ($\sigma$) is in the $\sigma_a \leq \sigma \leq \sigma_d$ range (Zone II) and overestimate the strength outside these limits (Zone I). The net effect can cause the factor of safety to be overestimated or underestimated depending on the percentage of the trial surface that is in Zone I or Zone II.

Generally, the parameters that define the density and strength of soil in and under a slope will vary with position. In order to analyze such cases, it is current practice in slope stability analysis to resort to a method of slices. Many such methods exist, each with its own set of assumptions to overcome the indeterminacy of the problem. These methods assume that the local factor of safety, i.e., the factor of safety on a particular slice, is equal to the global factor of safety of the soil mass above the trial surface. This is only exactly true at limit equilibrium. Consider the unsaturated slice in Figure 3.4. Assuming that the slices are of a negligible width, the forces on the sides of the slice are equal and opposite. The summation of forces in the direction normal to the bottom of the slice gives:
Zone I, III Mohr Coulumb envelope overestimates shear strength
Zone II Mohr Coulumb envelope underestimates shear strength

FIGURE 3.3 COMPARISON OF AN ACTUAL ENVELOPE AND A MOHR–COULUMB REPRESENTATION
\[ \Delta S_r = \left( \Delta N \tan \phi_a + \Delta C_0 \right) / FS \]

**FIGURE 3.4**

FORCES ON A SLICE (AFTER SIEGEL, 1975)
\[ \Delta N = (\Delta W - \Delta S_R \cdot \sin \alpha) \cos \alpha \]  

(3.2)

where

\[ \Delta N = \text{the force normal to the bottom of the slice} \]
\[ \Delta W = \text{the weight of the slice} \]
\[ \Delta S_R = \text{the resisting force along the bottom of the slice} \]
\[ \alpha = \text{the angle that the bottom of the slice makes with the horizontal axis.} \]

If a Mohr-Coulomb envelope is used, then

\[ \Delta S_R = (\Delta N \cdot \tan \alpha_a + \Delta C_a) / FS \]  

(3.3)

where

\[ \tan \alpha_a = \text{slope of the Mohr-Coulomb envelope} \]
\[ \Delta C_a = \text{the product of the arc length of the bottom of the slice and the Mohr-Coulomb cohesion intercept} \]
\[ FS = \text{local factor of safety of the slice.} \]

Substituting equation 3.3 into equation 3.2 and rearranging yields:

\[ \Delta N = \frac{\Delta W \cdot \cos \alpha - \Delta C_a \cdot \sin \alpha \cdot \cos \alpha / FS}{1 + \tan \alpha_a \cdot \sin \alpha \cdot \cos \alpha / FS} \]  

(3.4)

where \( C_a = c_a \cdot dx / \cos \alpha \) and \( c_a \) is the Mohr-Coulomb cohesion intercept.
Since the value of $\Delta N$ in equation 3.4 is dependent on a value of the local factor of safety that is exact only at limit equilibrium, slope stability methods inherently calculate incorrect values of the normal stress on the surface on which the factor of safety is computed.

The extent of the critical surface has been assumed to be infinite in the direction perpendicular to the slope cross-section (Figure 3.5). When the lateral extent of the slope is restricted, the critical surface will arc upwards near the lateral boundaries of the slope (Figure 3.6). An approach for accounting for the three-dimensional shape of the limit equilibrium surface on the factor of safety was developed by Chen (1981). His approach consists of the following steps:

1) Locate the critical circular surface for a slope using a slope stability method such as the Simplified Janbu\textsuperscript{1} factor of safety.

2) Calculate the factor of safety on the Simplified Janbu critical surface using the Spencer method. This is taken to be the exact value of the two-dimensional factor of safety.

3) Assume that the critical three-dimensional surface is an ellipsoid attached to a cylinder (Figure 3.6) with a cross-section defined by the two-dimensional surface.

\textsuperscript{1} The reader should employ the Simplified Janbu option for circles with caution (Boutrup, 1977).
4) Using the LEMIX program (Chen, 1981), calculate the factor of safety on the three-dimensional surface. This approach will be illustrated with the following example:

**Example 3.1**

It is desired to assess the three-dimensional factor of safety of the slope shown in Figure 3.7. Assume that the critical three-dimensional surface is an ellipsoid attached to a cylinder (Figure 3.6). The cylinder is defined by the radius of the critical surface for the Simplified Janbu factor of safety which is shown in Figure 3.8. The Simplified Janbu factor of safety on this surface is 1.28. The Spencer factor of safety on the same surface is 1.34. The width of the ellipsoids, $l_x$, is defined by a specified radius which passes through the endpoint of the cylinder. Using the program LEMIX it is possible to calculate the three-dimensional factor of safety for various values of the $l_c/H$ ratio where

$H = \text{height of the slope}$

$l_c = \text{half-width of the cylindrical portion of}$

the sliding mass, i.e., the portion which is identical to the two-dimensional rotational surface (see Figure 3.9).

The results are presented in Table 3.1 and Figure 3.10. As the $l_c/H$ ratio increases, the three-dimensional factor of safety approaches the conventional two-dimensional value.
Compacted clay
\[ \gamma = 18.87 \text{ kN/m}^3 \quad (120 \text{pcf}) \]
\[ c = 38.31 \text{ kPa} \quad (800 \text{ psf}) \]
\[ \phi = 5^\circ \]
\[ H = 12.19 \text{ m} \quad (40 \text{ ft}) \]

Sand-gravel foundation
\[ \gamma = 17.30 \text{ kN/m}^3 \quad (110 \text{pcf}) \]
\[ c = 0 \]
\[ \phi = 30^\circ \]

**FIGURE 3.7 EMBANKMENT—EXAMPLE 3.1**
$X = 14.63 \text{ m (48 ft)}$
$Y = 14.90 \text{ m (48.9 ft)}$
$R_{xy} = 37.42 \text{ m (91.6 ft)}$
$H = 12.19 \text{ m (40 ft)}$

$FS_{\text{janbu}} = 1.28$
$FS_{\text{spencer}} = 1.34$

FIGURE 3.8
CRITICAL CIRCLE FOUND WITH SIMPLIFIED JANBU
FACTOR OF SAFETY
Figure 3.9
Elevation View of Three-Dimensional Critical Surface
Table 3.1  Three-Dimensional Factor of Safety vs. $l_c/H$ ratio - Example 3.1

<table>
<thead>
<tr>
<th>$l_c/H$</th>
<th>FS (3-D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.00</td>
<td>1.34</td>
</tr>
<tr>
<td>50.00</td>
<td>1.34</td>
</tr>
<tr>
<td>25.00</td>
<td>1.35</td>
</tr>
<tr>
<td>12.50</td>
<td>1.35</td>
</tr>
<tr>
<td>6.25</td>
<td>1.37</td>
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<tr>
<td>3.12</td>
<td>1.39</td>
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<tr>
<td>1.56</td>
<td>1.43</td>
</tr>
<tr>
<td>0.78</td>
<td>1.50</td>
</tr>
<tr>
<td>0.39</td>
<td>1.58</td>
</tr>
<tr>
<td>0.19</td>
<td>1.67</td>
</tr>
</tbody>
</table>
FIGURE 3.10 THREE-DIMENSIONAL FACTOR OF SAFETY OF SLOPE IN

FIGURE 3.7 VS. $\frac{I_c}{H}$

$FS$ $1.7$ $1.6$ $1.5$ $1.4$

$\frac{I_c}{H}$ $2$ $4$ $6$ $8$ $10$ $12$ $14$ $16$ $18$ $20$ $22$ $24$ $26$
However, at values of $\frac{l_c}{H}$ less than approximately 3.0, the three-dimensional factor of safety is considerably higher than the two-dimensional value. This fact can be taken advantage of in embankments of small width because the most critical surfaces (i.e., those that have a long central cylinder) cannot develop, due to geometrical limitations.

Comments on the STABL Program

STABL is a general purpose slope stability computer program that was developed for the Indiana Department of Highways. It is designed to generate a number of trial surfaces that is specified by the user and to calculate the factor of safety on these surfaces. The user specifies the zone in which he desires the surfaces to be generated. This distinguishes STABL from other slope stability programs with searches specified by central coordinates and radii of circular area.

The STABL program does not actually find the critical surface since there is an infinite number of possible random shaped surfaces. Instead, STABL outputs the ten most critical surfaces that are found and their respective factors of safety. These surfaces are plotted automatically. An advantage that results from this methodology is that the factor of safety on various paths through a slope can be compared. For example, consider the embankment in Figure 3.11. In a complete stability analysis, the critical
surface may be found to pass through till and clay layers that underly the embankment. The corresponding factor of safety is an overall factor of safety. However, if the search is restricted to surfaces that pass exclusively through the embankment itself, the factor of safety on the most critical surface within the embankment is an "intrinsic" factor of safety that is unique to the embankment. The intrinsic factor of safety is frequently an upper bound to the actual factor of safety because surfaces with lower values of the factor of safety may be found through layers that underly the embankment.

STABL has several different surface generation options. A short description of these options is included in the following paragraphs.

The SURFAC option is a command that may be used to determine the factor of safety on a surface of general shape specified by the user. The factor of safety on the surface is calculated by the Simplified Janbu method. The SURFAC option is used to check the factor of safety calculated by STABL against documented solutions.

The SURBIS option is identical to the SURFAC option with the exception that it computes the factor of safety with the Simplified Bishop method. Care should be taken that a circular surface is input because the Simplified Bishop method calculates an incorrect value of the factor of safety.
if the coordinates of the limit equilibrium surface are not circular (Bishop, 1955).

The CIRCLE option randomly generates circular surfaces and evaluates the corresponding factors of safety with the Simplified Janbu factor of safety. Searching with circular surfaces generally yields a critical surface whose factor of safety is nearly as low as the factor of safety found on the critical noncircular surface.

The CIRCL2 option is identical to the CIRCLE option except that the factor of safety is calculated by the Simplified Bishop method. This option is generally preferred to the CIRCLE option because there is more experience with the Simplified Bishop method than with the Simplified Janbu method. Also, the Simplified Bishop method does not have the convergence problems sometimes encountered with the Simplified Janbu method for slopes that have high cohesion intercepts and low friction angles.

In nonhomogeneous slopes, the critical surface may be noncircular. The same situation may arise if a homogeneous slope is subjected to a surcharge load or pseudo-static earthquake loading. STABL supplies the RANDOM option for these cases. The RANDOM option pseudo-randomly generates noncircular surfaces and evaluates the factor of safety on them with the Simplified Janbu method. An example of a randomly generated noncircular surface is shown in Figure 3.12.
The BLOCK option specifies a straight line surface between randomly chosen points in boxes of a size specified by the user (Figure 3.13). The remaining portions of these trial surfaces are generated randomly to the left of the leftmost box and to the right of the rightmost box. The Simplified Janbu method is used to calculate the factor of safety on these surfaces. The BLOCK option is especially effective when there is a weak seam or bedding plane in the slope.

Using a method of slices for the analysis of block shaped surfaces is consistent with the factor of safety used for surfaces of other shapes because it imposes the condition that the local factor of safety and the global factor of safety are equal everywhere along the trial surface. This is different than ordinary methods of calculating the factor of safety of a block shaped surface. These methods implicitly assume that the local factor of safety on the passive wedge and the active wedge is unity although the factor of safety on the central block must be greater than unity for stability.

The BLOCK2 option is identical to the BLOCK option with the exception that the portions of the trial surface to the left of the leftmost box and to the right of the rightmost box are generated to simulate Rankine active and passive zones respectively. This option was developed because the BLOCK option yields a minimum factor of safety on surfaces.
that approximate the Rankine state. Therefore, use of the BLOCK2 option permits a savings in computational effort. The BLOCK option has been retained, however, for use in heterogeneous soil profiles.

While providing advice on the use of STABL in the capacity of STABL consultant at Purdue, the author noted that certain problems arose repeatedly. These problems included:

1. surfaces with unacceptable shapes
2. the direction of surface generation
3. direction limits on benched slopes
4. variability of results due to the random generation of surfaces
5. a lack of experience with the Simplified Janbu factor of safety.

These problems are discussed in the following paragraphs.

Occasionally, when the RANDOM option is used, one or more of the ten most critical surfaces that are generated will be concave or even have a reverse curvature as shown in Figure 3.14. Such surfaces are kinematically impossible. Therefore, the factors of safety computed on these surfaces should be disregarded.

STABL assumes that a slope rises from left to right. Therefore, it generates surfaces that progress from left to right (Figure 3.15a). If the user attempts to input a slope that rises from right to left, STABL would continue to generate
FIGURE 3.14

KINEMATICALLY IMPOSSIBLE SURFACES

Convex Surface

Doubly Curved Surface
Figure 3.15a  Example of slope input backwards

LIL  leftmost initiation limit
RIL  rightmost initiation limit
LTL  leftmost termination limit
RTL  rightmost termination limit
trial surfaces from left to right, which never reach the termination range. In this case, STABL outputs error message RC-06 and halts execution of the problem. This error can be avoided simply by inputting the data so that the slope rises from left to right (Figure 3.15b).

When a user desires the initiation limits to straddle a break in the ground surface where the inclination of the slope decreases from left to right, the direction limits must be compatible with each of the segments that lie between the leftmost initiation limit and the rightmost initiation limit. Otherwise, a situation may develop where STABL will generate a trial surface that goes outside the slope (see Figure 3.16). If the trial surface does not cross the ground surface between the termination limits, error message RC-10 is output and execution of the problem is halted. If the trial surface does cross the ground surface between the termination limits, the results are meaningless, although no error is detected by the program. To avoid this difficulty, the user should define left and right initiation points for each segment of the ground surface in the initiation range. Each set of initiation limits should be executed with appropriate direction limits on a separate run. This is illustrated in Figure 3.17.

The inclination of the first two segments of circular and random shaped surfaces generated by STABL and the inclination of subsequent segments of random shaped surfaces are
FIGURE 3.15b
EXAMPLE OF CORRECTLY INPUT SLOPE
FIGURE 3.17
EXAMPLE OF CORRECT INITIATION LIMITS FOR A BENCH SLOPE
pseudo-randomly generated with the aid of a random number
generator called RANF. RANF is a computer supplied function
on CDC 6000 series computers that generates uniformly dis-
tributed random numbers between 0 and 1. However, since the
generation sequence starts with the same built-in seed each
time that the program is used, it always yields identical
random number sequences. Therefore, STABL creates identical
surfaces each time it is run on a CDC computer unless the
seed is modified. When STABL is run on an IBM computer, the
user must supply a random number generator. Some users have
employed generators that yield a different sequence each
time the program is run. When this is the case, STABL will
create different trial surfaces each time the program is
run, and consequently, slightly different values of the fac-
tor of safety.

Originally, all the STABL options for calculating the
factor of safety on noncircular surfaces employed the Carter
method (Carter, 1971). This method makes the following
assumptions:

1) All interslice forces are equal in magnitude and
   opposite in sign. Therefore, they may be neglected
   in the analysis of the factor of safety.

2) The minimum factor of safety on any trial surface
   is obtained when moments of the driving and resist-
ing forces are taken about a point that is at an
   infinite height above the slope.
3) All other assumptions are identical to those made by the Simplified Bishop method.

The expression for the factor of safety given by the Carter method does not require the calculation of interslice forces. Therefore, the reasonableness of the line of thrust need not be checked as is required by methods such as the extended Spencer method (Spencer, 1973) and the Rigorous Bishop method (Bishop, 1955). This permits the Carter factor of safety to be calculated with relatively little computational effort.

Boutrup (1977) indicated that the Carter method is identical to the Rigorous Janbu method (Janbu, 1954) with the simplifying assumption that interslice forces may be neglected from the formulation. This implies that the Carter method (or the Simplified Janbu method) does not satisfy the requirements of statics for each slice, although horizontal force equilibrium is satisfied for the soil mass above a trial surface. Experience with other incomplete equilibrium formulations such as the Fellenius method of slices indicates that incomplete equilibrium techniques give values of the factor of safety that are lower than those found by complete equilibrium techniques. Since the Simplified Janbu method is conservative compared to more exact methods, it may be worthwhile to adjust it to correspond to methods that yield results that are closer to complete equilibrium solutions.
In order to provide a guide for performing this adjustment, values of the factor of safety have been computed for simple homogeneous slopes with various values of the sideslope, $B$, and the parameter, $\lambda = \tan\frac{\alpha}{2}$, for both the Friction Circle method and the Simplified Bishop method. The differences (in percent) of the factor of safety of these two methods relative to the Simplified Janbu factor of safety using circular surfaces were also computed. The results are given in Table 3.2 and Figures 3.18 and 3.19. The Simplified Janbu factor of safety may be adjusted to simulate these familiar methods of determining the factor of safety with the following expression:

$$FS = FS_{\text{STABL}} \left[1.0 + \frac{\% \text{ error}}{100}\right]$$

(3.5)

where

$FS$ = the value of the factor of safety that has been adjusted to simulate another method.

$FS_{\text{STABL}}$ = the minimum factor of safety obtained with the Simplified Janbu method coded in the STABL program.

\[
\% \text{ error} = \frac{FS_{\text{desired method}} - FS_{\text{STABL}}}{FS_{\text{STABL}}} \times 100
\]

Values of $\%$ error may be interpolated from Figures 3.18 and 3.19 for the Friction Circle and the Simplified Bishop methods, respectively. This adjustment was developed for simple, unsaturated, homogeneous slopes with circular trial
Table 3.2  Factor of Safety for Values of Sideslope and \( \lambda \). (after Boutrup, 1977)

<table>
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<tr>
<th>Method</th>
<th>( \lambda )</th>
<th>( \cot b )</th>
<th>( \lambda = \tan^\dagger/[c/(\gamma H)] )</th>
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<td>1.90</td>
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Method 1 - Friction Circle
Method 2 - STABL2, Simplified Janbu
% error

% error

\( \lambda = \tan \phi / (c/YH) \)

% error

\( \cot \beta = 1.5 \)
\( \cot \beta = 2.5 \)
\( \cot \beta = 3.5 \)

\( \text{FIGURE 3.18 COMPARISON OF STABILS SIMPLIFIED JANBU FACTOR OF SAFETY WITH THE FRICTION CIRCLE METHOD} \)
FIGURE 3.19

COMPARISON OF STABL'S SIMPLIFIED JANBU FACTOR OF SAFETY
WITH THE SIMPLIFIED BISHOP METHOD
surfaces. However, it may be used approximately on slopes that do not satisfy these conditions.

It should be emphasized that this adjustment is only an approximate procedure to correct an incomplete equilibrium technique, i.e., a method that does not satisfy equilibrium of the slices above the trial limit equilibrium surface. It would be more consistent to use a method of calculating the factor of safety that satisfies equilibrium of each of the slices.

**Corrections to STABL**

Originally, the STABL program calculated the Simplified Bishop factor of safety by multiplying the terms of the following summation which is used for calculating the Simplified Janbu factor of safety

\[
\sum_{i=1}^{n} \frac{A_i + FS \cdot A_2}{FS + A_3} = 0 \tag{3.6}
\]

by \( y = R \cdot \cos \alpha \) (see Boutrup, 1977)

where

\( y \) = vertical distance to the bottom of the slice from the moment center

\( R \) = radius of the trial circle

\( \alpha \) = slope of the bottom of the slice

\( n \) = number of slices
\[ A_1, A_2, A_3 = \text{terms reflecting the condition of slice (see Siegel, 1975).} \]

\[ FS = \text{the factor of safety} \]

Since the value of \( R \) is constant for circular surfaces, this reduces to:

\[
\sum_{i=1}^{n} \left[ \cos \alpha \frac{A_1 + FS \cdot A_2}{FS + A_3} \right] = 0 \tag{3.7}
\]

This procedure is very efficient for computer coding because the terms of the summation for the Simplified Bishop method may be obtained by multiplying the terms of the Simplified Janbu summation by their respective values of \( \cos \alpha \).

Unfortunately, this formulation assumes that all forces acting on a slice act along the base of the slice (Howland, 1982). This is not true if pseudo-static earthquake forces or boundary surcharge forces act on a slice or if the water table extends above a slice. Therefore, STABL gave incorrect values of the Simplified Bishop factor of safety in these circumstances. To rectify this problem, the author recoded the Simplified Bishop factor of safety to include the differences in the moment arms of these forces. The expression for the factor of safety is (details of the derivation are given in Appendix A):

\[
FS = \frac{n \sum \frac{A_1}{1 + A_2/FS}}{n \sum A_3 - \sum A_4 + \sum A_5} \tag{3.8}
\]
where

\[ \text{FS} = \text{factor of safety} \]

\[ A_1 = C' \frac{\tan \theta}{\tan \alpha} \sec \theta (\Delta W (1 - k_v) + \Delta Q \cos \delta + \Delta U_b \cos \theta - \Delta U_a \cos \alpha) \tag{3.9a} \]

\[ A_2 = \frac{\tan \theta}{\tan \alpha} \cos \theta \tag{3.9b} \]

\[ A_3 = (\Delta W (1 - k_v) + \Delta U_b \cos \theta + \Delta Q \cos \delta) \sin \alpha \tag{3.9c} \]

\[ A_4 = (\Delta U_b \sin \theta + \Delta Q \sin \delta)(\cos \alpha - h/R) \tag{3.9d} \]

\[ A_5 = k_h \Delta Q (\cos \alpha - h_{eq}/R) \tag{3.9e} \]

These \( A \)-terms are not the same as the terms used for determination of the Simplified Janbu factor of safety, although the variables in equations 3.9a through 3.9e are identical to the variables used when the program was originally developed (Siegel, 1975). The factor of safety must be calculated with an iterative procedure because equation 3.8 is implicit. In order to make the solution technique for the Simplified Janbu method analogous to that of the Simplified Bishop method, the Newton-Raphson method which was used for the Simplified Janbu factor of safety (Siegel, 1975) was replaced with an implicit iterative technique. The implicit formulation of the factor of safety is believed to be more efficient in achieving a convergence on the value of the factor of safety when incomplete equilibrium analyses are performed, especially when the factor of safety is much greater or much less than the value that is initially assumed (Boutrup, 1977).
Convergence of the iteration scheme is achieved if the assumed and back-calculated values of the factor of safety on a surface differ by less than 0.005. The maximum number of iterations is limited to ten. If the solution does not converge, the coordinates of the surface and the last back-calculated value of the factor of safety are output. The editing that is required to incorporate these changes into the STABL code is listed in Appendix A.

**Strength Parameters of Compacted Claus**

**The As-Constructed Condition**

To replicate the condition existing at the end of construction, a clay sample compacted to simulate field compactive effort must be tested according to UU procedures. This means that the sample is loaded quickly so that there is no time for the excess pore pressure induced by the loading to dissipate.

Ideally, the loading should be performed to simulate the stress path that the soil undergoes in the field. Also, the loading should reflect the rotation in the direction of the stresses that occurs in the embankment. Unfortunately, the initial stresses and stress changes in an embankment are not amenable to calculation. Even if this were possible, it would be necessary to perform a prohibitive number of tests along various stress paths in order to model the strength of
the soil as a function of position in the embankment. Consequently, the current practice is to simulate the UU shear strength of unsaturated, compacted soil under an ordinary triaxial loading. It has been shown that unsaturated, compacted clays tested in this fashion have a strength line that is uniquely defined by their water content and the compactive effort they have been subjected to (Weitzel, 1979). The shear strength data for St. Croix clay are given in Table 3.3 and in Figure 3.20. These data may be represented as linear strength lines by performing linear regressions on the data in Table 3.3. The Mohr-Coulomb parameters may be obtained from the strength lines through simple trigonometric relationships (Holtz and Kovacs, 1981). The results of the regression are given in Table 3.4 and Figures 3.21 and 3.22. The correlation coefficient, $r$, decreases as the water content of the soil increases. Even so, the correlations are relatively high except when the water content of the soil is well above the OMC. Using Figure 3.21 and 3.22 it is possible to estimate the UU Mohr-Coulomb strength parameters of St. Croix clay at any water content for the compaction level at which the soil samples were prepared.

Example 3.2

It is desired to calculate the intrinsic factor of safety of the St. Croix clay embankment shown in Figure 3.23. The water content of the soil is 19% and the
Table 3.3 UU Shear Strength of Compacted St. Croix Clay
(after Weitzel, 1979)

<table>
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<th>Compaction</th>
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<th>(\sigma_3) (kPa)</th>
<th>(\frac{(\sigma_1 + \sigma_3)}{2}) (kPa)</th>
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* Low Energy compaction has 60% of the compactive effort of standard compaction.
Table 3.3 (continued)

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<td>893</td>
</tr>
</tbody>
</table>
FIGURE 3.20a  UU STRENGTH LINES OF ST. CROIX CLAY—LOW ENERGY COMPACCIÓN

FIGURE 3.20b  UU STRENGTH LINES OF ST. CROIX CLAY—STANDARD COMPACCIÓN
FIGURE 3.20c

UU STRENGTH LINES OF ST. CROIX CLAY—MODIFIED COMPACTION
Table 3.4  UU Mohr-Coulomb Parameters of Unsaturated St. Croix Clay

<table>
<thead>
<tr>
<th>Compaction</th>
<th>OMC</th>
<th>w%</th>
<th>$\phi$ (degrees)</th>
<th>c (kPa)</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Energy*</td>
<td>24.0</td>
<td>20.75</td>
<td>17.1</td>
<td>95.8</td>
<td>.96</td>
</tr>
<tr>
<td></td>
<td>22.0</td>
<td>14.0</td>
<td>88.6</td>
<td>.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>24.0</td>
<td>5.9</td>
<td>89.8</td>
<td>.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>27.0</td>
<td>2.2</td>
<td>55.9</td>
<td>.62</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>21.6</td>
<td>19.0</td>
<td>24.1</td>
<td>108.5</td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>20.1</td>
<td>17.7</td>
<td>155.1</td>
<td>.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.6</td>
<td>8.8</td>
<td>152.4</td>
<td>.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.0</td>
<td>6.1</td>
<td>67.0</td>
<td>.78</td>
<td></td>
</tr>
<tr>
<td>Modified</td>
<td>16.3</td>
<td>13.8</td>
<td>26.0</td>
<td>455.4</td>
<td>.98</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>26.5</td>
<td>416.1</td>
<td>.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.3</td>
<td>19.0</td>
<td>461.8</td>
<td>.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>19.0</td>
<td>4.7</td>
<td>405.8</td>
<td>.47</td>
<td></td>
</tr>
</tbody>
</table>

*Low Energy compaction has 60% of the compactive effort of Standard compaction.
FIGURE 3.21 THE VARIATION OF $\phi$ OF COMPACTED ST. CROIX CLAY VS. WATER CONTENT
THE VARIATION OF THE COHESION INTERCEPT OF COMPACTED ST. CROIX CLAY VS. WATER CONTENT

- Modified Compaction
- Standard Compaction
- Low Energy Compaction

Water Content

Cohesion Intercept (KPa)

500
400
300
200
100

14.0
16.0
18.0
20.0
22.0
24.0
26.0
28.0

w %
H = 30.48 m (100 ft)  
X1 = 60.96 m (200 ft)  
X2 = 6.10 m (20 ft)  

\( \gamma_m = 19.02 \text{kN/m}^3 \) (121 pcf)  
\( c = 108.5 \) kPa (2266 psf)  
\( \phi = 24.1^\circ \)  
W\% = 19

**FIGURE 3.23** SLOPE - EXAMPLE 3.2
compactive effort is Standard. It can be seen from Figures 3.21 and 3.22 that \( \phi \) and \( c \) are \( 24.1^\circ \) and \( 108.5 \text{ kPa} \) (2266 psf) respectively. The moist density of St. Croix clay at this water content and compactive effort is \( 19.04 \text{ kN/m}^3 \) (121 pcf). Using STABL, the minimum factor of safety obtained on a random shaped surface through the embankment is 2.96.

The Long Term Condition

To replicate the behavior of an embankment long after it has been constructed, it is necessary to adjust strength test procedures. Long term conditions are bounded by two conditions. The first extreme is that long after construction the embankment remains unsaturated. The proper way to run a test on the soil for this situation is the consolidated-drained (CD) test. The soil is consolidated to the expected state of stress in the embankment and then sheared at a rate that is sufficiently slow to allow any excess pore pressure developed by the loading to dissipate. Ordinarily, the requirement that the soil should be consolidated to the expected state of stress is relaxed. Instead, the soil is isotropically consolidated to several different confining pressures and subsequently sheared. Experience indicates that this has little effect on the strength envelope that is obtained for many soils. The CD test is not performed on a routine basis because it must be run very slowly to allow excess pore pressures to dissipate.
The second extreme condition is that of the embankment becoming completely saturated sometime after construction. This case, like the case of the sudden drawdown of water behind a dam, requires the use of a consolidated-undrained (CU) test. The soil is consolidated to the expected state of stress in the field and then sheared quickly. As the soil is sheared, the excess pore pressure due to the stress changes is measured. As was the case with CD parameters, the requirement of consolidating the sample to the in situ state of stress is usually relaxed.

A major problem involved in analysis of the second extreme condition is the development of excess pore pressure. The excess pore pressure that develops in the laboratory is caused by the changes in the stresses that are applied to the sample. The excess pore pressure in the field is caused by the increase of density due to saturation and the gradual stress changes caused by displacements within the embankment that arise from changes from the UU to CU or CD conditions. Therefore, the excess pore pressures measured in a triaxial test cannot be used to predict excess pore pressures in an embankment.

An approximate method can be used to insure that the excess pore pressures will be zero or negative. It has been shown that the pore pressure parameter, \( A_p \), which relates excess pore pressure and deviator stress, varies with the overconsolidation ratio (Henkel, 1956). Typical results are
shown in Figure 3.24. The overconsolidation ratio (OCR) of a compacted clay may be estimated with the ratio of the compactive prestress to the overburden pressure. Therefore, the depth in the embankment above which positive pore pressure can not develop regardless of stress change (i.e., $A_f \leq 0$) is:

$$H = \frac{P_s}{OCR_o} \gamma_m$$  \hspace{1cm} (3.10)

where

- $H$ = depth beneath embankment surface to which $u \leq 0$
- $u$ = excess pore pressure due to the stress change
- $P_s$ = compactive prestress
- $\gamma_m$ = moist density of the soil
- $OCR_o$ = the OCR at which $u \leq 0$

Results in Figure 3.24 indicate that $OCR_o$ will vary between 4 and 5 for a natural saturated clay. It is reasonable to assume that the same is true for compacted clays. Typical results for an embankment built of compacted St. Croix clay are given in Table 3.5. These results assume that $OCR_o = 4.0$. Values of $P_s$ and $\gamma_m$ are taken from DiBernardo (1979).

Actually, the excess pore pressure caused by shear of compacted clays at strains smaller than failure strains can be larger than is predicted by use of the pore pressure parameter in Figure 3.24. Therefore, $OCR_o$ will not
FIGURE 3.24 \( A_f \) VS. OVERCONSOLIDATION RATIO
(AFTER HENKEL, 1956)
Table 3.5  Depth to Zone of Positive Pore Pressure Development

<table>
<thead>
<tr>
<th>Compactive Effort</th>
<th>GOMC(%)</th>
<th>w(%)</th>
<th>$p_s$ (kPa)</th>
<th>$\gamma_m$ (kN/m$^3$)</th>
<th>H* (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>22.0</td>
<td>19.5</td>
<td>430.6</td>
<td>18.70</td>
<td>5.76</td>
</tr>
<tr>
<td></td>
<td>22.0</td>
<td>271.8</td>
<td></td>
<td>19.51</td>
<td>3.48</td>
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<tr>
<td></td>
<td>25.0</td>
<td>68.6</td>
<td></td>
<td>19.42</td>
<td>.88</td>
</tr>
<tr>
<td>Modified</td>
<td>16.5</td>
<td>14</td>
<td>1166.3</td>
<td>20.17</td>
<td>14.47</td>
</tr>
<tr>
<td></td>
<td>16.5</td>
<td>717.9</td>
<td></td>
<td>20.97</td>
<td>8.56</td>
</tr>
<tr>
<td></td>
<td>19.5</td>
<td>212.7</td>
<td></td>
<td>20.63</td>
<td>2.58</td>
</tr>
</tbody>
</table>

*$H = \text{depth to the bottom of the zone of positive pore pressure development}$
correspond exactly to a state of zero excess pore pressure generation. In any event, the depth above which the excess pore pressure is taken to be less than zero will sometimes be only a small portion of the embankment height. Therefore, stress changes will cause an increase in pore pressures. Unfortunately, the stress changes and hence the pore pressure changes can not be predicted easily. Consequently, the only convenient approach is to assume that pore pressure in an embankment is equal to the head of water above the location in question.

The CU friction angle and cohesion intercept of laboratory compacted and saturated St. Croix clay are approximately 20° and 15 kPa regardless of compaction variables or stress path (Johnson, 1979). Therefore, it is possible to develop a graph of the intrinsic factor of safety for every possible geometry. For the unsaturated case, the moist density of the embankment soil is used. For the saturated case, the saturated density of the embankment soil is used and a water table is assumed to run along the free surface of the embankment. In these examples the effective stress parameters are assumed to be the same for both saturated and unsaturated compacted clay. The results are given in Table 3.6 and Figures 3.25 and 3.26. These results were developed using the Simplified Janbu factor of safety on circular surfaces that were restricted to remain above the elevation of the toe and to exit the slope less than 10 meters from the crest of the slope.
Table 3.6 Intrinsic Simplified Janbu Factors of Safety of Compacted St. Croix Clay Embankments Using Laboratory Compacted CU Shear Strength

<table>
<thead>
<tr>
<th>Unsaturated Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>5m</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>10m</td>
</tr>
<tr>
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<td></td>
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<tr>
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</tr>
<tr>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>20m</td>
</tr>
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<td></td>
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<tr>
<td></td>
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<tr>
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<tr>
<td></td>
</tr>
<tr>
<td>30m</td>
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</tr>
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</table>
Table 3.6 (continued)

<table>
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<tr>
<th>H</th>
<th>θ</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 m</td>
<td>25</td>
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<td>1.56</td>
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<td>45</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>0.99</td>
</tr>
<tr>
<td>10 m</td>
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<td>1.95</td>
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<td></td>
<td>20</td>
<td>1.47</td>
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<tr>
<td></td>
<td>25</td>
<td>1.16</td>
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<td></td>
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<td>0.96</td>
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<td>1.66</td>
</tr>
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<td></td>
<td>15.0</td>
<td>1.37</td>
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<td>17.5</td>
<td>1.16</td>
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<tr>
<td></td>
<td>20.0</td>
<td>0.99</td>
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<tr>
<td>30 m</td>
<td>10</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>12.5</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>17.5</td>
<td>0.97</td>
</tr>
<tr>
<td>40 m</td>
<td>10.0</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>12.5</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>1.05</td>
</tr>
</tbody>
</table>
FIGURE 3.25

UNSATURATED LONGTERM INTRINSIC FACTOR OF SAFETY OF ST. CROIX CLAY EMBANKMENTS

\( \gamma = 19.38 \text{ kn/m}^3 \)
\( \phi = 20^\circ \)
\( C = 15 \text{ kn/m}^2 \)
$Q_{sat} = 20.80 \text{ kN/m}^3$

$\phi' = 20^\circ$

$C' = 15 \text{ kN/m}^3$

**FIGURE 3.26** SATURATED LONGTERM INTRINSIC FACTOR OF SAFETY OF ST. CROIX CLAY EMBANKMENTS.
**Interpretation of the Factor of Safety**

This section addresses the engineering interpretation of a computed factor of safety and the choice of a minimum factor of safety to achieve embankment stability. The general rule of thumb in the past has been that a value of the factor of safety for earthworks between 1.3 and 1.5 is acceptable (Terzaghi and Peck, 1967). The lower limit is for maximum loading conditions and the upper limit is for service conditions (Meyerhoff, 1970). If the engineer designs an embankment slope with conventional methods of analysis and these values of the factor of safety, it is expected that the embankment will perform satisfactorily. Up to this point, the factor of safety that has been discussed has been a strength factor of safety. Since factors such as the dependence of the position of the critical surface on the factor of safety make the interpretation of the strength factor of safety difficult, supplemental methods of evaluating the proximity of a slope to limit equilibrium are useful. Two such methods are the geometric interpretation and the probabilistic approach.

**The Geometric Interpretation**

The factor of safety is usually defined as the ratio of the strength in the limit equilibrium state to the actual stress. The choice of strength as the variable for this comparison is arbitrary, however. It is equally valid to
define the factor of safety in terms of some relevant dimension of the slope geometry. Consider a typical slope in Figure 3.27a. Generally, the material parameters of the soil, $\phi$, $c$, and $\gamma$ are known. The height, $H$, and the width, $W$, are specified by need. The only variable that is not fixed is the sideslope, $B$. It seems appropriate, therefore, to define the factor of safety as the ratio of the sideslope at limit equilibrium, $B_{cr}$, to the actual sideslope, $B$, or, i.e.,

$$FS_B = \frac{B_{cr}}{B}$$  \hspace{1cm} (3.11)

The value of the side slope at limit equilibrium, $B_{cr}$, may be obtained graphically by plotting the strength factor of safety vs. the side slope for fixed values of $\phi$, $c$, $\gamma$ and $H$. $B_{cr}$ is the point on the curve that corresponds to a strength factor of safety of 1.0 (Figure 3.27b).

**Example 3.3**

It is desired to determine the relationship between the intrinsic strength factor of safety of an embankment and the $B_{cr}/B$ ratio. The height of the embankment is 15.24 m (50 ft). The soil density, friction angle, and cohesion intercept are taken to be 19.34 kN/m$^3$ (123 pcf), 21°, and 14.85 kPa (310 psf), respectively, to simulate long-term behavior.
Earthen slope considered for defining the side slope factor of safety.
FIGURE 3.27 b

PROCEDURE TO DETERMINE THE LIMIT EQUILIBRIUM VALUES OF THE SIDESLOPE AND HEIGHT
The first step is to calculate the values of the intrinsic strength factor of safety for various side slopes. This can be done with the computer program in Appendix C. The results are given in Table 3.7 and Figure 3.28. The value of $B_{cr}$ obtained from Figure 3.28 is $39.1^\circ$. The ratio, $B_{cr}/B$, is plotted vs. the strength factor of safety in Figure 3.29. For this case, the ratio $B_{cr}/B$ is always greater than the value of the strength factor of safety. One shortcoming of this approach occurs when there is no value of sideslope for which the factor of safety reduces to 1.0. In such a case the factor of safety can not be defined as $B_{cr}/B$. Fortunately, in such cases, the shear strength is so high that a stability analysis usually is not necessary.

A factor of safety based on the ratio of the height at limit equilibrium to the actual height, $H_{cr}/H$, may be defined in a manner analogous to the development of the side slope factor of safety by holding $\phi$, $c$, $\gamma$, and $B$ constant. The actual geometry of the embankment and the hypothetical geometry at limit equilibrium are shown in Figure 3.30. The graphical method for finding the critical height is illustrated in Figure 3.28.

Example 3.4

It is desired to find the variation of the ratio $H_{cr}/H$ of an embankment vs. the intrinsic strength factor of safety. The side slope of the embankment is 1 to 2
Table 3.7  Sideslope Factor of Safety Calculation - Example 3.3

<table>
<thead>
<tr>
<th>$\theta^o$</th>
<th>FS</th>
<th>$\theta_{cr}/\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>1.670</td>
<td>1.777</td>
</tr>
<tr>
<td>24</td>
<td>1.535</td>
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<td>26</td>
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<td>1.331</td>
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<td>30</td>
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<td>36</td>
<td>1.069</td>
<td>1.086</td>
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<tr>
<td>38</td>
<td>1.024</td>
<td>1.027</td>
</tr>
<tr>
<td>39</td>
<td>1.002</td>
<td>1.003</td>
</tr>
<tr>
<td>40</td>
<td>.984</td>
<td>.978</td>
</tr>
</tbody>
</table>
FIGURE 3.30  EARTHEN SLOPE CONSIDERED TO DEFINE THE FACTOR OF SAFETY BASED ON THE HEIGHT CRITERION
Assume the values of the soils material parameters are the same as in example 3.3. The height is variable. The strength factor of safety was obtained for various heights with the computer program in Appendix C. The results are given in Table 3.8 and Figure 3.31. The critical height is approximately 44.2 m (145 ft). The critical value is divided by each height corresponding to a value of the strength factor of safety. The results are given in Table 3.8 and Figure 3.32. The ratio, \( H_{cr}/H \), is substantially larger than the value of the strength factor of safety. This is not always the case. For example, if \( \gamma, t, \) and \( c \) were taken to be 19.59 kN/m\(^3\) (124.6 pcf), 7.3\(^0\), and 95.02 kPa (1984 psf), respectively, to simulate the unsaturated short term strength of St. Croix clay compacted 2% wet of OMC, the results would be those shown in Figure 3.33. In this case the difference between the ratio \( H_{cr}/H \) and the intrinsic strength factor of safety is smaller. This example demonstrates that the strength factor of safety corresponds to different values of the \( H_{cr}/H \) ratio, depending on the values of \( t \) and \( c \). An analogous remark applies to the side slope ratio.

The Probabilistic Approach

The conventional definition of the factor of safety overlooks the variability of material parameters because it uses deterministic (single-valued) soil properties. The
Table 3.8  Factor of Safety Calculated Based on Height Criterion - Example 3.4

<table>
<thead>
<tr>
<th>H (ft)</th>
<th>$H_{cr}/H$</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14.500</td>
<td>3.456</td>
</tr>
<tr>
<td>20</td>
<td>7.250</td>
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<td>25</td>
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<td>35</td>
<td>4.143</td>
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<td>40</td>
<td>3.625</td>
<td>1.533</td>
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<td>45</td>
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<td>2.90</td>
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<td>2.636</td>
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<tr>
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<td>2.071</td>
<td>1.235</td>
</tr>
<tr>
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<td>1.933</td>
<td>1.207</td>
</tr>
<tr>
<td>80</td>
<td>1.813</td>
<td>1.163</td>
</tr>
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<td>85</td>
<td>1.706</td>
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<tr>
<td>90</td>
<td>1.611</td>
<td>1.143</td>
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<td>1.526</td>
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<td>140</td>
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</tbody>
</table>
FIGURE 3.31  DETERMINATION OF $H_{cr}$  — EXAMPLE 3.4
Figure 3.32  Factor of Safety Based on the Height Criterion vs. The Strength Factor of Safety—Example 3.4
FIGURE 3.33  EXERCISE 3.4 REPEATED WET OF OPTIMUM
variability is only taken into account through the personal judgment exercised in the selection of the soil properties. Variability in soil properties arises through:

(1) material and sample non-homogeneities that do not represent the whole soil ("true" variability)
(2) sampling errors caused by disturbance during the sampling process
(3) errors that occur when tests are not performed according to a standard.

Uncertainty, however, is not limited to the variability observed in the basic variables. Analytical models and laboratory and field experiments are often only an idealized representation of reality. Predictions made on the basis of these models and experiments may be inaccurate and contain uncertainty. Therefore, the capacity of a slope to resist loading will not have a unique value. Similarly, the load (or demand) on the trial surface will have a distribution of values. These distributions can be represented by probability density functions. The probability that a slope will reach a state of limit equilibrium equals the probability that the capacity will be less than the demand (Yao, 1982), i.e.,

\[ P_f = p(R<S) = \int_{R_{\text{min}}}^{S_{\text{max}}} f_R(s) \cdot F_S(s) \, ds \]  \hspace{1cm} (3.12)
where

- $R$ denotes strength (or capacity)
- $S$ denotes load (or demand)
- $f_S$ is the probability density function (PDF) of the load
- $F_R$ = cumulative distribution function (CDF) of the resistance.
- $R_{\text{min}}$ = minimum value of the strength (see Figure 3.34)
- $S_{\text{max}}$ = maximum value of the load (see Figure 3.34)

Capacity-demand problems are simplified in the case of slope stability analysis because the capacity-demand ratio for a slope of specified geometry and material parameters is identical to the factor of safety. Therefore, the probability of failure, $P_f$, is also equal to the probability that the factor of safety is less than one, or

$$P_f = P(\text{FS} < 1.0)$$  \hspace{1cm} (3.13)

This calculation is performed by integrating the probability density function of the factor of safety up to a value of one. The density function of the factor of safety depends on the distributions of the soil density and shear strength variables. When the distribution of these variables is known, the density function of the factor of safety may be obtained by Simulation (Yao, 1982).

Simulation is essentially a controlled statistical sampling technique which can be used to study complex
FIGURE 3.34  PROBABILITY DENSITY (OR DISTRIBUTION)
FUNCTIONS OF CAPACITY AND DEMAND
stochastic systems when analytical and/or numerical techniques do not suffice. A necessary part of any such procedure is an algorithm for random number generation. A random number generator produces sequences with probability density functions that are uniformly distributed between 0 and 1 and that possess the appearance of randomness. Most computer systems have a built-in random number generator. The inverse transformation method, often called the Monte-Carlo method of simulation, is used to generate non-uniform random number X, with cumulative distribution function F_X(x). The algorithm is very simple:

1. Generate number Q uniformly distributed between 0 and 1.
2. Return X = F_X^{-1}(Q)

(F_X^{-1} is the inverse function corresponding to F_X)

This algorithm assumes that the equation F_X(x) = Q can be solved explicitly. Other distributions can be simulated by direct generation methods such as composition methods and rejection-acceptance methods (Fishman, 1978). The probability of failure equals the cumulative distribution function of the factor of safety up to a value of one, i.e.,

\[ P_f = F_{FS} (FS = 1.0) \]

(3.14)

Since the Monte-Carlo method uses the actual distributions
of the variables that affect the factor of safety, it can, in principle, generate the actual density function of the factor of safety. Unfortunately, this requires a very large number of simulations and consequently, a great amount of computational effort.

A simpler approach used in practice to obtain the probability of failure is to assume a distribution of the factor of safety and to generate statistical moments of this distribution from the statistical moments of the dependent variables (density and shear strength parameters). Two procedures to obtain these moments are the Taylor Series expansion method and the Point Estimates Method (Yao, 1982).

The Taylor Series expansion requires partial derivatives of the factor of safety with respect to each of the variables that affect the factor of safety (Harr, 1977). The difficulty of evaluating these derivatives limits the usefulness of the Taylor Series expansion for modelling the density function of the factor of safety.

The Point-Estimates Method is an approximate procedure for calculating the statistical moments of the factor of safety which does not require evaluation of the derivatives of the factor of safety relative to the variables affecting it (Rosenblueth, 1975). This method approximates the statistical moments of a function Y (i.e., the factor of safety) by "replacing" the distribution of the variables
affecting the function with point estimates (or weights) at properly selected values of the variables. For example, the estimated value of the $n$th statistical moment of the function $Y$ that depends on two random variables may be found with the following two-point procedure:

$$E(Y^n) = P_{++}Y_{++} + P_{+-}Y_{+-} + P_{-+}Y_{-+} + P_{--}Y_{--}$$  \hspace{1cm} (3.15)$$

$$y_{\pm \pm} = Y[\bar{x}_1 \pm \sigma_{x_1}, \bar{x}_2 \pm \sigma_{x_2}]$$ \hspace{1cm} (3.16)$$

and

$$P = \text{weighting factor}$$

$$x_1, x_2 = \text{random variables}$$

$$\sigma_{x_1}, \sigma_{x_2} = \text{standard deviations of the random variables}$$

$$\bar{x} = \text{mean value of the variable, } x$$

If $x_1$ and $x_2$ have symmetric distribution, the weighting factor is $1/4$. The point estimates method can be adapted to handle any number of correlated and/or uncorrelated random variables (Rosenblueth, 1975).

To illustrate the application of the probabilistic approach to slope stability problems, the friction-circle slope stability program (Appendix C) was adapted to calculate the probability of failure of a simple slope. The resulting program is given in Appendix E. This program assumes that the material parameters defining the soil strength and density have symmetric distributions. These
distributions are defined with mean values, coefficients of variation, and upper and lower bounds. The material parameters are assumed to be uncorrelated. The probability density function of the factor of safety is assumed to be beta distributed (Appendix D). The mean and variance (i.e., the first two statistical moments) of the factor of safety are obtained with a two-point point estimates procedure. The lower bound of the distribution of the factor of safety is the factor of safety that is obtained using the maximum value of the soil density and the minimum values of the Mohr-Coulomb parameters, \( \phi \) and \( c \). The upper bound is the factor of safety that is obtained using the minimum value of the density and maximum values of the Mohr-Coulomb parameters. Finally, the probability of failure is calculated by integrating equation 3.13 numerically.

**Example 3.5**

It is desired to determine the probability of failure of the slope shown in Figure 3.35. The average ratio \( \bar{y}H/c \) is 25/3

where

\[ \bar{y} = \text{the mean value of the soil density} \]
\[ \bar{c} = \text{the mean value of the cohesion intercept of the soil} \]
\[ H = \text{the height of the slope} \]

The variability of the material parameters is given in Table
Figure 3.3.5: Slope — Example 3.5

- $\phi = 5^\circ$
- $\gamma = 20 \text{ kN/m}^3$
- $c = 100 \text{kPa}$
- $H = 41.66 \text{ m}$
- $\beta = 20^\circ$
3.9. The mean factor of safety corresponding to the mean values of the soil density and the Mohr-Coulomb strength parameters is found to be 1.28 using the friction circle method. The program in Appendix E is used to quantify the effect of the variability of the cohesion intercept on the probability of failure. The results are reported in Table 3.10 and Figure 3.36. An increase in the variability of the cohesion intercept increases the probability of slope failure. Therefore, slopes with equal mean factors of safety are not necessarily equally safe. Historical records indicate that earth dams designed with ordinary techniques for a factor of safety of 1.3 to 1.5 have a rate of failure on the order of one in two thousand (Meyerhof, 1970). However, further work needs to be done in order to recommend design values of the probability of failure for various situations such as sudden drawdown and long-term conditions.

Frequently, the probability of failure that is calculated based on ordinary estimates of material variability is much higher than the values reported by Meyerhoff. This discrepancy arises because designers typically use a lower bound on their strength parameters rather than mean values when they calculate the factor of safety. If mean values of the strength parameters were used, a higher factor of safety has to be employed to assure adequate performance.
Table 3.9 Material Parameters - Example 3.5

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>( \phi_{\min} )</td>
<td>0.6 °</td>
</tr>
<tr>
<td>( \phi_{\max} )</td>
<td>1.4 °</td>
</tr>
<tr>
<td>( \gamma_{\min} )</td>
<td>0.85 °</td>
</tr>
<tr>
<td>( \gamma_{\max} )</td>
<td>1.15 °</td>
</tr>
<tr>
<td>( c_{\min} )</td>
<td>0.66 c</td>
</tr>
<tr>
<td>( c_{\max} )</td>
<td>1.33 c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>( \gamma )</td>
<td>0.1</td>
</tr>
<tr>
<td>( c )</td>
<td>variable</td>
</tr>
</tbody>
</table>

Note: The notation \( \hat{\phi} \) represents the angle, \( \phi_{\min} \) and \( \phi_{\max} \) are the minimum and maximum values of the angle, \( \gamma_{\min} \) and \( \gamma_{\max} \) are the minimum and maximum values of another parameter, and \( c_{\min} \) and \( c_{\max} \) are the minimum and maximum values of a third parameter.
Table 3.10 Probability of Failure vs. Variability of Cohesion - Example 3.5

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>FS</th>
<th>$\mu_c$</th>
<th>$P_f$</th>
</tr>
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<td>.016</td>
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<td></td>
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<td>.253</td>
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<td></td>
<td></td>
<td>.33</td>
<td>.284</td>
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<td>.01</td>
<td>.226</td>
</tr>
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<td></td>
<td>.05</td>
<td>.292</td>
</tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>.20</td>
<td>.401</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.30</td>
<td>.449</td>
</tr>
<tr>
<td></td>
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<td>.33</td>
<td>.462</td>
</tr>
</tbody>
</table>

$\beta$ = sideslope
FS = factor of safety based on the mean value of $\phi$, $c$, and $\gamma$.
$\mu_c$ = coefficient of variation of the cohesion
$P_f$ = probability of failure.
Coefficient of variation of cohesion

FIGURE 3.36 PROBABILITY OF FAILURE VS. COEFFICIENT OF VARIATION OF THE COHESION INTERCEPT - EXAMPLE 3.5
IV - SETTLEMENT CONSIDERATIONS FOR EMBANKMENT DESIGN

Once an embankment has been checked for intrinsic and overall stability of its sideslopes, the settlement of the embankment should be investigated. Embankment settlement is comprised of displacements that occur within the embankment itself as well as compression of fine grained soil layers that underlie the embankment. Displacements within the embankment itself are caused principally by:

1) partially saturated compression under the body forces of the fill. This occurs as rapidly as the fill is constructed (DiBernardo, 1979).
2) volume change due to an increase in moisture content of the compacted embankment. This is thought to be the major source of displacements in compacted clay embankments.

Compression of the fine grained soil layers below the embankment consists of:

1) immediate settlement that occurs at constant volume. Immediate settlement is completed at the end of
embankment construction (consequently, it has no effect on the embankment performance and may be neglected).

2) time dependent consolidation settlement that occurs as excess pore pressure induced by the embankment construction dissipates

3) secondary compression settlement that occurs after time dependent consolidation is complete (calculation of the secondary compression settlement is frequently omitted because it is assumed to be small compared to the consolidation settlement).

Saturation Induced Displacement of Compacted Clay Embankments

When a sample of compacted clay becomes saturated, it may either swell or settle depending on the mineralogy of the clay, the presaturation water content, the compactive effort, and the overburden pressure. Prediction models have been developed for the volume change due to saturation for laboratory compacted St. Croix clay (DiBernardo, 1979). An extension of this model for field compacted soils with a range of plasticity index values was proposed by Lin (1981). However, since the data base for these models is limited, it is recommended that the volume change due to saturation be estimated from tests on the soil that will be used in the embankment.
To insure that the testing will best simulate the volume change in the embankment due to saturation, the following guidelines should be followed:

1) The soil sample should be compacted to the expected state of compaction in the field according to the procedure described in Chapter II.

2) The presaturation water content of the soil should be the same as is expected in the actual embankment.

3) The soil sample should be subjected to a pressure equal to the overburden pressure in the embankment. Therefore, to simulate the variation of volume change with depth, the test samples must be subjected to a range of overburden pressures.

4) Measurement of volume change of the soil sample should be made from the time that the soil is back-pressure saturated until the volume change ceases. For details of the test procedure, see DiBernardo (1979).

If it is assumed that all volume change occurs vertically, it is possible to estimate the settlement of the surface of the embankment with the following expression:

\[ S = \frac{1}{100} \int_{0}^{H} U(z) \, dz \]  

(4.1a)

where

\[ S = \text{the settlement due to saturation} \]
\( U(z) = \) the % volume change at the depth \( z \)

\( z = \) the depth beneath the top of the embankment

\( H = \) the height of the embankment

If the distribution \( U(z) \) can be idealized as a sequence of strata each with its own uniform value of \( U \), then equation 4.1a may be evaluated numerically with the following expression

\[
S = \frac{1}{100} \sum_{i=1}^{n} U_i \cdot \Delta z_i \tag{4.1b}
\]

where

\( U_i = \) the % volume change of stratum \( i \)

\( \Delta z_i = \) the thickness of stratum \( i \)

\( n = \) the number of strata

It is interesting to note that it is possible for the upper portion of an embankment to be swelling while the lower portion is settling. In fact, it is theoretically possible to build an embankment whose net saturation settlement is nil by specifying the compaction so that there are compensating zones of swelling and settling soil within the embankment. Generally, however, it is best to design the embankment to settle slightly because swelling of soil beneath the road bed can cause severe pavement distress.
Consolidation Settlement of Compressible Soil Layers Beneath the Embankment

Magnitude of Settlement

When a saturated clay sample is subjected to an axial stress change in a standard consolidation test, an excess pore pressure equal to the stress change is induced in the sample. As time proceeds, the excess pore pressure will dissipate and the sample will settle by a volume equal to the volume of the dissipated pore water. The relationship between the axial stress and the void ratio after all of the excess pore pressure has been dissipated is described in Figure 4.1a. For purposes of analysis, the relationship may be replaced with the representation shown in Figure 4.1b which is defined by the compression index, $C_c$, the recompression index, $C_r$, and the preconsolidation pressure, $\sigma'_p$.

If the existing axial pressure, $\sigma'_v$, is equal to the preconsolidation pressure, the soil is normally consolidated. The settlement of the soil due to the axial pressure change is:

$$ S = \frac{H}{1 + e_o} C_c \log \frac{\sigma'_v + \Delta \sigma}{\sigma'_p} \quad (4.2) $$

where

- $S = \text{the settlement of the clay layer}$
- $H = \text{the thickness of the clay layer}$
FIGURE 4.1a  TYPICAL $e - \log \sigma'$ RELATIONSHIP

FIGURE 4.1b  SIMPLIFICATION OF TYPICAL $e - \log \sigma'$ RELATIONSHIP FOR ANALYTICAL PURPOSES
\[ \Delta \sigma = \text{the increase in axial stress of the clay layer} \]

\[ e_o = \text{the initial void ratio of the clay layer} \]

\[ \sigma'_w = \text{the existing overburden pressure} \]

If the existing overburden pressure is less than the preconsolidation pressure, the soil is overconsolidated. The settlement due to the axial pressure change is:

\[ S = \frac{H}{1 + e_o} \left[ C_r \cdot \log \frac{\sigma'_v + \Delta \sigma}{\sigma'_v} \right] \quad (4.3) \]

when \((\sigma'_v + \Delta \sigma) \leq \sigma'_p\), or

\[ S = \frac{H}{1 + e_o} \left[ C_r \cdot \log \frac{\sigma'_v + \Delta \sigma}{\sigma'_v} + C_c \cdot \log \frac{\sigma'_v + \Delta \sigma - \sigma'_p}{\sigma'_p} \right] \quad (4.4) \]

when \((\sigma'_v + \Delta \sigma) > \sigma'_p\).

If the existing overburden pressure is greater than \(\sigma'_p\), the soil is underconsolidated. The settlement due to the axial pressure change is:

\[ S = \frac{H}{1 + e_o} \left[ C_c \cdot \log \frac{\sigma'_v + \Delta \sigma}{\sigma'_p} \right] \quad (4.5) \]

These expressions for settlement are exact provided that

1) the values of \(e_o\), \(C_r\), \(C_c\), \(\sigma'_v\), and \(\sigma'_p\) are the same in the consolidating layer as measured in
the consolidation test

2) the stress change in the consolidating layer does not vary with depth

3) the excess pore pressure caused by the embankment loading is the same as in the consolidation test.

Deviations from the first two assumptions may be accounted for by dividing the consolidating layer into artificial strata. Deviations from the third assumption may be accounted for by use of a correction factor that will be discussed later.

To determine the value of $e_o$ in each stratum, one can assume that the difference in the initial void ratio between the center of the stratum and the position of the soil sample, $\Delta e_o$, will be equal to the change in void ratio that would occur if the soil sample were to move along the $e - \log\sigma'$ curve by a stress change equal to the difference in overburden pressure between the position of the soil sample and the center of the stratum.

If the center of the stratum is above the elevation of the sample, $\Delta e_o$ will be positive. If, in addition, the soil is overconsolidated above the elevation of the sample

$$\Delta e_o = C_r \log\left(\frac{\sigma_{\text{sample}}}{\sigma'_{vo}}\right)$$

(4.6a)

where
\( \sigma_{\text{sample}} \) = the overburden pressure in the soil sample

\( \sigma_{\text{vo}} \) = the overburden pressure in the stratum

If the soil above the elevation of the sample is considered to be underconsolidated:

\[
\Delta \varepsilon_0 = 0 \quad (4.6b)
\]

If the soil is underconsolidated at pressures above the preconsolidation pressure of the sample, \( \sigma'_{\text{p}} \), and overconsolidated at pressures below \( \sigma'_{\text{p}} \)

\[
\Delta \varepsilon_0 = C_r \cdot \log \left( \frac{\sigma'_{\text{p}}}{\sigma'_{\text{vo}}} \right) \quad (4.6c)
\]

If the soil is normally consolidated at pressures above \( \sigma'_{\text{p}} \) and overconsolidated at pressures beneath \( \sigma'_{\text{p}} \)

\[
\Delta \varepsilon_0 = C_c \cdot \log \left( \frac{\sigma'_{\text{sample}}}{\sigma'_{\text{p}}} \right) + C_r \cdot \log \left( \frac{\sigma'_{\text{p}}}{\sigma'_{\text{vo}}} \right) \quad (4.6d)
\]

If the center of the stratum is below the elevation of the sample, \( \Delta \varepsilon_0 \) will be negative. If the soil is overconsolidated beneath the elevation of the sample

\[
\Delta \varepsilon_0 = - C_r \cdot \log \left( \frac{\sigma'_{\text{vo}}}{\sigma'_{\text{sample}}} \right) \quad (4.6e)
\]

If the soil is overconsolidated at pressures below \( \sigma'_{\text{p}} \) and normally consolidated at pressures above \( \sigma'_{\text{p}} \).
\[ \Delta \varepsilon_o = - C_r \cdot \log \left( \frac{\sigma'_p}{\sigma_{\text{sample}}} \right) - C_c \cdot \log \left( \frac{\sigma'_w}{\sigma'_p} \right) \]  

(4.6f)

If the soil is overconsolidated throughout its stress path:

\[ \Delta \varepsilon_o = - C_r \cdot \log \left( \frac{\sigma'_p}{\sigma_{\text{sample}}} \right) \]  

(4.6g)

If the soil is underconsolidated throughout its stress path:

\[ \Delta \varepsilon_o = 0 \]  

(4.6h)

Like the void ratio, \( \sigma'_p \) varies with depth in a clay layer. Typically, \( \sigma'_p \) decreases with depth until the soil is normally consolidated. Thereafter it assumes a value equal to the overburden pressure (Holtz and Kovacs, 1981). This is illustrated in Figure 4.2a. The variation of \( \sigma'_p \) with depth may be represented by two straight-line segments (Figure 4.2b). Once the values of \( \sigma'_p \) at the endpoints of the two segments are specified, the value of \( \sigma'_p \) may be obtained by interpolation in any stratum. Developing the \( \sigma'_p \) profile requires more consolidation tests than are normally run. This is not prohibitive, however, because these consolidation tests can be run as constant gradient tests and completed in one work day.

The change in vertical pressure due to the embankment load can be approximated with the expressions presented in Appendix F. \( C_c \) and \( C_r \) are assumed constant for a given clay layer.
FIGURE 4.2a
TYPICAL PRECONSOLIDATION PRESSURE PROFILE

FIGURE 4.2b
SIMPLIFIED PRECONSOLIDATION PRESSURE PROFILE
Once the values of $o_0$, $C_r$, $C_c$, $\sigma'_v$, $\sigma'_p$ and $\Delta \sigma$ are determined in each stratum, the settlement of the consolidating layer may be calculated by evaluating equations 4.2 - 4.5 for each stratum. The computer program in Appendix G has been provided to facilitate computations.

**Example 4.1**

It is desired to make a preliminary estimate of the consolidation settlement of the embankment built over the clay layer shown in Figure 4.3 without performing consolidation tests. The compression index may be estimated using an appropriate correlation with standard index tests (Terzaghi and Peck, 1967). The recompression index is assumed to be one tenth of the compression index. For purposes of this example, $\sigma'_p$ was taken to be 95.76 kPa (2000 psf) at the depth of the sample. $\sigma'_p$ was assumed to equal the overburden pressure when the overburden pressure exceeds 95.76 kPa.

It is instructive to consider the stress changes in the clay layer caused by the embankment before actually calculating settlement. This was done with the computer program in Appendix F. The results, which are shown in Table 4.1, indicate that:

1) The increase in vertical stress at a given depth in the clay layer is approximately constant across the embankment

2) The vertical stress increase does not attenuate
FIGURE 4.3
EMBANKMENT – EXAMPLE 4.1
Table 4.1 Stress Change Beneath Embankment – Figure 4.3

<table>
<thead>
<tr>
<th>x (m)</th>
<th>z (m)</th>
<th>( \Delta \sigma_z ) (kPa)</th>
<th>( \Delta \sigma_x ) (kPa)</th>
<th>( \Delta \tau_{xz} ) (kPa)</th>
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</tr>
<tr>
<td>10.67</td>
<td>13.72</td>
<td>196.8</td>
<td>33.0</td>
<td>-2.1</td>
</tr>
<tr>
<td>10.67</td>
<td>15.24</td>
<td>192.2</td>
<td>27.4</td>
<td>-2.9</td>
</tr>
<tr>
<td>10.67</td>
<td>16.76</td>
<td>187.7</td>
<td>22.9</td>
<td>-3.5</td>
</tr>
<tr>
<td>10.67</td>
<td>18.29</td>
<td>183.2</td>
<td>19.3</td>
<td>-3.9</td>
</tr>
</tbody>
</table>
appreciably with depth.

3) The increase in horizontal stress is small compared to the increase in vertical stress everywhere except near the top of the clay layer.

4) Although the clay layer is moderately overconsolidated, the vertical stress increase occurs principally at values higher than the preconsolidation pressure.

Since most of the parameters affecting the consolidation vary with depth in the clay layer, the layer must be divided into a sufficient number of strata to insure adequate accuracy. This can be achieved with the computer program in Appendix G. The settlement at the centerline of the embankment shown in Figure 4.3 is given in Table 4.2 as a function of the number of strata, n. Five or more strata will suffice in this case. The settlement at the embankment centerline is $1.3 \text{ m}$. Once the number of strata to be used in the analysis is known, the lateral variation of the settlement across the embankment can be calculated. The results are shown in Table 4.3.

As noted before, the difference in the stress paths of the consolidation test and the clay layer beneath the embankment will give rise to the generation of different amounts of excess pore pressure, and consequently to different amounts of settlement. An approximate method for
Table 4.2 Settlement vs. Number of Strata - Example 4.1

<table>
<thead>
<tr>
<th>n</th>
<th>S(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.372</td>
</tr>
<tr>
<td>2</td>
<td>1.305</td>
</tr>
<tr>
<td>3</td>
<td>1.295</td>
</tr>
<tr>
<td>4</td>
<td>1.308</td>
</tr>
<tr>
<td>5</td>
<td>1.295</td>
</tr>
<tr>
<td>6</td>
<td>1.298</td>
</tr>
<tr>
<td>x(m)</td>
<td>S(m)</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>0.000</td>
<td>1.298</td>
</tr>
<tr>
<td>1.524</td>
<td>1.298</td>
</tr>
<tr>
<td>3.048</td>
<td>1.292</td>
</tr>
<tr>
<td>4.572</td>
<td>1.289</td>
</tr>
<tr>
<td>6.096</td>
<td>1.280</td>
</tr>
<tr>
<td>7.620</td>
<td>1.268</td>
</tr>
<tr>
<td>9.144</td>
<td>1.250</td>
</tr>
<tr>
<td>10.668</td>
<td>1.228</td>
</tr>
</tbody>
</table>
dealing with this discrepancy is discussed in the following paragraphs.

The settlement of a compressible layer can be defined as:

\[ S_{\text{field}} = \int_0^H a_v u \, dz \]  \hspace{1cm} (4.8)

where

\[ a_v = \text{the coefficient of vertical compressibility of the soil} \]
\[ u = \text{the excess pore pressure due to loading} \]
\[ H = \text{the thickness of the soil layer} \]
\[ dz = \text{the thickness increment in the soil layer} \]

In the field the excess pressure will be (Holtz and Kovacs, 1981):

\[ u = B [\Delta \sigma_{\text{oct}} + a \cdot \Delta \tau_{\text{oct}}] \]  \hspace{1cm} (4.9)

where

\[ \sigma_{\text{oct}} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} \]
\[ \tau_{\text{oct}} = \frac{1}{3} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \]
\[ \sigma_1, \sigma_2, \sigma_3 = \text{principal stresses} \]
\[ a, B = \text{pore pressure parameters that are determined experimentally} \]
The excess pore pressure generated in a consolidation test is equal to the change in vertical stress, i.e.,

\[ u = \Delta \sigma_v \] (4.10)

It follows that the value of the settlement in the field will be

\[ S_{\text{field}} = \int_0^H a_v \cdot B \left( \Delta \sigma_{\text{oct}} + a \cdot \Delta \tau_{\text{oct}} \right) \, dz \] (4.11)

and that the value of settlement if excess pore pressures are equal to those in a consolidation test will be

\[ S_{\text{lab}} = \int_0^H a_v \cdot \Delta \sigma_v \cdot dz \] (4.12)

By taking the ratio of the settlements in equations 4.11 and 4.12, a correction factor to apply to settlements computed with the laboratory pore pressures may be obtained (Skempton and Bjerrum, 1957). This correction factor is:

\[ \mu = \frac{S_{\text{field}}}{S_{\text{lab}}} = \frac{\int_0^H a_v \cdot B \cdot \left( \Delta \sigma_{\text{oct}} + a \cdot \Delta \tau_{\text{oct}} \right) \, dz}{\int_0^H a_v \cdot \Delta \sigma_v \cdot dz} \] (4.13)

Ordinarily, the parameter, B, is 1.0 for saturated clays. The parameter, a, depends on the stress path. The results of Example 4.1 show that the stress path beneath the
embankment is essentially triaxial compression. For this case, the parameter, \( a \), may be obtained with the following expression

\[
a = 3(A_{ac} - \frac{1}{3})^{1/2}
\]  

(4.14)

where \( A_{ac} = \Delta u / (\Delta \sigma_1 - \Delta \sigma_3) \), is measured in a triaxial test. The parameter, \( a \), depends on the amount of strain. When the soil is treated as an isotropic elastic solid (which incidentally, is the assumption that is made in calculating the stress changes caused by the embankment) the parameter, \( a \), is zero.

Assuming that the stress-strain behavior of a soil may be idealized as in Figure 4.1b, the value of the coefficient of axial compressibility in the normally consolidated range may be taken to be

\[
a_v = \frac{C_e / \ln 10}{(1 + e) \sigma' v}
\]  

(4.15)

Equation 4.13 may be simplified by assuming that \( a_v \) is constant and that \( B \) equals unity. The resulting expression is

\[
\mu = \frac{H \int (\Delta \sigma_{oct} + a \Delta \tau_{oct}) \, dz}{\int_{0}^{H} \Delta \sigma_v \, dz}
\]  

(4.16)
Assuming that the soil may be considered to be an isotropic elastic solid, this may be further simplified to

\[ \mu = \frac{\frac{1}{3} \int \mathbf{H} \left( \Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3 \right) \cdot dz}{\int \mathbf{H} \Delta \sigma_y \cdot dz} \]  

\text{(4.17)}

**Example 4.2**

It is desired to calculate the settlement correction factor for the centerline of the embankment in example 4.1. The values of the stress changes due to the embankment load (obtained with the computer program in Appendix F) are presented in Table 4.4. Using equation 4.17, the resulting correction factor is 0.66. This means that the expected settlement will be:

\[ S_{\text{field}} = \mu \cdot S_{\text{lab}} = 0.66 \times 1.30m = 0.86m \]

At positions other than the centerline, the stress changes and hence, the correction factor, will be slightly different.

**Time-Rate of Settlement**

When the magnitude of consolidation settlement is large enough to be of concern, it is worthwhile to predict how much of this settlement will occur during the service life
Table 4.4 Calculation of Correction Factor $\mu$ - Example 4.2

<table>
<thead>
<tr>
<th>Depth Interval (meters)</th>
<th>Depth (meters)</th>
<th>$\Delta\sigma_v$ (kPa)</th>
<th>$\Delta\sigma_1$ (kPa)</th>
<th>$\Delta\sigma_2$ (kPa)</th>
<th>$\Delta\sigma_3$ (kPa)</th>
<th>$\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.048 - 6.096</td>
<td>4.572</td>
<td>233.0</td>
<td>282.0</td>
<td>200.7</td>
<td>119.4</td>
<td>602</td>
</tr>
<tr>
<td>6.096 - 9.144</td>
<td>7.620</td>
<td>233.8</td>
<td>257.4</td>
<td>151.4</td>
<td>45.5</td>
<td>454.3</td>
</tr>
<tr>
<td>9.144 - 12.192</td>
<td>10.668</td>
<td>227.0</td>
<td>245.8</td>
<td>140.7</td>
<td>35.6</td>
<td>422.1</td>
</tr>
<tr>
<td>12.192 - 15.240</td>
<td>13.716</td>
<td>218.3</td>
<td>233.2</td>
<td>130.6</td>
<td>28.1</td>
<td>391.9</td>
</tr>
<tr>
<td>15.240 - 18.288</td>
<td>16.764</td>
<td>208.6</td>
<td>220.4</td>
<td>121.3</td>
<td>22.2</td>
<td>364.0</td>
</tr>
</tbody>
</table>

$\Sigma \Delta\sigma_v = 1125.8$

$\Sigma (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) = 2234.3$

$$\mu = \frac{1}{3} \frac{\Sigma (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) \Delta z}{\Sigma (\Delta\sigma_v) \Delta z} = \frac{1}{3} \frac{(2234.3)(3.048)}{(1125.8)(3.048)} = 0.66$$
of the embankment. For a homogeneous soil layer beneath a long linear embankment, consolidation occurs due to the dissipation of excess pore pressures in the vertical direction as well as in the horizontal direction parallel to the cross section of the embankment. The governing equation for such a condition is:

$$c_v \frac{\partial^2 u}{\partial z^2} + c_h \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \tag{4.18}$$

where

- $c_v = \text{coefficient of consolidation in the vertical direction}$
- $c_h = \text{coefficient of consolidation in the horizontal direction}$
- $u = \text{excess pore pressure}$
- $x, z = \text{the horizontal and vertical coordinate directions}$
- $t = \text{time after the excess pore pressures were created}$

The solution for equation 4.18 depends on the distribution of the excess pore pressure. Therefore, an exact solution is not possible for the general case. An approximate solution is possible, however, by specifying the distribution of pore pressure on a grid of evenly spaced points in the consolidating layer (Figure 4.4). By replacing the partial derivations in equation 4.18 with finite difference approximations on this grid, it is possible to derive the following
FIGURE 4.4  FINITE DIFFERENCE GRID FOR TWO-DIMENSIONAL CONSOLIDATION EQUATION
A finite difference solution for the two-dimensional consolidation equation:

\[ u_{i,j,k+1} = \alpha_x [u_{i+1,j,k} + u_{i-1,j,k}] + \alpha_z [u_{i,j+1,k} + u_{i,j-1,k}] + [1 - 2\alpha_x - 2\alpha_z] u_{i,j,k} \]  

(4.19)

where

\[ i, j \] are column and row identifiers of the nodal points on the grid illustrated in Figure 4.4.

\[ k \] is the number of the time step.

\[ \alpha_x = \frac{c_h \Delta t}{\langle \Delta x \rangle^2} \quad \alpha_z = \frac{c_v \Delta t}{\langle \Delta z \rangle^2} \]

\( \Delta x, \Delta z \) are the horizontal and vertical spacings of the grid shown in Figure 4.4.

\( \Delta t \) is the time increment used in the analysis.

It is not common practice to evaluate equation 4.19 because \( c_h \) is not usually measured. Therefore, it is generally assumed that consolidation occurs solely due to vertical drainage. The governing equation for this condition is the well-known Terzaghi one-dimensional consolidation equation (Terzaghi, 1943):

\[ c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \]  

(4.20)
The finite difference expression for the solution of this equation is

\[ u_{i,k+1} = \alpha_z u_{i+1,k} + (1 - 2\alpha_z) u_{i,k} + \alpha_z u_{i-1,k} \]  \hspace{1cm} (4.21)

The accuracy of this expression is maximized when \( \Delta z \) approaches zero and when \( \alpha_z = 1/6 \) (Perloff and Baron, 1976).

At the boundary of the grid, equation 4.21 must be modified to account for drainage conditions. When a boundary is drained, the excess pore pressure is assumed to have an ambient value at the onset of consolidation equal to half of the initial excess pore pressure. Thereafter, the excess pore pressure at the drained boundary is set equal to zero. To calculate the excess pore pressure at an undrained boundary, it is necessary to assume a "mirror" node just outside the grid with a value of excess pore pressure equal to that of a node just inside the grid. The resulting expressions for nodes at the top and bottom boundary respectively are:

\[ u_{i,k+1} = 2\alpha_z u_{i+1,k} + (1 - 2\alpha_z) u_{i,k} \] \hspace{1cm} (4.21a)

and

\[ u_{i,k+1} = 2\alpha_z u_{i-1,k} + (1 - 2\alpha_z) u_{i,k} \] \hspace{1cm} (4.21b)

When the consolidating layer is composed of contiguous soil layers with different values of \( c_v \), continuity of flow
must be satisfied across the layer interfaces. Invoking Darcy's law, the value of the pore pressure at the layer interface may be obtained with the following expression (Harr, 1966):

\[
u_{i,k} = u_{i+1,k} - \frac{u_{i+1,k} - u_{i-1,k}}{1 + \frac{k_2}{k_1}\frac{\Delta z_1}{\Delta z_2}}
\] (4.22)

where

\[k_1, k_2\] are the permeabilities of the soil above and below the layer interface, respectively

\[\Delta z_1, \Delta z_2\] are the grid spacing above and below the layer interface, respectively.

As a first approximation, the ratio of the permeabilities between the upper and lower layers may be taken to be:

\[k_1/k_2 = c_{v1}/c_{v2}
\] (4.23)

where \(c_{v1}\) and \(c_{v2}\) are the coefficients of consolidation in the upper and lower soil layers, respectively.

Once the excess pore pressure is calculated at all the nodes for a given time after the onset of consolidation, the percent consolidation at that time may be calculated by evaluating the following expression:

\[UX = \left[1 - \frac{\int_0^H u_i(z) \, dz}{\int_0^H u(z) \, dz}\right] \times 100
\] (4.24)
where

\[ u_i(z) = \text{the initial distribution of excess pore pressure with depth} \]

\[ u(z) = \text{the distribution of excess pore pressure with depth at the time in question} \]

\[ H = \text{the thickness of the consolidating layer} \]

\[ z = \text{coordinates in the direction of the depth} \]

Equation 4.24 can be used to calculate the percent consolidation in each layer of a series of contiguous soil layers as well as the overall percent consolidation. A program for performing these computations is provided in Appendix H.

Values of the initial excess pore pressure that are to be used in the analysis should be calculated with equation 4.9. This insures that the distribution of excess pore pressures used to calculate the percent consolidation is the same as the distribution used to calculate the magnitude of consolidation settlement.

The coefficient of consolidation is defined by the following relationship:

\[ c_v = \frac{k(1 + e)}{\gamma_v \gamma_w} \]  \hspace{1cm} (4.25)

where \( \gamma_w \) is the unit weight of water.
Since the values of the permeability, the void ratio, and the vertical compressibility of the soil change as the consolidation progresses, it is necessary to simplify the characterization of the value of \( c_v \) during consolidation. This can be done with the controlled gradient consolidation test which is used in developing the preconsolidation pressure profile. The value of \( c_v \) obtained in a controlled gradient test is calculated with the following expression (Lowe, Jonas and Obrician, 1969):

\[
 c_v = \frac{\delta \sigma}{\delta t} \frac{H^2}{2u}
\]  

(4.26)

where

\[
 \frac{\delta \sigma}{\delta t} = \text{time rate of change of applied stress}
\]

\( H = \) the sample thickness

\( u = \) excess pore pressure maintained at the undrained end of the sample

This expression allows the calculation of \( c_v \) continuously during consolidation without recourse to either the logarithm or square root of time curve fitting methods. Other advantages of this testing procedure include:

1) the test may be run at low strain rates that approach consolidation rates in the field

2) the excess pore pressure is approximately constant across the sample

3) secondary compression does not occur
Once the value of $c_v$ of a soil layer is known along a compression curve like the one shown in Figure 4.1a, it is possible to choose a representative value of $c_v$ corresponding to the average value of vertical effective stress during consolidation. As was the case for calculating the magnitude of settlement, this procedure should be repeated in a number of artificial strata within the soil layer because the soil parameters and the stress changes will vary with depth.

Budget and time constraints may prohibit the type and amount of testing necessary to perform the analysis just described. When this is the case, an estimate of the variation of $c_v$ with depth needs to be made. In general, the value of $c_v$ in a soil that is overconsolidated will be substantially higher than $c_v$ of a normally consolidated sample of the same soil. If it is possible to estimate which portions of the soil layer will be normally consolidated and overconsolidated during the consolidation process, the entire layer can be reduced to a two-strata system. Normally, $c_v$ of the upper portion will correspond to an overconsolidated state and $c_v$ of the lower portion will correspond to a normally consolidated state.

Example 4.3

It is desired to estimate the time rate of settlement of the consolidation settlement at the centerline of the
embankment in Example 4.1. The liquid limit of the soil in the consolidating layer is 45. Typical values of \( c_v \) are 8.61 m\(^2\)/day (0.8 ft\(^2\)/day) and 2.15 m\(^2\)/day (0.2 ft\(^2\)/day) for the overconsolidated and normally consolidated portions of the layer, respectively (Holtz and Kovacs, 1981). The initial excess pore pressures were estimated using equation 4.10. The values of the stress changes were taken from Table 4.1. It was assumed that the overconsolidated portion occupies the upper 6.1m (20 ft) of the soil layer. In this example the bottom of the consolidating layer is undrained. The results, obtained with the computer program in Appendix H, are shown in Table 4.5. The time rate of settlement in the normally consolidated stratum proceeds at a much slower rate than the rate of the entire layer. Since most of the settlement occurs in the normally consolidated stratum, it would be unconservative to estimate the time rate of settlement on the basis of the overall rate of consolidation for the soil layer. A better estimate of the time rate in this case can be made by assuming that all of the settlement occurs in the normally consolidated stratum and proceeds at the rate computed for this stratum. This calculation is included in Table 4.5.
Table 4.5 Percent Consolidation of Clay Layer - Example 4.3

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Over Consolidated Strata</th>
<th>Normally Consolidated Strata</th>
<th>Entire Layer</th>
<th>Centerline Settlement (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>25.3</td>
<td>0.0</td>
<td>11.4</td>
<td>0.0000</td>
</tr>
<tr>
<td>60</td>
<td>43.8</td>
<td>0.0</td>
<td>20.6</td>
<td>0.0000</td>
</tr>
<tr>
<td>120</td>
<td>60.6</td>
<td>2.1</td>
<td>30.2</td>
<td>0.0183</td>
</tr>
<tr>
<td>230</td>
<td>75.8</td>
<td>8.2</td>
<td>40.6</td>
<td>0.0704</td>
</tr>
<tr>
<td>420</td>
<td>85.5</td>
<td>17.3</td>
<td>50.0</td>
<td>0.1484</td>
</tr>
<tr>
<td>820</td>
<td>91.2</td>
<td>31.5</td>
<td>60.1</td>
<td>0.2694</td>
</tr>
<tr>
<td>1480</td>
<td>94.0</td>
<td>48.0</td>
<td>70.0</td>
<td>0.4109</td>
</tr>
<tr>
<td>2480</td>
<td>96.1</td>
<td>65.3</td>
<td>80.0</td>
<td>0.5590</td>
</tr>
<tr>
<td>4200</td>
<td>98.0</td>
<td>82.6</td>
<td>90.0</td>
<td>0.7074</td>
</tr>
<tr>
<td>5920</td>
<td>99.0</td>
<td>91.3</td>
<td>95.0</td>
<td>0.7818</td>
</tr>
</tbody>
</table>
V - CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Conclusions

1. A hybrid method of compaction specification which makes the insitu water content of the embankment soil equal to the OMC is introduced.

2. The Simplified Bishop factor of safety option in the STABL program has been recoded to correct various difficulties.

3. A methodology for adjusting the Simplified Janbu factor of safety that is used by several STABL options to definitions of the factor of safety that are more familiar is presented.

4. Embankment side slope design has been illustrated for short and long term situations using laboratory compacted shear strength data.

5. Geometric and probabilistic interpretations of the factor of safety are introduced to illustrate their usefulness in the selection of an appropriate factor of safety for design.

6. A methodology of predicting settlement of embankment foundations has been illustrated. Computer programs
to compute the magnitude and time-rate of settlement are included. These programs, in conjunction with STABL, form an analysis package for the design of embankments.

**Recommendations**

1. To use the hybrid approach for specifying compaction, values of the coefficient of compaction that reflect the influence of soil type, compactor type, the operating procedure, and the number of passes need to be developed.

2. The current version of STABL should be augmented by adding a complete equilibrium method of calculating the factor of safety such as the Spencer method (Spencer, 1973).

3. The LEMIX program, which calculates the factor of safety on a three-dimensional surface, demonstrates the importance of three-dimensional effects. The next step in this field should be the development of a program that allows the random generation of general shaped three-dimensional surfaces as well as the subsequent calculation of the factor of safety on these surfaces.

4. The geometric and probabilistic approaches which were introduced should be further developed to determine acceptable design values of the geometric factor of
safety and the probability of failure for use in slope design.

5. Procedures should be developed to apply the statistical relationships between the material parameters of lab and field compacted clays in settlement predictions within an embankment and in slope stability analysis.
REFERENCES
REFERENCES


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Yao, J. T. P., Class notes, 1982.
APPENDIX A

DERIVATION OF
THE SIMPLIFIED BISHOP FACTOR OF SAFETY

Notation after Siegel (1975).

STEP 1 - Enforce moment equilibrium of sliding circular mass divided into n slices:

\[ \sum M_o = 0 \]

\[ = \sum_{1}^{n} \left[ (\Delta W(1-k_V) + \Delta U_B \cos \theta + \Delta Q \cos \phi)(R \sin \alpha) \right] - \sum_{1}^{n} \left[ \Delta S_r \cdot R - \sum_{1}^{n} \left[ (\Delta U_B \sin \theta + \Delta Q \sin \phi)(R \cos \alpha - h) \right] \right] + \sum_{1}^{n} \left[ K_n \Delta W(R \cos \alpha - h_{eq}) \right] \quad (A.1) \]

where

\[ R = \text{radius of the circle} \]

\[ \Delta S_r = \frac{1}{F_S} \left[ c_a' + \Delta N' \tan \theta_a' \right] \quad (A.2) \]

\[ c_a' = c \cdot dx/cos \alpha \]

Dividing equation A.1 by R yields:
\[ \Sigma M_0 = \sum_{1}^{n} \left[ (\Delta W (1-k_V) + \Delta U_B \cos \delta + \Delta Q \cos \delta) \sin \alpha \right] \\
- \sum_{1}^{n} \left[ (\Delta U_B \sin \delta + \Delta Q \sin \delta) (\cos \alpha - h/R) \right] \\
+ \sum_{1}^{n} \left[ k_h \Delta W (\cos \alpha - \frac{h_{eq}}{R}) \right] \quad (A.3) \]

**STEP II - Substitute** \( \Delta S \), into equation A.3 and assume the factor of safety is equivalent in each slice:

\[ FS = \frac{1}{n} \sum_{1}^{n} \left[ C'_{a} + \Delta N \tan \gamma_{a} \right] \quad (A.4) \]

where

\[ A_3 = (\Delta W (1-k_V) + \Delta U_B \cos \delta + \Delta Q \cos \delta) \sin \alpha \quad (A.4a) \]
\[ A_4 = (\Delta U_B \sin \delta + \Delta Q \sin \delta) (\cos \alpha - h/R) \quad (A.4b) \]
\[ A_5 = k_h \Delta W (\cos \alpha - \frac{h_{eq}}{R}) \quad (A.4c) \]

**STEP III - Sum forces in vertical direction for each slice:**

\[ \Sigma F_V = \Delta W (1-k_V) - (C'_{a} + \Delta N \tan \gamma_{a}) \sin \alpha / FS - \Delta N \cos \alpha \]
\[ + \Delta Q \cos \delta + \Delta U_B \cos \delta - \Delta U_B \cos \delta \quad (A.5) \]

Rearranging equation A.5 yields:

\[ \Delta N \tan \gamma_{a} \sin \alpha / FS + \cos \alpha = \]
\[ \Delta W (1-k_V) - C'_{a} \sin \alpha / FS + \Delta Q \cos \delta + \Delta U_B \cos \delta - \Delta U_B \cos \delta \quad (A.6) \]
Finally,

$$\Delta N' = \frac{\Delta W(1-k_v) - C_a \sin \alpha/FS + \Delta Q \cos \delta + \Delta U_\beta \cos \delta - \Delta U_\alpha \cos \alpha}{\cos \alpha + \tan^+ a \cdot \sin \alpha/FS}$$  \hspace{1cm} (A.7)$$

Substituting equation A.7 into equation A.4 yields:

$$FS = \left[ C_a + \tan^+ a \cdot (\Delta W(1-k_v) - C_a \sin \alpha/FS \\
+ \Delta Q \cos \delta + \Delta U_\beta \cos \delta - \Delta U_\alpha \cos \alpha)/ \\
(\cos \alpha + \tan^+ a \cdot \sin \alpha/FS)/(E A_3 - E A_4 + E A_5) \right]$$  \hspace{1cm} (A.8)$$

or rearranging

$$FS = \left[ \frac{n C_a + \tan^+ a \cdot \sec \alpha (\Delta W(1-k_v) + \Delta Q \cos \delta + \Delta U_\beta \cos \delta - \Delta U_\alpha \cos \alpha)}{\Sigma \left[ \frac{1}{1 + \tan^+ \alpha \cdot \tan \alpha/FS} \right]} \right]$$  \hspace{1cm} (A.9)$$

For simplicity of coding equation A.9 may be written as follows:

$$FS = \left[ \frac{\Sigma A_1}{1 + A_2/FS} \right]$$  \hspace{1cm} (A.10)$$

where

$$A_1 = C_a + \tan^+ a \cdot \sec \alpha (\Delta W(1-k_v) + \Delta Q \cos \delta \\
+ \Delta U_\beta \cos \delta - \Delta U_\alpha \cos \alpha)$$  \hspace{1cm} (A.10a)$$
\[ A_2 = \tan \theta + \tan \alpha \quad (A.10b) \]

This expression is programmed in STABL3.

The changes in the STABL code necessary to program equation A.10 are included in the following pages.
Changes to the STABL Program

11 if rec=0, rec=100, end=15) mkeyw
100 format(a6) check if recfil is initial command

ff(if recfil, ec=1) set tc=23
ff(mkeyw, ec(key)=1) go to 24
Changes to Subroutine SURFAC:

<p>| c         | radius      | distance between the coordinate points and the center of the limit equilibrium surface (mb=1)      | surf 120   |
|          | reacer      | subroutine that reads integer or real data in free                                                  | mgoodman   |
| c         | xcntr       | x coordinate of the center of a circular limit equilibrium surface                                  | mgoodman   |
| c         | xhalf2      | x coordinate of the midpoint of the second segment on the limit equilibrium surface                 | mgoodman   |
| c         | x1          | x coordinate of the first point on the limit                                                      | mgoodman   |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x2</td>
<td>x coordinate of the second point on the limit equilibrium surface</td>
</tr>
<tr>
<td>x3</td>
<td>x coordinate of the third point on the limit equilibrium surface</td>
</tr>
<tr>
<td>ycntr</td>
<td>y coordinate of the center of the limit equilibrium surface</td>
</tr>
<tr>
<td>yhalf2</td>
<td>y coordinate of the midpoint of the second segment on the limit equilibrium surface</td>
</tr>
<tr>
<td>y1</td>
<td>y coordinate of the first point on the limit equilibrium surface</td>
</tr>
<tr>
<td>y2</td>
<td>y coordinate of the second point on the limit equilibrium surface</td>
</tr>
<tr>
<td>y3</td>
<td>y coordinate of the third point on the limit surface</td>
</tr>
</tbody>
</table>

surf 138

common /nxsu/nsurf,surf(100,2)
cmmon /blk15/ m,mh
cmmon /blk20/ racius
cmension error(6)
103 format(12x,13,2x,2f12.2)
calculate the radius of the limit equilibrium surface if the
c the simplified bishop method is being used
if (mb.ne.1) go to 104
x1 = surf(1,1)
y1 = surf(1,2)
x2 = surf(2,1)
y2 = surf(2,2)
x3 = surf(3,1)
y3 = surf(3,2)
xhalf2 = (x2 + x3)/2.0
yhalf2 = (y2 + y3)/2.0
xcrtr = ((x1**2-x2**2)*(y3-y2) - (x2**2-x3**2)*(y2-y1) + (y3-y1)
        *(y2-y1)*(y3-y2))/(2.0*((x1-x2)*(y3-y2) - (x2-x3)**
        2
        *(y2-y1)))
ycrtr = (x2-x3)/(y3-y2)*(xcrtr - xhalf2) + yhalf2
radius = scrt((xcrtr - x2)**2 + (ycrtr - y2)**2)
      print ',', radius, 'feet'
104 return
Changes to Subroutine RANSUF:

| c | mb | control variable that indicates whether the simplified bishop or the simplified Janbu is to be used if the simplified bishop method is used |
| c | n | index variable for array subscripting. |

| c | radius | the radius of the points on the simplified bishop limit equilibrium surface |
| c | ranf | function subprogram that generates a pseudo-random |

22 ctheta=thetae+(thetab-thetae)*ranf(0.)*1.02

calculate the radius of the limit equilibrium surface if the simplified bishop factor of safety is being employed

if (mb.eq.1) radius = tsurf/2.0/sin(dtheta/2.0)

theta=thetas
Changes to Subroutine WEIGHT:

- **hight**: array containing the heights of the slices
- **hghtec**: array of the values of the height of the centroid of the horizontal earthquake forces above the base of each slice
- **i**: index variable for array subscripting.
- **mb**: variable use to discriminate between the simplified Bishop arc and the simplified Janbu methods
  - \( mb = 1 \) if the simplified Bishop method is used
- **n**: index variable for array subscripting.
- **wtheq**: the product of the weight of a slice and the distance between its base and the centroid of its horizontal earthquake force.
- **wt**: weight of a slice subsection.

```
cmmon /blk11/cavt,kcecf,vccecf
cmmon /blk15/m,mb
cmmon /blk21/hight(200),hghteq(200)
cmension y(20),yw(20),jw(10),y(10)
```

wght 112  mgoodman
wght 112  mgoodman
wght 112  mgoodman
wght 112  mgoodman
wght 114  mgoodman
wght 216  mgoodman
wght 216  mgoodman
wght 216  mgoodman
wght 219  mgoodman
wght 340  mgoodman
wght 340  mgoodman
wght 340  mgoodman
wght 342  mgoodman
wght 422  mgoodman
wght 422  mgoodman
wght 424  mgoodman
wght 424  mgoodman
js=1
if (npz * ec * 0) go to 110
cc 12 j=1, npz
jk(j)=2
12 crctinue
110 ccrtinue
rtcc1=rtcc+1

cc 2 i=1, nslice
initialize the slice height and the height of the centroid of the
earthquake force above the base of the slice

htgh(f) = 0.0
htgec(f) = 0.0
cx(f)=x(f+1)-x(f)

10 if (mb * eq * 1) wtheq = 0.0
    if (k * ec * 1) go to 14
call scilwt(yt,yj(1),itp(jt),yw(1),wtt(i),i)
if (mb.ec.1) wtheq = wtheq + ((yt + yj(1))/2.0 - yb)*wtt(i)
c c to 15
14 call scilwt(yt,yb,itp(jt),yw(1),wtt(i),i)
if (mb.ec.1) wtheq = wtheq + (yt-yb)/2.0*wtt(i)
c c to 21
15 if(k. ec.2) go to 17
k1=k-1
c c 11 j=2,k1
call scilwt(yi(j-1),yi(j),soil(j),yw(j),wt,i)
if (mb. ec.1) wtheq = wtheq + ((yi(j-1) + yi(j))/2.0 - yb)*wtt(i)=wtt(i)+wt
11 continue
17 call scilwt(yi(k-1),yb,soil(k),yw(k),wt,i)
if (mb. ec.1) wtheq = wtheq + (yi(k-1) - yb)/2.0*wt
wtt(i)=wtt(i)+wt

velt(i)=0.
if(isurc. ec.0) go to 120

20 js=jtn
120 if (mb. re.1) go to 2

calculate the (1) the slice height and (2) the height of the
vertical of the horizontal earthquake force component above the
ease of the slice

eight(i) = yt - yb
hgetec(i) = wtheq/wtt(i)
2 continue
Changes to subroutine FACTR:

- a1: array used in factor of safety calculation
- a2: array used in factor of safety calculation
- a3: array used in factor of safety calculation
- a4: term used in factor of safety calculation
- a5: term used in factor of safety calculation
- beta: array containing values of the angle of the top of
  simplified Bishop limit equilibrium surface
- radius: the length of the radius of the points on the
  simplified Bishop limit equilibrium surface
- rd: factor for conversion of degrees to radians.

```
common /blk15/ n,mb
common /blk20/radius
common /blk21/hight(200),hghteq(200)
cimension a1(200),a2(200),a3(200)
```
if (mb*ec*1) go to 40

simplify jantl a-terms

al = cslcex*(1) + tp*(wtt*(1)*(1.0-vkcoeff) - ualpha(i)*ca
1 + ubeta(i)*co + p(i)*cd)
αι(i) = a0/ca**2
a2*(i) = wtt*(i)*(ta+kcoeff-vkcoeff*ta)*ubeta(i)*(cb*ta-sb)+p(i)*cd*ta-fct 464
1 sc)
a3*(i) = ta*tp
sumb = sumb + a2*(i)
go to 2

simplify bishop a-terms

40 al*(i) = cslcex*(1)/ca + tp/ca*(wtt*(1)*(1.0-vkcoeff) + p(i)*cd +
1 ubeta(i)*ct - ualpha(i)*ca)
a2*(i) = tp*ta
a3*(i) = (wtt*(1)*(1.0-vkcoeff) + ubeta(i)*cb + p(i)*cd)*sa
a4 = (ubeta(i)*sb + p(i)*sd)*(ca-hight(i)/radius)
a5 = kcoeff*wtt*(i)*(ca-hghteal(i)/radius)
sumb = sumb + a2*(i) - a4 + a5

2 cccrtinue

if (mb*ec*1) go to 50
sumt = al*(i)/(1.0 + a3*(i)/fold) + sumt

go to 6
sumt = sumt + al*(i)/(1.0 + a2*(i)/fold)
6 cccrtinue
APPENDIX B

CALCULATION OF THE RADIUS OF A SPECIFIED CIRCULAR SURFACE

STABL randomly generates circular surfaces by generating successive chords of a circle which are inclined at a deflection angle with one another. Since all of the chords are of equal length, the radius may be determined with the following equation:

\[ R = \frac{T}{2} \sin \left(\frac{\phi}{2}\right) \quad (B.1) \]

where

- \( R \) = radius of the circle
- \( T \) = chord length of the segments circumscribing the circle
- \( \phi \) = deflection angle of the segments circumscribing the circle

However, when a specified surface is input, the chord lengths and deflection angles will not have constant values. In this case a more general expression for calculating the radius is required. To do this any two adjacent segments on a known circular surface may be taken (Figure B.1).
FIGURE B.1

GEOMETRY OF FIRST 2 CHORDS OF A SPHERICAL CIRCULAR SURFACE
The equation of line \( P_1 \) which is the perpendicular bisector of the chord defined by \((x_1, y_1)\) and \((x_2, y_2)\) is

\[
y = -\frac{(x_2 - x_1)}{(y_2 - y_1)}(x - x_{c1}) + y_{c1}
\]

where \((x_1, y_1)\) are the coordinates of the first point on the first chord.

\((x_2, y_2)\) are the coordinates of the second point on the first chord and the first point on the second chord.

\((x_{c1}, y_{c1})\) are the coordinates of the intersection of line \( P_1 \) and the first chord.

Similarly, the equation of line \( P_2 \), which is the perpendicular bisector of the chord defined by \((x_2, y_2)\) and \((x_3, y_3)\), is

\[
y = -\frac{x_3 - x_2}{y_3 - y_2}(x - x_{c2}) + y_{c2}
\]

where \((x_3, y_3)\) are the coordinates of the second point on the second chord.

\((x_{c2}, y_{c2})\) are the coordinates of the intersection of line \( P_2 \) and the second chord.

Lines \( P_1 \) and \( P_2 \) intersect at the center of the circle.

The coordinates of the center of the circle may be found by
setting equations B.2 and B.3 equal to each other. The resulting expression is:

\[
\frac{x_1 - x_2}{y_2 - y_1} (x_0 - \frac{x_1 + x_2}{2}) + \frac{y_1 + y_2}{2} = \frac{x_2 - x_3}{y_3 - y_2} (x_0 - \frac{x_2 + x_3}{2}) + \frac{y_2 + y_3}{2} \quad (B.4)
\]

Rearranging,

\[
x_o = \frac{[\langle x_1^2 - x_2^2 \rangle - \langle x_2^2 - x_3^2 \rangle (y_2 - y_1) + \langle y_3 - y_1 \rangle (y_2 - y_1) (y_3 - y_2)]/2}{[\langle x_1^2 \rangle (y_3 - y_2) - \langle x_2^2 \rangle (y_2 - y_1)]} \quad (B.5)
\]

The value of \( y_o \) may be obtained by substituting the value of \( x_o \) in equation B.3. The result is

\[
y_o = \frac{x_2 - x_3}{y_3 - y_2} (x_0 - x_c) + y_c \quad (B.6)
\]

where

\[
x_c = \frac{x_2 + x_3}{2}
\]

\[
y_c = \frac{y_2 + y_3}{2}
\]

Finally, the radius may be found with the following expression:

\[
R = \sqrt{\frac{1}{2} (x_0^2 - x_2^2 + (y_0 - y_2)^2)} \quad (B.7)
\]

These equations are coded in the current version of the STABL program.
APPENDIX C

THE FRICTION CIRCLE FACTOR OF SAFETY

The stability number, \( c/(FS'V'H) \), of a trial circular surface through a homogeneous slope such as the one shown in Figure C.1 may be determined by the following sequence of calculations (Taylor, 1940):

\[
n = \frac{1}{2} \left( \cot x - \cot y - \cot \beta + \sin \phi \csc x \csc y \right) \quad (C.1)
\]

where

\( \beta = \) sideslope

\( \phi = \) friction angle of the slope

\( x, y = \) angles shown in Figure C.1

If \( n > 0 \), the trial circle passes under the toe of the slope as shown by surface III in Figure C.2. If \( n \leq 0 \), the trial surface starts at the toe as shown by surface I or II in Figure C.2.

When \( n \leq 0 \), equations C.2 through C.5 are evaluated.

\[
\frac{H}{2d} = \frac{\frac{1}{2} \csc^2 x (y' \csc^2 y - \cot y) + \cot x - \cot \beta}{\frac{1}{3}(1-2\cot^2 \beta) + \cot \beta (\cot x - \cot y) + \cot x \cot y} \quad (C.2)
\]
GEOMETRY REQUIRED TO SPECIFY A CIRCULAR SLIP SURFACE FOR A SIMPLE SLOPE
\[
\cot u = \frac{H}{2d} \cdot \sec x \cdot \csc \cdot \csc^2 y - \tan x \quad (C.3)
\]

\[
\sin(u-v) = \frac{H}{2d} \cdot \sin u \cdot \csc x \cdot \csc y \cdot \sin \phi \quad (C.4)
\]

\[
\frac{c}{FS'Y'H} = \frac{\frac{1}{2} \csc^2 x (\csc^2 y - \cot y) + \cot x - \cot \phi}{2 \cot x \cdot \cot v + 2} \quad (C.5)
\]

When \( n > 0 \), equations C.6 and C.7 replace equations C.2 and C.5.

\[
\frac{H}{2d} = \frac{\frac{1}{2} \csc^2 x \cdot (\csc^2 y - \cot y) + \cot x - \cot \phi - 2n}{\left[ \frac{1}{3}(1 - 2 \cot^2 \phi) + \cot \phi \cdot (\cot x - \cot y) + \cot x \cdot \cot y + 2n^2 - 2n \cdot \sin \phi \cdot \csc x \cdot \csc y \right]} \quad (C.6)
\]

\[
\frac{c}{FS'Y'H} = \frac{\frac{1}{2} \csc^2 x (\csc^2 y - \cot y) + \cot x - \cot \phi - 2n}{2 \cot x \cdot \cot v + 2} \quad (C.7)
\]

Given \( \phi \) and \( \phi \), it is possible to search for the surface on which the stability number has a maximum value by checking all combinations of the angle \( x \) from 0 up to \( \phi \) and all the values of the angle \( y \) from 0 to 90°. The search may be limited to \( y \leq x \) if the limit equilibrium surface does not go beneath the elevation of the toe. The value of the stability number so obtained is almost identical to that which is presented in the stability chart that Taylor developed for simple slopes in homogeneous material except that the
equations presented here do not reflect the correction for the error associated with the assumption that the resultant of the normal and frictional forces acts tangent to the friction circle. The maximum value of this correction is approximately 7% (Taylor, 1937).

To calculate the factor of safety of a slope, the factor of safety must be the same on the cohesion intercept and the friction angle, i.e.,

$$ \text{FS}_c = \text{FS}_\phi $$  \hfill (C.8)

This condition may be satisfied with the following iterative procedure which is used by the computer program included hereafter:

1. Assume a value of \( \text{FS}_\phi \).
2. Calculate the required value of \( \phi \) as follows:

$$ \phi_{\text{req}} = \tan^{-1} \left( \frac{\tan \phi}{\text{FS}_\phi} \right) $$ \hfill (C.9)

3. Calculate the maximum stability number, \( N_s \), for \( \phi_{\text{req}} \) with equation C.7.
4. Calculate \( \text{FS}_c \) with the expression:

$$ \text{FS}_c = \left( \frac{c/vH}{N_s} \right) $$ \hfill (C.10)

5. Compare \( \text{FS}_c \) and \( \text{FS}_\phi \):
   a) If \( \phi = 0 \), then \( \text{FS} = \text{FS}_c \).
   b) If \( \phi \neq 0 \) and \( \text{FS}_\phi = \text{FS}_c \), then convergence has been obtained.
   c) If \( \phi \neq 0 \) and \( \text{FS}_\phi \neq \text{FS}_c \), repeat steps 2 through 5.
This procedure works except when the stability numbers close to zero cause numerical instability.
User Manual - Friction Circle Factor of Safety Program

The following program was developed in 77 Fortran on a CDC 6000 series computer. All input is in English units. However, any dimensionally homogeneous set of units will work. All input is unformatted.

1. Input on one record:
   a) the friction angle of the slope (degrees)
   b) the slope angle (degrees)
   c) the cohesion intercept (psf)
   d) the slope height (ft)
   e) the density of the soil (pcf)

2. Input on one record:
   a) an integer variable controlling the limits of the search:
      if the slip circle is completely above the elevation of the toe input '1'
      if the slip surface intersects the toe of the slope and may descend down to a specified depth limit, input '2'
      if the slip surface may pass under the toe of the slope, input '3'

3. If the value of the integer variable on record #2 is '2', input on one record:
   a) the depth factor of the search.
      The depth limit is the depth beneath the top of the slope to the bottom of the deepest circle that is geometrically possible.
The depth factor is obtained by dividing the depth limit by the slope height. The depth limit, DH, is illustrated in Figure D.2.

4. If the value of the integer on record #2 is '3', input on one record:
   a) the depth factor of the search.
   b) the toe factor of the search.

   The toe factor is the multiple of the slopes height that the slip circle can extend beyond the slopes toe (see Figure D.2).

The program listing is included in the following pages.
common/hate/ numx, numy, toe, slope, dangle,
1       depthf, nlimit, nsmax
common/lost/ coti, sinphi
real nsmax, nlimit
integer toe, cycles
data tol1, tol2, cycles, fsphi/1.0e-8,0.01,100.1.0/

pi = acos(-1.0)

read (5,*) phi, slope, cohes, hight, gamma
write (6,10) phi, cohes, gamma, hight, slope
10 format (' friction angle=',f6.1,' degrees',/,
1     ' cohesion=',f8.1,/, 
2     ' density=',f6.1,/, 
3     ' hight=',f6.1,/, 
4     ' side-slope=',f6.1, ' degrees'/)
read (5,*) toe
if (toe.eq.1) write(6,12)
12 format (' circles start at the toe with depth factor=1 ',/)
if (toe.eq.2) read(5,*) depthf
if (toe.eq.3) read(5,*) depthf, nlimit
if (toe.eq.2) write(6,14) depthf
14 format (' D =',f5.1,/')
if (toe.eq.3) write(6,16) depthf, nlimit
16 format (' D =',depthf,/, ' n =',nlimit,/')

c
c
numx = int(slope) - 1
numy = 89
if (toe.eq.1) numy = numx
slope = slope*pi/180.0
phi = phi*pi/180.0
if (abs(slope-pi/2.0).lt.tol1) then
  coti = 0.0
else
  coti = 1.0/tan(slope)
endif
dangle = pi/180.0
sinphi = sin(phi)
cgamh = cohes/(gamma*hight)

c
do 100 i = 1,cycles
phireq = atan(tan(phi)/fsphi)
sinphi = sin(phireq)
call phicir.
fscohs = cgamh/nsmax
if (phi.lt.tol2) then
  print*, 'fs=', fscohs
  print*, 'stability number=', nsmax
100 continue
stop
endif
fsdiff = abs(fscohs-fsphi)
print*, fsphi, fscohs, nsmax
if (fsdiff.lt.tol2) then
  print*, ' fs = ', fscohs
  stop
elseif (fsdiff.ge.tol2) then
  fsphi = (fsphi + fscohs)/2.0
endif
if (i.eq.cycles) then
  print*, ' convergence not obtained'
  stop
endif
100 continue

end

subroutine phicir
common/hate/ numx, numy, toe, slope, dangle,
1 depthf, nlimit, nsmax
common/lost/ coti, sinphi
common/love/ en, x, y, cscx, cscy, cotx, coty, stbnum
real nsmax, nlimit
integer toe
y = 0.0
nsmax = 0.0

do 25 itery = 1, numy
  y = y + dangle
  coty = 1.0/tan(y)
  cscy = 1.0/sin(y)
  x = 0.0
  do 25 iterx = 1, numx
    x = x + dangle
    if (toe.eq.1.and.x.lt.y) go to 25
    cotx = 1.0/tan(x)
    cscx = 1.0/sin(x)
    en = 0.5*(cotx - coty - coti + sinphi*cscx*cscy)
    if ((toe.eq.1.or.toe.eq.2).and.en.gt.0.0) go to 25
    if (toe.eq.3.and.en.gt.nlimit) go to 25
    d = 0.5*(cscx*cscy - cotx*coty + 1.0)
    if (toe.ne.1.and.d.gt.depthf.and.y.gt.x) go to 25
    call stabnm
    nsmax = amax1(nsmax, stbnum)
    continue
  25 continue

return
end

subroutine stabnm
common/love/ en, x, y, cscx, cscy, cotx, coty, stbnum
common/lost/ coti, sinphi

determine the stability number of a specified slip 
surface on a specified slope.

                  secx = 1.0/cos(x)
if (en. le. 0.0) then
toe circle
param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx
          - coti)/(1.0/3.0*(1.0 - 2.0*coti**2) + coti
          *(cotx - coty) + cotx*coty)
param2 = param1*y*secx*cscx*cscy**2 - tan(x)
u = atan(1.0/param2)
param3 = param1*sin(u)*cscx*cscy*sinphi
uv = asin(param3)
v = u - uv
stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx -
1        coti)/(2.0*cotx/tan(v) + 2.0)
elseif (en. gt. 0.0) then
slip surface beneath the toe of the slope
param1 = (0.5*cscx**2*(y*cscy**2 - coty) + cotx -
          1        coti - 2.0*en)/(1.0/3.0*(1.0 - 2.0*coti
          **2) + coti*(cotx - coty) + cotx*coty +
          3        2.0*en**2 - 2.0*en*sinphi*cscx*cscy)
param2 = param1*y*secx*cscx*cscy**2 - tan(x)
u = atan(1.0/param2)
param3 = param1*sin(u)*cscx*cscy*sinphi
uv = asin(param3)
v = u - uv
stbnum = (0.5*cscx**2*(y*cscy**2 - coty) + cotx -
1        coti - 2.0*en)/(2.0*cotx/tan(v) + 2.0)
endif
return
end
APPENDIX D

THE BETA DISTRIBUTION

The beta distribution for the variable \( x \) is defined by the following density function (Harr, 1977):

\[
f(x) = \frac{1}{b-a} \frac{\Gamma(\alpha + \beta + 2)}{\Gamma(\alpha + 1) \Gamma(\beta + 1)} (x-a)^{\alpha} (b-x)^{\beta}
\]

where

- \( b \) = upper bound of the density function
- \( a \) = lower bound of the density function
- \( \Gamma \) = Gamma function

The expected value of the variable \( x \) is

\[
E(x) = a + \frac{\alpha + 1}{\alpha + \beta + 2} (b-a)
\]

and the variance of the variable \( x \) is

\[
V(x) = \frac{(b-a)^2 (\alpha + 1) (\beta + 1)}{(\alpha + \beta + 2)^2 (\alpha + \beta + 3)}
\]

When \( a, b, E(x) \) and \( V(x) \) are known, the constants, \( \alpha \) and \( \beta \), may be defined by the following

\[
\alpha = \frac{\beta^2}{\beta}
\]

\[
\beta = \frac{\alpha + 1}{\alpha} - (\alpha + 2)
\]
where

\[ x = \frac{E(x) - a}{b - a} \]  \hspace{1cm} (D.6)

\[ \varphi = \frac{V(x)}{(b - a)^2} \]  \hspace{1cm} (D.7)
APPENDIX E

PROBABILISTIC SLOPE STABILITY ANALYSIS BY
THE POINT-ESTIMATES METHOD

The following program was developed to determine the probability of failure of a simple slope. Failure is defined to be a condition when there is any surface along which equilibrium cannot be maintained. Limit equilibrium is checked with the Taylor friction-circle method. The values of the soils friction angle, $\phi$, cohesion intercept, $c$, and density, $\gamma$, are assumed to be symmetrically distributed. The mean value and the standard deviation of the strength factor of safety of the slope are obtained with a two-point estimate (Rosenblueth, 1975). A beta distribution is fitted to these statistical moments and the upper and lower bounds of the factor of safety. The probability of failure is computed by numerically integrating equation 3.13.
User Manual - Probabilistic Slope Stability Program

This program was developed on a Vax computer. All units are in English units, but any dimensionally homogeneous set of units will work. All input is unformatted.

Parameters Defining the Distribution of the Values of the Material Properties of the Slope

1. Input on one record:
   a) mean value of \( \beta \) (degrees)
   b) the coefficient of variation of \( \beta \)
   c) the lower bound of \( \beta \) (degrees)
   d) the upper bound of \( \beta \) (degrees)

2. Input on one record:
   a) the mean value of \( c \) (psf)
   b) the coefficient of variation of \( c \)
   c) the lower bound of \( c \) (psf)
   d) the upper bound of \( c \) (psf)

3. Input on one record:
   a) the mean value of \( \gamma \) (pcf)
   b) the coefficient of variation of \( \gamma \)
   c) the lower bound of \( \gamma \) (pcf)
   d) the upper bound of \( \gamma \) (pcf)

Parameters Defining the Geometry of the Slope

4. Input on one record:
a) the height of the slope (ft)
b) the slope angle (degrees)

Parameters Defining the Type of Limit Equilibrium Surface

5. Follow instructions 2-4 in the User Manual for the Taylor friction circle factor of safety which is found in Appendix C.

The program listing is included in the following pages.
common/groovy/ nlimit, depthf
common/hate/ numx, numy, toe, dangle, nsmax
real mean(3), stddev(3), cv(3), bound(3,2), nlimit, nsmax
integer toe

data fsx, iout /1.0,1/

c
c
c
input material parameters
c  
a) mean value  b) coefficient of variation
c  
c) lower and upper bounds of
c  1) the friction angle
c  2) the cohesion intercept
c  3) the soil density
c
do 5 i = 1,3
read (5,*) mean(i), cv(i), bound(i,1), bound(i,2)
stddev(i) = mean(i)*cv(i)
5 continue
write (6,8)
8 format (15//), t46, 'soil properties', //,
  1   t20, 'mean', t40, 'coeff. of var.', t60, 'lower bound',
  2   t80, 'upper bound', //
write (6,12) mean(1), cv(1), bound(1,1), bound(1,2)
12 format ('phi', t18, f6.2, t42, f5.3, t62, f6.2, t82, f6.2)
write (6,14) mean(2), cv(2), bound(2,1), bound(2,2)
14 format ('cohesion', t16, f8.1, t42, f5.3, t62, f8.1, t80, f8.1)
write (6,16) mean(3), cv(3), bound(3,1), bound(3,2)
16 format ('density', t19, f5.1, t42, f5.3, t63, f5.1, t83, f5.1, 5(/),
  1   t45, 'slope geometry', //)

c
input geometry of the slope
c
read (5,*) hight, slope
write (6,17) hight, slope
17 format (t37, 'slope hight = ', f5.1, //, t37, 'slope = ', f5.2)
numx = int(slope) - 1
numy = 89
pi180 = acos(-1.0)/180.0
dangle = pi180
slope = slope*pi180

c
choose the type of failure surface

c
if the depth factor = 1.0  set toe = 1
if the depth factor > 1.0  set toe = 2
if the failure surface passes under the toe  set toe = 3
if toe option = 2  input the depth factor, depthf
if toe option = 3  input the depth factor and the toe factor
nlimit
read (5,*) toe

if (toe.eq.1) then
  write (6,18)
  format (t37, 'circles start have depth factor=1.0',//)
  numy = numx
  depthf = 1.0
  nlimit = 1.0
endif
if (toe.eq.2) then
  read (5,*) depthf
  write (6,19) depthf
  format (t37, 'circles start at the toe with depth factor=',f6.1,//)
  nlimit = 1.0
endif
if (toe.eq.3) then
  read(5i*) depthf
  write (6,20) depthf, nlimit
  format (t37, 'circles extend beneath the toe',//)
  depth factor =',f6.1,/
  , toe factor =',f6.1,//)
endif

calculate the value of the central factor of safety

write (6,21)
format (1h1,10(/),t20, 'calculation of central fs')
phmean = mean(1)*pi180
call fsafty (slope, hight, phmean, mean(2), mean(3), fscntr)
write (6,22) fscntr
format (/,t22, 'central fs =',f6.3)

cestablish the lower bound of the capacity-demand functional
c
phimin = bound(1,1)*pi180
cmin = bound(2,1)
denmax = bound (3,2)
write (6,23)
format (5(/),t20, 'calculation of min fs')
phi1 = phimin/pi180
write (6,25) phi1,cmin,denmax
format (/,t22, 'phi = ',f6.2,/,t22, 'cohesion = ',f8.1,/,t22, 'density = ',f6.1)
call fsafty (slope, hight, phimin, cmin, denmax, fsmin)
write (6,26) fsmin
format (t22, 'min fs =',f6.3)
if (fsmin.ge.1.0) then
  print*, 'probability of failure = 0'
  stop
endif
establish the upper bound of the capacity-demand functional

\[
\text{phimax} = \text{bound}(1,2) \times \pi180
\]
\[
\text{cmax} = \text{bound}(2,2)
\]
\[
\text{denmin} = \text{bound}(3,1)
\]

format (5/.), t20, 'calculation of max fs')

\[
\text{phi2} = \text{phimax}/\pi180
\]
\[
\text{cmax}, \text{denmin}
\]

write (6, 27)

format (t22, 'max fs =', f6.3)

if (fsmx.1t.1.0) then
  print*, 'probability of failure = 1.0' stop
endif

c use the rosenbleuth approximation to approximate the mean value and the variance of the capacity-demand functional.

c the following expressions for the point estimates of the capacity-demand functional assume that the coefficient of skewness of the phi,c and gamma parameters are all = 0 and that phi,c and gamma are uncorrelated

c if (iout.eq.1) then
  write (6, 40)
format (1h1, t30, 'rosenbleuth point estimates')
  icount = 0
endif

sumfs = 0.0
smfsqr = 0.0

do 50 i = 1, 2
  if (i.eq.1) phi = (mean(1) + stddev(1)) * pi180
  if (i.eq.2) phi = (mean(1) - stddev(1)) * pi180

do 50 j = 1, 2
  if (j.eq.1) cohes = mean(2) + stddev(2)
  if (j.eq.2) cohes = mean(2) - stddev(2)

do 50 k = 1, 2
  if (k.eq.1) gamma = mean(3) + stddev(3)
  if (k.eq.2) gamma = mean(3) - stddev(3)
  if (iout.eq.1) then
    icount = icount + 1
    write (6, 45) icount
    format (5/., t20, 'point estimate', i3, /)
    \[
    \text{phi}3 = \phi3/\pi180
    \]
    write (6, 25) phi3, cohes, gamma
  endif

  format (5/., t20, 'point estimate', i3, /)
  \[
  \text{phi}3 = \phi3/\pi180
  \]
  write (6, 25) phi3, cohes, gamma
endi

c calculate the point estimator of the factor of safety

c
call fsafety (slope, hight, phi, cohes, gamma, fs)
write (6, 46) fs
format (t20, 'factor of safety=', f6.3)
sumfs = sumfs + fs
smfsqr = smfsqr + fs**2
continue

calculate the mean value of the point estimators of the fs
fsmean = sumfs/8.0

calculate the variance of the point estimators of the fs
varfs = smfsqr/8.0 - fsmean**2
sigma = sqrt(varfs)

calculate the probability that the fs will be < fsx=1.0, i.e.,
the probability that the slope can not maintain limit equilibrium

call beta (fsmean, sigma, fsmin, fsmax, fsx, pfail)
write (6, 60) fscntr, fsmax, fsmin, v, pfail
format (1h1,20(/), t20,
1 'properties of the capacity-demand functional',//,
2 t30, 'mean value =', f6.3, /,
3 t30, 'max value =', f6.3, /,
4 t30, 'min value =', f6.3, /,
5 t30, 'coef. of var. =', f6.3, /,
6 t30, 'prob. of failure=', f7.5)
stop
end

c subroutine fsafety (slope, hight, phi, cohes, gamma, fs)
common/hate/ numx, numy, toe, dangle, nsmax
common/lost/ coti, sinphi
real nsmax
integer toe, cycles
data tol1, tol2, cycles, iout/1.0e-8, 0.01, 15, 1/

c pi = acos(-1.0)
if (iout.eq.1) write(6,10)
format (///, t10, 'fs phi', t30, 'fs cohes', t50, 'stability number',//
20 format (t10, f5.3, t30, f5.3, t53, f7.4)

c if (abs(slope-pi/2.0) > tol1) then
  coti = 0.0
else
  coti = 1.0/tan(slope)
endif
sinphi = sin(phi)
cgamh = cohes/(gamma*hight)
fsphi = 1.0
fscohs = 0.0
fc1old = 0.0
fp1old = 0.0
c
do 100 i = 1,cycles
cestimate phi required for limit equilibrium
cphireq = atan(tan(phi)/fsphi)
sinphi = sin(phireq)
cfc2old = fc1old
cp2old = fp1old
cfc1old = fscohs
cfp1old = fsphi
c
call phicir
cback calculate the fs on the cohesion
cfscohs = cgamh/nsmax
fratio = abs(fscohs-fsphi)/fscohs
ccompare the fs assumed on phi to the fs calculated on the cohesion
cif (fratio.lt.to12) then
  if (iout.eq.1) write (6,20) fsphi,fscohs,nsmax
  fs = (fscohs + fsphi)/2.0
  return
elseif (fratio.ge.to12) then
  if (iout.eq.1) write (6,20) fsphi,fscohs,nsmax
  if (phi.lt.to11) then
    fsphi = fscohs
  else
    diff1 = abs(fp2old-fsphi)
    diff2 = abs(fc2old-fscohs)
    if (diff1.lt.to12.and.diff2.lt.to12) then
      fsphi = fscohs
    else
      fsphi = (fsphi + fscohs)/2.0
    endif
  endif
endif
if (i.eq.cycles) then
  print*, 'convergence not obtained'
  stop
endif
100 continue
end

subroutine phicir
common/hate/ numx, numy, toe, dangle, nsmax
common/groovy/ nlimit, depthf
common/lost/ coti, sinphi
common/love/ en, x, y, cscx, cscy, cotx, coty, stbnum
real nsmax, nlimit
integer toe

y is 1/2 the angle swept out by the circle in question

y = 0.0
nsmax = 0.0
do 25 itery = 1, numy
  y = y + dangle
  coty = 1.0/tan(y)
  cscy = 1.0/sin(y)
  x = 0.0
  do 25 iterx = 1, numx
    x = x + dangle
    if (toe.eq.1.and.x.lt.y) go to 25
    cotx = 1.0/tan(x)
    cscx = 1.0/sin(x)
    calculate the extent of the limit equilibrium surface beyond the toe
    en = 0.5*(cotx - coty - coti + sinphi*cscx*cscy)
    if (toe.ne.3.and.en.gt.0.0) go to 25
    if (toe.eq.3.and.en.gt.nlimit) go to 25
    calculate the depth factor
    d = 0.5*(cscx*cscy - cotx*coty + 1.0)
    if (toe.ne.1.and.d.gt.depthf.and.y.gt.x) go to 25
    calculate the value of the stability number for the angles x and
    call stabnm
    choose the maximum value of the stability number
    nsmax = amax1(nsmax, stbnum)
    continue
  return
end
subroutine stobnm
common/love/ en, x, y, cscx, cscy, cotx, cod, stbnum
common/lost/ cod, sinphi

use the friction circle method to determine the stability number of a specified surface on the slope question
equations are from taylor (1937)

\[ \sec x = 1.0 / \cos(x) \]

toe circle

if (en.le.0.0) then
  calculate \( h/2d \) eq. 9#

\[ \text{param1} = (0.5 \times \csc x^2 \times (y \times \csc y^2 - \cot y) + \cot x \]

1 - cod)/(1.0/3.0*(1.0 - 2.0*cod**2) + cod
2 *(cotx - coty) + cotx*coty)

  calculate \( \cot(u) \) eq. 10#

\[ \text{param2} = \text{param1} \times y \times \sec x \times \csc x \times \csc y^2 - \tan(x) \]

v = atan(1.0/param2)

  calculate \( \sin(u-v) \) eq. 11#

\[ \text{param3} = \text{param1} \times \sin(u) \times \csc x \times \csc y \times \sinphi \]

uv = asin(param3)

\[ \nu = u - uv \]

  calculate the stability number eq. 12#

\[ \text{stbnum} = (0.5 \times \csc x^2 \times (y \times \csc y^2 - \cot y) + \cot x - \]

1 - cod)/(2.0*cotx/tan(v) + 2.0)

slip surface beneath the toe of the slope

elseif (en.gt.0.0) then

  eq. 14#

\[ \text{param1} = (0.5 \times \csc x^2 \times (y \times \csc y^2 - \cot y) + \cot x - \]

1 - 2.0*en)/(1.0/3.0*(1.0 - 2.0*cod**2) + cod*(cotx - coty) + cotx*coty +

2 - 2.0*en**2 - 2.0*en*\sinphi*\csc x*\csc y)

  eq. 15#

\[ \text{param2} = \text{param1} \times y \times \sec x \times \csc x \times \csc y^2 - \tan(x) \]
\[ u = \text{atan}(1.0/\text{param2}) \]

\[ \text{param3} = \text{param1} \times \sin(u) \times \csc x \times \csc y \times \sin \phi_1 \]

\[ \text{uv} = \text{asin(param3)} \]

\[ v = u - uv \]

Stability number \quad \text{eq. 17#}

\[ \text{stbnum} = (0.5 \times \csc x \times \csc y \times \cot y + \cot x - 1 \times \cot i - 2.0 \times \text{en}) / (2.0 \times \cot x / \tan(v) + 2.0) \]

endif

return

end

subroutine beta (xbar, sx, xmin, xmax, x, pf)

real const(5)

subroutine to find the probability of given points
by using a beta distribution fitted knowing the
mean, standard deviation and range of all data.
formulas from 'mechanics of particulate media' by m.e.harr

const(1) = xmin
const(2) = xmax

calculate x-tilda \quad \text{pg. 496 \text{eq. c-30}}

\[ x_t = (xbar - \text{const}(1)) / (\text{const}(2) - \text{const}(1)) \]

calculate variance-tilda \quad \text{eq. c-30}

\[ v_t = (sx / (\text{const}(2) - \text{const}(1)))^2 \]

calculate alpha \quad \text{eq. c-31a}

\[ \text{const}(3) = x_t^2 \times (1.0 - x_t) / v_t^2 - (1.0 + x_t) \]

calculate beta \quad \text{eq. c-31b}

\[ \text{const}(4) = (\text{const}(3) + 1.0) / x_t - (\text{const}(3) + 2.0) \]
\[ \text{const}(5) = 1.0 \]
\[ \text{cal} = \text{qudrtr}(\text{const}(1), \text{const}(2), \text{const}) \]
\[ \text{const}(5) = 1.0 / \text{cal} \]
\[ \text{pf} = \text{qudrtr}(\text{const}(1), x, \text{const}) \]

return

end
function qudrtr(a, b, coeff)
real xg(8), wg(8), coeff(5)
data xg, wg
1 /0.0150125098, 0.2816035507, 0.458016776, 0.6178762444,
2 0.75540444083, 0.8656312023, 0.9455750230, 0.9894069349,
3 0.1894506104, 0.1826034150, 0.1691565193, 0.1495959888,
4 0.1246289712, 0.0951585116, 0.062535239, 0.0271524594/

integrates the function betaf between the limits a and b
by sixteen point symmetric gaussian quadrature.
coeff is a real array of coefficients
for use in the function betaf, assumed independent of the
variable over which the integration is taking place.

c c c
sum=0.0
amb=(b-a)*0.5
apb=(b+a)*0.5
do 1 i=1,8
    xp=apb+amb*xg(i)
    xm=apb-xg(i)*amb
    sum=sum+wg(i)*(betaf(xp, coeff)+betaf(xm, coeff))
1 continue
sum=sum*amb
qudrtr=sum
return
end c c c

function betaf(x, coeff)
real coeff(5), x

c c c
computes beta function
coeff(1)=a
coeff(2)=b
coeff(3)=alpha
coeff(4)=beta
coeff(5)=normalizing constant at the point x,
where a < x < b

c c c
eq. c-27a

betaf = coeff(5)*(x-coeff(1))**coeff(3)*(coeff(2)-x)**coeff(4)
return
end
APPENDIX F

STRESSES CAUSED BY CONSTRUCTION OF AN EMBANKMENT

The following expressions may be used to calculate the stress changes caused by a semi-embankment loading in a semi-infinite, weightless, linear elastic half-space (Jurgenson, 1940):

\[ \Delta \sigma_z = \frac{P}{\pi} \left( \beta + \frac{z}{a} \alpha - \frac{z}{r_2^2} (x-b) \right) \]  \hspace{1cm} (F.1)

\[ \Delta \sigma_x = \frac{P}{\pi} \left( \beta + \frac{z}{a} \alpha + \frac{z(x-b)}{r_2^2} + \frac{2z}{a} \ln \frac{r_1}{r_0} \right) \]  \hspace{1cm} (F.2)

\[ \Delta \tau_{xz} = \frac{P}{\pi} \left( \frac{z}{a} \alpha - \frac{z}{r_2^2} \right) \]  \hspace{1cm} (F.3)

where

\[ P = \gamma \cdot H \]

\[ \gamma = \text{embankment soils density} \]

\[ H = \text{embankment height} \]

\[ \beta, \alpha, a, b, r_0, r_1, r_2 \] are illustrated in Figure F.1.

Equations F.1 to F.3 assume:

1) The ground surface is horizontal.
FIGURE F.1
INFINITELY LONG, PERFECTLY FLEXIBLE, SEMI-EMBANKMENT LOADING
OVER A SEMI-INFINITE LINEAR ELASTIC MEDIUM
2) The embankment is perfectly flexible. This is equivalent to saying that the embankment - halfspace interface is frictionless.

3) Strains are infinitesimal.

4) The embankment is infinite in its linear direction.

The stress changes for an entire embankment may be obtained by summing the stress contributions of each half of the embankment with equations F.1 to F.3. It should be noted that the stresses cannot be calculated at positions A, B, or C in Figure F.1.

Given \( \Delta \sigma_x, \Delta \sigma_z, \) and \( \Delta \tau_{xz}, \) the principal stress changes, \( \Delta \sigma_1 \) and \( \Delta \sigma_3 \) are calculated. The stresses in the linear direction depend on the values of elastic constants. Since the problem is one of plane strain:

\[
\Delta \sigma_2 = \nu (\Delta \sigma_1 + \Delta \sigma_3)
\]

where \( \nu = \) Poisson's ratio.

The value of Poisson's ratio does not affect the values of \( \Delta \sigma_x, \Delta \sigma_z, \Delta \tau_{xz}, \Delta \sigma_1, \) or \( \Delta \sigma_3. \)

Instructions for a computer program that facilitates computation of equations F.1 to F.4 are included on the following pages.
User Manual - Stress Under an Embankment

The following program contained hereafter was developed on a CDC 6000 series computer using 77 FORTRAN. Input is intended to be in English units, but other dimensionally homogeneous units may be employed. All input is unformatted.

1. Read in on one record:
   a) the side slope of the right hand side of the embankment (degrees)
   b) the side slope of the left hand side of the embankment (degrees)
   c) the embankment height (ft)
   d) the crest to crest width (ft)
   e) the density of the embankment soil (pcf)

2. Read in on one record:
   a) the Poisson's ratio of the soil

3. Read in on one record:
   a) the number of \((X, Z)\) coordinate pairs at which stresses are desired (integer)

4. Read in on one record:
   a) the X coordinate of the point at which stresses are desired (ft)
   b) the Z coordinate of the point at which stresses are desired (ft)
Repeat for each coordinate pair.

X is measured from the center line of the crest of the embankment (see Figure F.2).

Z is measured downwards from the ground surface (see Figure F.2).

The program listing is included in the following pages.
FIGURE F.2
EMBANKMENT GEOMETRY FOR STRESS PROGRAM
program main (input, output, tape5=input, tape6=output)

common x, z, hembnk, width, densy, sigma2, sigmaz, sigmax, sigmaz, sigma1, 
1 sigma2, sigma3, theta, mu, slope1, slope2, pi

real mu

this program calculates the values of stress change caused 
by an embankment on a linear elastic half-space.

read in 
1) the righthand side side-slope of the embankment in 
degrees. 
2) the lefthand side side-slope of the embankment in 
degrees. 
3) the embankment hight 
4) the crest width 
5) the embankment's density

read (5,*) slope1, slope2, hembnk, width, densy 
write (6,5) slope1, slope2, hembnk, width, densy
format ('1',5(/),t50,'right side-slope = ',f8.2,' degrees.',/, 
1 t50,'left side-slope = ',f8.2,' degrees.',/, 
2 t50,'embankment hight = ',f10.2,' feet.',/, 
3 t50,'crest width = ',f6.2,' feet.',/, 
4 t50,'embankment density = ',f7.2,' lb./ft**3')

pi = acos(-1.0)

read in the value of poisson's ratio for the elastic 
half-space.

read (5,*) mu 
write (6,3) mu
format (//'t50,'mu = ',f4.2)

read in the number of coordinate pairs that the stress 
change is desired at.

read (5,*) npairs 
write (6,10)
format ('1',t8,'x',t22,'z',t38,'sigma z',t52,'sigma x',t66, 
1 'sigma xz',t81,'sigma1',t95,'sigma2',t109,'sigma3', 
2 t124,'theta',//}}

calculation loop input an (x,z) coordinate pair
for each loop

do 100 i =1,npairs
    read (5,*) x,z
    call stress
    write (6,20) x,z,sigmax,sigmaz,sigmax,sigmax,sigma1,sigma2,sigma3,theta
    format (2(f9.1,6x),6(5x,f9.1),5x,f9.2)
100 continue
end

subroutine stress

common x,z,hembnk,width,densy,sigmax,sigmaz,sigmax,sigmax,sigma1,sigma2,sigma3,theta,mu,slope1,slope2,pi

real mu

p = densy * hembnk

contribution to the stresses from the right half of the embankment loading

sloper = slope1 * pi/180.0
ar = hembnk/tan(sloper)
br = ar + width/2.0
x1 = br - x
r2 = sqrt((width/2.0 + ar - x1)**2 + z**2)
r1 = sqrt((x1 - ar)**2 + z**2)
r0 = sqrt(x1**2 + z**2)

betar = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
gammarr = acos((r1**2 + r0**2 - ar**2)/(2.0*r1*r0))
ciflr = betar + x1/ar * gammarr - z*(x1-br)/r2**2
rightx = betar + x1/ar*gammarr + z*(x1-br)/r2**2 + 2.0*z/ar*log(r1/r0)
rihtxz = z/ar*gammarr - z**2/r2**2

contribution to the stresses from the left side.

slope1 = slope2*pi/180.0
al = hembnk/tan(slope1)
b1 = al + width/2.0
x2 = b1 + x
r2 = sqrt((width/2.0 + al - x2)**2 + z**2)
r1 = sqrt((x2 - al)**2 + z**2)
r0 = sqrt(x2**2 + z**2)

betal = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
gammal = acos((r1**2 + r0**2 - al**2)/(2.0*r1*r0))
cifll = betal + x2*gammal/al - z*(x2-b1)/r2**2
leftx = betal + x2/al*gammal + z/r2**2*(x2 - b1) + 2.0*z/al*log(r2/r1)

\[ \frac{r_1}{r_0} \]
\[
\text{leftxz} = z/a_1*\text{gamma1} - z**2/r_2**2 \\
\text{sigmax} = p/\pi*(\text{ciflr} + \text{cifll}) \\
\]
\[
\text{sigmax} = p/\pi*(\text{rightx} + \text{leftx}) \\
\text{sigmax} = p/\pi*(\text{rightxz} + \text{leftxz}) \\
\text{descr} = \sqrt{((\text{sigmax} - \text{sigmax})**2/4.0 + \text{sigmaxz}**2)} \\
\text{center} = (\text{sigmax} + \text{sigmax})/2.0 \\
\text{sigma1} = \text{center} + \text{descr} \\
\text{sigma3} = \text{center} - \text{descr} \\
\text{sigma2} = \mu*(\text{sigma1} + \text{sigma3}) \\
\text{theta} = (0.5*\text{atan}(-2.0*\text{sigmaxz}/(\text{sigmax} - \text{sigmax})))\times180.0/\pi \\
\]
\[
\text{return} \\
\text{end}
APPENDIX G

MAGNITUDE OF CONSOLIDATION SETTLEMENT

The program contained hereafter computes the magnitude of consolidation settlement of compressible soil layers caused by construction of an embankment. Up to ten layers are permitted. Each layer may be automatically divided into any number of strata. The preconsolidation pressure profile is input as shown in Figure 4.2b. The void ratio is automatically corrected for depth with equations 4.6a to 4.6h. Settlement is computed with equations 4.2 to 4.5.

The program was developed on a CDC 6000 series computer using 77 FORTRAN.
Input is intended to be in English units, but other dimensionally homogeneous units may be employed.

All input is unformatted.

Specify the Number and Type of Layers in the Soil Profile:

1. Read in on one record:
   a) the total number of soil layers (integer)
   b) the number of compressible soil layers (integer)
   c) the number of layers with a portion above the ground water table (integer)
   d) the number of layers with a portion below the ground water table (integer)

Specify the Compressibility Models for Each Layer:

2. For each soil layer, read in on one record:
   a) the layer number (integer)
   b) the thickness of the layer (feet)
   c) An identifier variable that indicates
      if the soil layer is compressible (integer).
      If the soil is not considered to be compressible, input '0'.
      If the soil is considered to be compressible, input '1'.
   d) If the soil is considered to be compressible, read
in the following variables on a separate record
prior to inputting (a)-(c) for the subsequent soil
layer.
i) An option variable that is used to control the
manner by which \( e_o \) and \( \sigma'_p \) are assumed
to vary with depth in the soil layer (integer)
If one value of \( e_o \) and \( \sigma'_p \) are used to repre-
sent the entire soil layer, input '1'.
If \( e_o \) and \( \sigma'_p \) are specified at three depths
within the soil layer, input '2'.

ii) An option variable that specifies if the soil
is to be treated as normally consolidated or
underconsolidated at depths beneath which the
calculated value of \( \sigma'_v \) is greater than
the value of \( \sigma'_p \) (integer)
If the soil layer is to be treated like normally
consolidated soil at depths where \( \sigma'_v \) is
greater than \( \sigma'_p \), input '1'.
If the soil layer is to be treated as an under-
consolidated soil at depths where \( \sigma'_v \) is greater
than the value of \( \sigma'_p \), input '2'.

Specify the Ground Water Table:

3. Read in on one record:
   a) the depth of the groundwater table beneath
      the ground surface (feet).
The depth is measured downwards from the ground surface (see Figure F.2).

Specify the Saturated Density of the Soil Layers:

4. For each layer with a saturated zone, read in on one record:
   a) the layer number (integer)
   b) the saturated density of the soil layer (pcf)

Specify the Unsaturated Density of the Soil Layers:

5. For each layer with an unsaturated zone, read in on record:
   a) the layer number (integer)
   b) the density of the soil layer (pcf)

Specify the Compressibility Parameters of Each Layer:

6. For each compressible layer, read in on one record:
   a) the layer number (integer)

   If one value of $e_o$ and $\sigma'_p$ represent the entire layer, read in on a separate record:
   b) the initial void ratio
   c) the preconsolidation pressure (psf)
   d) the compression index
   e) the recompression index
   f) the depth beneath the ground surface from which the tested sample was taken (ft)
If the soil may be considered to be underconsolidated, input on a separate record:

b) the compression index
c) the recompression index

Also, if the soil is underconsolidated, input the following items on the subsequent record:

d) $e_0$ at depth 1#
e) $\sigma'_p$ at depth 1# (psf)
f) Depth 1# (ft)
g) $e_0$ at depth 2#
h) $\sigma'_p$ at depth 2# (psf)
i) Depth 2# (ft)
j) $e_0$ at depth 3#
k) $\sigma'_p$ at depth 3# (psf)
l) Depth 3# (ft)

Specify the Embankment Load:

7. Read in on one record:

a) the height of the embankment (feet)
b) the sideslope angle of the embankment (degrees)
c) the width of the embankment crest (feet)
d) the density of the embankment soil (pcf)
Specify the Lateral Limits of the Settlement Calculations:

8. Read in on one record:
   a) the leftmost bound for which settlements will be calculated (feet)
   b) the rightmost bound for which settlements will be calculated (feet)
   c) the number of evenly spaced points along the embankments cross section at which settlement will be calculated (integer)

Coordinates for these bounds are measured (+) and (-) to the right and left of the embankment centerline respectively.

If the settlement of only one profile is desired, the program calculates settlement at the left boundary.

Specify the Number of Strata Compressible Layers are Divided into:

9. For each compressible layer, read in on one record:
   a) the layer number (integer)
   b) the number of strata into which the layer will be divided into for purposes of analysis (integer)

The program listing is contained in the following pages.
parameter (nprfil = 11, nlyrs = 10, nlyrs1 = 11)

integer comprs(nlyrs), strata(nlyrs), ocr(nlyrs), iprfil(nlyrs), iout

real thick(nlyrs1), densat(nlyrs), denmst(nlyrs),
cc(nlyrs), total(nprfil), settle(nprfil, nlyrs),
rc(nlyrs), xil(nprfil), eo1(nlyrs), eo2(nlyrs),
eo3(nlyrs), pc1(nlyrs), pc2(nlyrs), pc3(nlyrs), zsamp1(nlyrs),
zsamp2(nlyrs), zsamp3(nlyrs)

real gammaiu/62.4/, tol/.001/

program assumes e-logp behaviour only in saturated zone.

units of length - (feet)
units of force - (pounds)
units of pressure - (psf)

write(6,1)
write(6,2)

pi = acos(-1.0)

read in:
1) total # of soil layers
2) # of compressible soil layers
3) # of layers with portions above the ground water table.
4) # of layers with portions below the ground water table.

read (5,*) nlayer, nclayr, nabove, nbelow

do 4 i=1,nlayer
    ocr(i) = 0
    iprfil(i) = 0
    continue

read in geometry of problem

for each layer read in:
1) layer #
2) the thickness of the i'th layer.
3) if the layer is compressible.
   if, yes then input '1'
   if, no then input '0'

4) if the layer is compressible, read in on a separate card:

   comprs
   if the variation of pc and eo is defined by
   two line segments, set iprfil = 2
   if only one value of pc and eo are specified
   for the layer, set iprfil = 1

   ocr
   if pc is to be automatically corrected to
   the overburden pressure when pc < overburden
   pressure, set ocr = 1
   if pc is to be considered underconsolidated
   when pc < overburden pressure,
   set ocr = 2

   ocr = 1 if the soil is normally consolidated
   at depths beneath which
   the overburden pressure is greater than the
   value of the preconsolidation pressure that
   was input

   ocr = 2 if the preconsolidation pressure is constant
   with depth.

do 5 i=1,nlayer

read (5,*) il, thick(il), comprs(il)

if (comprs(il).eq.1) read (5,*) iprfil(il), ocr(il)

5 continue

do 10 i=1,nlayer

write (6,6) i, thick(i)
   if (comprs(i).eq.1) then
      if (ocr(i).eq.1.or.iprfil(i).eq.2) then
         write (6,7)
      elseif (ocr(i).eq.2) then
         write (6,8)
      endif
   elseif (comprs(i).eq.0) then
      write (6,9)
   endif

10 continue
read in elevation of ground water surface

read (5,*) zwater
write (6,11) zwater

write (6,20)

initialize moist and saturated densities of each layer.

do 24 i=1,nlayer
   denmst(i) = 0.0
   densat(i) = 0.0
24  continue

for each layer with a saturated zone, read in:
  1) layer #
  2) saturated density

do 25 i=1,nbelow

   read (5,*) ii,densat(ii)
25  continue

for each layer with an unsaturated zone, read in:
  1) layer #
  2) moist density

do 30 i=1,nabove

   read (5,*) ii,denmst(ii)
30  continue
continue

read in compressibility properties

**do 40 i = 1, nclayr**

input the number of the compressible layer

**read (5,*) i1**

**if (iprfil(i1).eq.1) then**

for each compressible layer, read in:
1) initial void ratio
2) preconsolidation pressure (psf)
3) compression index (log 10)
4) recompression index (log 10)
5) sample depth (feet)

**read (5,*) eo1(i1), pc1(i1), cc(i1), rc(i1), zsamp1(i1)**

**elseif (iprfil(i1).eq.2) then**

**read (5,*) cc(i1), rc(i1)**

**read (5,*) eo1(i1), pc1(i1), zsamp1(i1), eo2(i1), pc2(i1),**

1. zsamp2(i1), eo3(i1), pc3(i1), zsamp3(i1)

**endif**

**40 continue**

output material parameters

**write (6,50)**

**do 60 i=1, nlayer**

**if (abs(denmst(i)).lt.tol) then**

**write (6,54) i, densat(i)**

**elseif (abs(densat(i)).lt.tol) then**

**write (6,56) i, denmst(i)**
elseif (densat(i).gt.0.0. and. denmst(i).gt.0.0) then
  write (6, 57) i, densat(i), denmst(i)
endif
if (comprs(i).eq.1) then
  if (iprfil(i).eq.1) then
    write (6, 58) eo1(i), pc1(i), cc(i), rc(i), zsamp1(i)
  elseif (iprfil(i).eq.2) then
    write (6, 61) eo1(i), pc1(i), cc(i), rc(i), zsamp1(i),
                 eo2(i), pc2(i), zsamp2(i), eo3(i), pc3(i),
                 zsamp3(i)
  endif
elseif (comprs(i).eq.0) then
  endif
endif
continue
write (6, 67)

read in dimensions of embankment load:

1) embankment height
2) sideslope of embankment (degrees)
3) crest width
4) density of embankment material during period of consolidation.

read (5, *) hembnk, slopel, width, densy
write (6, 68) hembnk, slopel, width, densy

read in:
1) the lateral limits between which settlement is calculated
2) the # of points between the bounds for which consolidation is to be calculated.

read (5, *) bound1, boundr, nntrv1

nntrv = nntrv1 - 1
write (6, 72) nntrv1, bound1, boundr
if (nntrv1.gt.1)then
  dx = (boundr - bound1)/float(nntrv)
else if (nntrv1.eq.1) then
  dx = 0.0
endif
write (6, 74)
for each compressible layer, read in:
1) layer #
2) the # of strata the layer will be divided into for purposes of analysis

```
do 80 i=1,nclayr
    read (5,*) i,l, strata(i,)
write (6,75) i,l, strata(i,)
80 continue
```

if strains are to be output for each strata
set iout = 1
otherwise set iout = 0

```
read (5,*) iout
```

```
x1 = boundl - dx
```

```
if (iout.eq.1) write (6,85)
    this loop sums settlement from each layer beneath each point for which settlement is desired.
```

```
do 200 np= 1,nntrvl
    ztop = 0.0
    prtop = 0.0
    x1 = xl + dx
    if (iout.eq.1) write (6,90) x1
    x = abs (x1)
    total(np) = 0.0
```

```
    this loop calculates settlement of each layer beneath the np'th layer.
    do 190 i=1,nlayer
    if (iout.eq.1) write (6,91) i, thick(i)
    settle(np,i) = 0.0
190 continue
```

```
zbottm = ztop + thick(i)
```
calculate the pressure at the bottom of the 1st layer.

if (zwater.gt.ztop. and. zwater.lt.zbottm) then
    prbotm = prtop + (zwater-ztop)*denmst(i) + (zbottm-zwater)
   *(densat(i)-gammaw)
else if (zwater.le.ztop) then
    prbotm = prtop + (zbottm-ztop)*(densat(i)-gammaw)
else if (zwater.ge.zbottm) then
    prbotm = prtop + (zbottm-ztop)*denmst(i)
endif

if (comprs(i).eq.1) then
    dz = (zbottm-ztop)/float(strata(i))
endif

if (iprfil(i).eq.1) then

    calculate the sample pressure

    if (zwater.le.ztop) then
        psampl = prtop + (densat(i) - gammaw)*(zsamp1(i)-ztop)
    else if (zwater.gt.ztop. and. zwater.lt.zsamp1(i)) then
        psampl = prtop + denmst(i)*(zwater-ztop) + (denmst(i)-gammaw)*
        (zsamp1(i)-zwater)
    endif
endif

nstrta = strata(i)

distinct strata for purposes of calculating settlements.
do 100 n=1,nstrta
    z = ztop + float(2*n-1)/2.0*dz
if (iout.eq.1) write (6,92) n,z

    calculate the insitu pressure at the midpoint of the n'th strata.

if (zwater.le.ztop) then
    pr1 = prtop + (densat(i)-gammaw)*(z-ztop)
else if (zwater.gt.ztop. and. zwater.lt.z) then
    pr1 = prtop + (zwater-ztop)*denmst(i) + (z-zwater)*
    (densat(i)-gammaw)
endif

if (z.le.zwater) then
set the settlement of the strata = 0 above the ground water table.

\[ s_{\text{squeeze}} = 0.0 \]

elseif (\text{zwater} < \text{z}) then

calculate \( e_0 \) at the center of each strata beneath the water table.

if (\text{iprfil}(i) \neq 1) then

if (\text{ocr}(i) \neq 2) then

\[ \text{precon} = \text{pc1}(i) \]

elseif (\text{ocr}(i) \neq 1 \text{ and } \text{pr1} > \text{pc1}(i)) then

\[ \text{precon} = \text{pr1} \]

elseif (\text{ocr}(i) \neq 1 \text{ and } \text{pr1} \leq \text{pc1}(i)) then

\[ \text{precon} = \text{pc1}(i) \]
endif

center of strata above the samples elevation

if (\text{z} < \text{zsamp1}(i)) then

if (\text{psamp1} \leq \text{precon}) then

\text{case i}

the soil is overconsolidated above the sample

\[ \text{estrat} = e_{01}(i) + r(i) \times \log_{10}(\text{psamp1}/\text{pr1}) \]

e elseif (\text{psamp1} > \text{precon}) then

if (\text{ocr}(i) \neq 2) then

if (\text{pr1} > \text{precon}) then

\text{case ii}

the soil is underconsolidated and therefore will have a constant void ratio at pressures above the preconsolidation pressure.

\[ \text{estrat} = e_{01}(i) \]

e else if (\text{pr1} \leq \text{precon}) then

\text{case iii}

the soil is underconsolidated at pressures above the p'c and overconsolidated at pressures below the p'c

\[ \text{estrat} = e_{01}(i) + r(i) \times \log_{10}(\text{precon}/\text{pr1}) \]
if (ocr(i) .eq. 1) then
  case iv
  the soil is normally consolidated at pressures above the p’c and over-consolidated at pressures beneath the p’c
  estrat = eo1(i) + cc(i) * alog10(psampl/precon) + rc(i) * alog10(precon/pr1)
endif
endf

center of the strata below the samples elevation
else if (z .ge. zsampl(i)) then
  if (psampl < precon) then
    if (pr1 .le. precon) then
      case v
      the soil is over-consolidated
      estrat = eo1(i) - rc(i) * alog10(pr1/psampl)
    endif
  endif
endf
else if (pr1 > precon) then
  if (ocr(i) .eq. 2) then
    case vi a)
    the soil is over-consolidated below the p’c and normally consolidated above the p’c
    estrat = eo1(i) - rc(i) * alog10(precon/psampl) -
  endif
elseif (ocr(i) .eq. 2) then
  case vi b)
  the soil over-consolidated below the p’c and overconsolidate above the p’c
  estrat = eo1(i) - rc(i) * alog10(precon/psampl)
endif
else if (psampl.ge.precon) then
    case vii
    the soil is underconsolidated
    estrat = eo1(i)
endif
endif
elseif (iprfil(i).eq.2) then
    if (z.le.zsamp2(i)) then
        precon = pc2(i) + (pc2(i) - pc1(i))/(zsamp2(i) -
        zsamp1(i))*(z - zsamp2(i))
        estrat = eo2(i) + (eo2(i) - eo1(i))/(zsamp2(i) -
        zsamp1(i))*(z - zsamp2(i))
    elseif (z.gt.zsamp2(i)) then
        precon = pc2(i) + (pc3(i) - pc2(i))/(zsamp3(i) -
        zsamp2(i))*(z - zsamp2(i))
        estrat = eo2(i) + (eo3(i) - eo2(i))/(zsamp3(i) -
        zsamp2(i))*(z - zsamp2(i))
    endif
    if (precon.lt.prl.and.ocr(i).eq.1) precon = prl
endif
call stress (x,z,hembnk,slope1,width,densy,sigmaz,pi)
sumsig = sigmaz + prl
endif
calculate the compression of the strata
if (sumsig.le.precon) then
    sqeeze = dz/(1.0 + estrat)*rc(i)*alog10(sumsig/prl)
else if (sumsig.gt.precon) then
    if (pr1.le.precon) then
        sqeeze = (dz/(1.0 + estrat))*rc(i)*alog10(precon/prl) -
        cc(i)*alog10(sumsig/precon))
    elseif (pr1.gt.precon) then
        if (ocr(i).eq.1) then
            sqeeze = dz/(1.0 + estrat)*(cc(i)*alog10(sumsig/prl))
        elseif (ocr(i).eq.2) then
            sqeeze = dz/(1.0 + estrat)*(cc(i)*alog10
            (sumsig/precon))
        endif
    endif
endif
cendif
```plaintext
c  sum the settlements of each strata in the layer.
c  settle (np, i) = settle (np, i) + squeeze

c 100  continue

c  endif

c  reset the elevation and pressure at the top
  of the layer beneath the current layer.

c  ztop = zbottom
  prtop = probottom

c  sum the settlements of each layer beneath the
  the point at which the settlement is desired.

c  total(np) = total(np) + settle(np, i)

c 190  continue

c 200  continue

c  output statements

c  write (6, 210)
x11(1) = bound1

c  if (nntrv1.ge.2) then
    do 300 np = 2, nntrv1
         x11(np) = x11(np - 1) + dx
  300  continue
  endif

c  write (6, 250) (x11(np), np=1, nntrv1)

c  write (6, 305)

c  do 400 i=1, nlayer
    if (comprs(i).eq.1) then
      write (6, 310) i, (settle(np, i), np=1, nntrv1)
  400  continue

c  write (6, 410) (total(np), np = 1, nntrv1)

c  formats

c  format('1', 25(/), t30, 66(''), /,
  1 t30, '*', t95, '*', /, t30, '*', t95, '*', /, t30, '*', t95, '*', /,
```
t30, '*, t55, 'program elogp', t95, '*', 
4 t30, '*, t40, 'this program calculates the settlement beneath 
5 t95, '*, /
6 t30, '*, t40, 'an infinitely long embankment load due to the 
7 t95, '*, /
8 t30, '*, t44, 'compression of saturated clay layers.', t95, 
9 t95, '*, /
, t30, '*, t95, '*, /
, t30, '*, t95, '*/
1 t30, '*, t95, '*, /
format(1, 5/), t58, 'problem geometry', 5(/),
1 t38, 'layer', 15x, 'thickness (feet)', 8x, 'compressible?'
2 /
1 format(t39, i2, t60, f6.1)
2 format('+', t88, 'yes')
3 format('+', t82, 'underconsolidated')
4 format('+', t88, 'no')
5 format(5(/), t52, 'properties of the soil profile', /////)
6 format(t2, 'layer', t10, 'sat. density',
1 5x, 'moist density', 7x, 'void ratio',
2 5x, 'preconsolidation', 5x, 'compression', 1x, 'recompression'
3 5x, 'sample depth', /
, t49, 'initial', t64, 'pressure', t86, 'index', t100,
5 'index', /, t13, '(pcf)', t31, '(pcf)', t67, '(psf)', t116,
6 '(feet)', 5(/))
7 format(t3, i2, t13, f5.1, t31, '****')
8 format(t3, i2, t13, '****', t31, f5.1)
9 format(t3, i2, t13, f5.1, t31, f5.1)
10 format('+', t50, f5.3, t66, f8.1, t86, f5.3, t100, f5.3, t117, f6.1)
11 format('+', t50, '****', t66, '*******', t86, '******', t100, '****'
1 t117, '*****')
11 format(3(/), 35x, 'the phreatic surface lies', f8.1, 'feet beneath 
1 the ground surface.')
12 format('+', t50, f5.3, t66, f8.1, t86, f5.3, t102, f5.3, 
1 t117, f6.1, /, t50, f5.3, t66, f8.1, t117, f6.1, /,
2 t50, f5.3, t66, f8.1, t117, f6.1)
13 format('1', 5(/), t50, 'dimensions of embankment load', //)
14 format('1', 5(/), t50, 'embankment hight = ', f4.1, 'feet.', /,
1 t50, 'sideslope = ', f4.1, 'degrees', /,
2 t50, 'crest width = ', f5.1, 'feet.', /,
3 t50, 'density of the applied load = ', f5.1, 'pcf.')
15 format(3(/), t18, 'settlement will be calculated at ', i2, 
1 'equally spaced positions between x= ', f5.1, 'and x= ',
2 f5.1, 'feet.', 3(/))
16 format(t45, 'layer #', 15x, 'no. of divisions', //)
17 format(t47, i2, t74, i2)
18 format('1', t25, 'x (feet)', t40, 'layer', t50, 'strata',
1 t66, 'depth (feet)', t84, 'thickness (feet)',
2 t105, 'strain (percent)', 3(/)
2 t50, 'settlement (feet)', //, t55, 'x (feet)', //)
2 t6, f6.1)
2 format('+', t41, i2, t88, f8.1)
2 format('+', t52, i2, t63, f7.1)
2 format('+', t110, f7.2)
end subroutine stress (x, z, hembnk, slope1, width, densy, sigmaz, pi)

p = densy * hembnk

geometry

slope = slope1 * pi/180.0
a = hembnk/tan(slope)
b = a + width/2.0
x1 = b - x

c
contribution to vertical stress from right half of the load

c
r2 = sqrt((width/2.0 + a - x1)**2 + z**2)
r1 = sqrt((x1 - a)**2 + z**2)
r0 = sqrt(x1**2 + z**2)
betar = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
gammar = acos((r1**2 + r0**2 - a**2)/(2.0*r1*r0))
ciflr = betar + x1/a * gammar - z*(x1-b)/r2**2

c
contribution to vert. stress from the left side.

c
r1 = sqrt((width + a - x1)**2 + z**2)
x2 = 2.0 * a + width - x1
r0 = sqrt(x2**2 + z**2)
betal = acos((r1**2 + r2**2 - (width/2.0)**2)/(2.0*r1*r2))
gamma1 = acos((r1**2 + r0**2 - a**2)/(2.0*r1*r0))
cifll = betal + x2*gamma1/a - z*(x2-b)/r2**2

c
sigmaz = p/pi*(ciflr + cifll)
return
end
APPENDIX H

TIME-RATE OF CONSOLIDATION SETTLEMENT

The FORTRAN IV program contained hereafter solves the well known Terzaghi one-dimensional consolidation equation

\[ c_v \frac{\delta^2 w}{\delta z^2} = \frac{\delta w}{\delta t} \]  \hspace{1cm} (H.1)

by means of the finite-difference approximation discussed in Chapter IV. Up to 10 contiguous layers with differing values of \( c_v \) may be input. Any distribution of initial excess pore pressure may be input because the solution is obtained numerically. The distribution of initial excess pore pressures is input at discrete points. These points are assumed to be evenly spaced within each separate layer. The spacing in each layer is determined with the following equation:

\[ \Delta z_i = \sqrt{\frac{c_v i \Delta t}{\alpha_i}} \]  \hspace{1cm} (H.2)

where

\[ 0 \leq \alpha \leq 1/2 \]
\[ i = \text{layer number} \]
\[ c_{vi} = \text{coefficient of consolidation of the } i^{\text{th}} \text{ layer} \]
\[ t = \text{time increment to be used in the analysis} \]
\[ \Delta z_i = \text{spacing of nodes in the } i^{\text{th}} \text{ layer}. \]

Whenever possible, \( \alpha \) should be approximately \( 1/6 \).

Generally the solution will be more accurate if a large number of nodal points is chosen in each layer.
User Manual - Time Rate of Settlement Program

All input is unformatted.
Units are feet, pounds, and days unless otherwise specified.

1. Read in the number of contiguous consolidating layers (integer)

2. For each layer read in:
   a) the layer number (integer)
   b) $c_v$ of the layer ($\text{ft}^2/\text{day}$)
   c) thickness of the layer (ft)
   d) nodal spacing in the layer (ft)

3. Read in the time increment at which excess pore pressures are to be calculated (days)

4. Read in the values of the initial excess pore pressures at each of the nodes from top to bottom (psf)

5. Read in the output scheme: (integer)
   a) If the values of the excess pore pressure at the nodes are desired, input 1. Due to format restrictions this output scheme may be used only if 15 or fewer nodes are used.
   b) If the value of the percent consolidation
is desired in each layer, \texttt{input 2}

c) If the value of the percent consolidation is desired for a one layer problem, \texttt{input 0}

6. Read in the:

a) code for the drainage condition of the top boundary (integer)

b) code for the drainage condition of the bottom boundary (integer)

c) depth of the top boundary beneath the ground surface (ft)

If a boundary is drained, set its drainage condition code = 0

If a boundary is undrained, set its drainage condition code = 1

7. Read in the termination limits:

a) the maximum number of years at which the program will cease to calculate the excess pore pressures.

b) the maximum overall percent consolidation at which the program should cease to calculate excess pore pressures.

The program listing is contained in the following pages.
INTEGER RTOP, RBOTTM, CYCLES, IOUT, NSTRAT(10)
REAL U(101), UINIT(101), CV(10), THICK(10), DELZ(10), ALPHA(10),
1 Z11(20), AINTL(10), APRSNL(10), CONSOL(10)

PRINT TITLE

WRITE (6,1)
WRITE (6,2)

DIMENSIONS OF UNITS IN INPUT
LENGTH -- FEET
PRESSURE -- PSF
TIME -- DAYS
EXCEPT WHERE SPECIFIED OTHERWISE AS IN THE CASE
OF 'NYEARS' WHICH IS INPUT IN YEARS.

READ IN THE NUMBER OF SEQUENTIAL COMPRESSIBLE LAYERS

READ (5,*) LAYERS

FOR EACH LAYER, READ IN:
1) THE LAYER NUMBER 'I1'
2) THE LAYERS COEFFICIENT OF CONSOLIDATION 'CV(I1)',
3) THICKNESS 'THICK(I1)',
4) THICKNESS INCREMENT 'THICK (I1)'

DO 10 I = 1, LAYERS
     READ (5,*) I1, CV(I1), THICK(I1), DELZ(I1)
10 CONTINUE

CALCULATE THE NUMBER OF STRATA IN EACH LAYER FOR
COMPUTATIONAL PURPOSES. THE USER SHOULD CHECK THE OUTPUT
TO INSURE THAT THE PROGRAM DIVIDES EACH LAYER INTO THE
SAME # OF STRATA THAT WAS ASSUMED.

DO 20 I = 1, LAYERS
     VALUE1 = THICK(I)/DELZ(I)
     NSTRAT(I) = INT(VALUE1)
     VALUE2 = FLOAT(NSTRAT(I))
     IF (VALUE2.LT.VALUE1) NSTRAT(I) = NSTRAT(I) + 1
20 CONTINUE

READ IN THE TIME INCREMENT 'DELT' TO BE USED IN THE ANALYSIS

READ (5,*) DELT

CALCULATE THE VALUE OF 'ALPHA' FOR EACH LAYER
```
DO 30 I = 1,LAYERS
   ALPHA(I) = CV(I)*DELT/DELZ(I)**2
30 CONTINUE

CALL THE TOTAL # OF STRATA 'NS' IN ALL OF THE LAYERS,
   AND THE NUMBER OF POINTS 'NPTS' FOR WHICH THE FINITE DIFFER- 
   ENCE PROCEDURE WILL BE PERFORMED.

NS = 0
DO 40 I = 1,LAYERS
   NS = NS + NSTRAT(I)
40 CONTINUE
NPTS = NS + 1

READ IN THE VALUES OF THE INITIAL EXCESS PORE PRESSURE WHICH 
   ARE ASSUMED TO BE EQUAL TO THE TOTAL STRESS CHANGE AT THE 
   POINT FOR EACH OF THE POINTS. DO NOT REPEAT VALUES AT 
   INTERLAYER BOUNDARIES.

READ (5,*5) (UNIT(I),I=1,NPTS)

PRINT HEADINGS
WRITE (6,55)
PRINT INPUT DATA
DO 70 I = 1,LAYERS
   WRITE (6,60) I,THICK(I),NSTRAT(I),DELZ(I),CV(I),ALPHA(I)
70 CONTINUE

CHECK IF 0.0 < ALPHA < 0.5
DO 80 I = 1,LAYERS
   IF (ALPHA(I),LE,0.5) GO TO 80
   WRITE (6,75) I
   STOP
80 CONTINUE

READ IN OUTPUT SCHEME
OUTPUT ONLY THE TIME AND THE PERCENT CONSOLIDATION
SET IOUT = 0
OUTPUT THE TIME, THE EXCESS PORE PRESSURE AT EACH POINT 
   AND THE PERCENT CONSOLIDATION
SET IOUT = 1
```
OUTPUT THE TIME, THE PERCENT CONSOLIDATION IN EACH LAYER
AND THE OVERALL PERCENT CONSOLIDATION
SET IOUT = 2

THE USER MUST BE CAREFUL TO NOT USE MORE THAN 15 POINTS IN
ORDER TO PREVENT THE OUTPUT FORMATS FROM BLOWING UP WHEN
USING IOUT = 1

READ (5,*) IOUT

INPUT BOUNDARY CONDITIONS OF THE TOP AND BOTTOM OF THE
SOIL LAYER WHOSE TIME RATE OF SETTLEMENT IS BEING
STUDIED.

IF THE TOP OR BOTTOM BOUNDARY IS DRAINED THEN INPUT
RTOP = 0 OR RBOTTM = 0 RESPECTIVELY.
IF THE TOP OR BOTTOM BOUNDARY IS UNDRAINED, THEN INPUT
RTOP = 1 OR RBOTTM = 1 RESPECTIVELY.

READ (5,*) RTOP, RBOTTM, ZTOP

INITIALIZATIONS

CNS = 0.0
T = 0.0
AINIT = 0.0
APRSNT = 0.0
IF (LAYERS.EQ.1) GO TO 82
    DO 81 I=1,LAYERS
        AINTL(I) = 0.
        APRSNL(I) = 0.
    CONTINUE
81 CONTINUE
82 CONTINUE

OUTPUT THE INITIAL PORE PRESSURES AT EACH POINT FROM TOP
TO BOTTOM AS IT WAS INPUT. THIS FORMAT REPEATS THE
PRESSURE AT INTER-STRATA BOUNDARIES.

IF (IOUT.GT.0) GO TO 89
NSTART = 1
WRITE (6,85)
DO 95 I = 1,LAYERS
    NEND = NSTRAT(I) + NSTART
    WRITE (6,90) I,(UINIT(J),J=NSTART,NEND)
    NSTART = NEND
95 CONTINUE
GO TO 101
89 IF (IOUT.GT.1) GO TO 239
WRITE (6,140)
Z11(1) = ZTOP
NP = 1

...
DO 97 I = 1, LAYERS
   NEND = NSTRAT(I)
DO 96 J = 1, NEND
   NP = NP + 1
   Z11(NP) = Z11(NP-1) + DELZ(I)
   CONTINUE
97 CONTINUE
WRITE (6, 98) (Z11(NP), NP=1, NPTS)
WRITE (6, 99)
WRITE (6, 150) T, (UINIT(J), J=1, NPTS)
WRITE (6, 165) CNS
GO TO 101

C C C
C CALCULATE THE INITIAL AREA OF THE ISOCRONNE, I.E., THE
C INTEGRAL OF THE ORDINATES OF EXCESS PORE PRESSURE *
C THE DEPTH INTERVAL FOR WHICH THAT VALUE OF PRESSURE
C IS AN AVERAGE VALUE.
C C C
239 WRITE (6, 339) (I, I=1, LAYERS)
101 NSTART = 1
DO 110 I = 1, LAYERS
   NEND = NSTRAT(I) + NSTART
   NEND1 = NEND - 1
   DO 100 J = NSTART, NEND
      IF (J .NE. NSTART) GO TO 102
      AINIT = AINIT + UINIT(J) * DELZ(I) / 2.0
      IF (LAYERS .GE. 2) AINTL(I) = AINTL(I) + UINIT(J) * DELZ(I) / 2.0
      GO TO 100
102 IF (J .GT. NEND1) GO TO 103
      AINIT = AINIT + UINIT(J) * DELZ(I)
      IF (LAYERS .GE. 2) AINTL(I) = AINTL(I) + UINIT(J) * DELZ(I)
      GO TO 100
103 AINIT = AINIT + UINIT(J) * DELZ(I) / 2.0
   IF (LAYERS .GE. 2) AINTL(I) = AINTL(I) + UINIT(J) * DELZ(I) / 2.0.
100 CONTINUE
   NSTART = NEND
110 CONTINUE

C C C
C CHANGE THE EXCESS PORE PRESSURE AT UNCONFINED BOUNDARIES
C TO THE AMBIENT VALUE = 0.5 * INITIAL EXCESS PRESSURE.
C C C
   IF (RTOP .EQ. 0 .AND. RBOTTM .EQ. 1) GO TO 111
   IF (RTOP .EQ. 1 .AND. RBOTTM .EQ. 0) GO TO 112
   IF (RTOP .EQ. 0 .AND. RBOTTM .EQ. 0) GO TO 113
   IF (RTOP .EQ. 1 .AND. RBOTTM .EQ. 1) GO TO 114
111 UINIT(1) = UINIT(1) / 2.0
   APRSNT = AINIT - UINIT(1) * DELZ(1) / 2.0
IF (LAYERS.GE.2) APRSNL(1) = AINTL(1) - UINIT(1)*DELZ(1)/2.0
GO TO 120

112 UINIT(NPTS) = UINIT(NPTS)/2.0
APRSNT = AINIT - UINIT(NPTS)*DELZ(LAYERS)/2.0
IF (LAYERS.GE.2) APRSNL(LAYERS) = APRSNL(LAYERS) - UINIT(NPTS)
*DELZ(LAYERS)/2.0
GO TO 120

113 UINIT(1) = UINIT(1)/2.0
APRSNT = AINIT - (UINIT(1)*DELZ(1) + UINIT(NPTS)*DELZ(LAYERS))/2.
IF (LAYERS.GE.2) APRSNL(1) = AINTL(1) - UINIT(1)*DELZ(1)/2.0
IF (LAYERS.GE.2) APRSNL(LAYERS) = APRSNL(LAYERS) - UINIT(NPTS)
GO TO 120

116 WRITE (6,117)
STOP

120 CNS = (1.0 - APRSNT/AINIT)*100.0
IF (LAYERS.EQ.1) GO TO 126
DO 125 I = 1,LAYERS
    CONSOL(I) = (1.0 - APRSNL(I)/AINTL(I))*100.
125 CONTINUE
126 CONTINUE

C
IF (IOUT.EQ.0) WRITE (6,130)
IF (IOUT.NE.0) GO TO 114
WRITE (6,145) T,CNS
GO TO 115

114 IF (IOUT.NE.1) GO TO 115
WRITE (6,150) T,(UINIT(J),J=1,NPTS)
WRITE (6,165) CNS

C
C INPUT THE ITERATION LIMITS
C NYEARS = # OF YEARS THE PROGRAM WILL CALCULATE EXCESS PORE
C PRESSURES AND CONSOLIDATIONS.
C CMAX = THE MAXIMUM PERCENT OF CONSOLIDATION THE PROGRAM
C WILL CALCULATE PRESSURES AND CONSOLIDATIONS.
C
115 READ(5,*) NYEARS,CMAX
C
C CYCLES = NYEARS*365/INT(DELT)

C
FINITE DIFFERENCE LOOP
C
C CALCULATE EXCESS PORE PRESSURES AND PERCENT CONSOLIDATION
C FOR EACH TIME STEP
C
DO 200 ICYCLE = 1,CYCLES
T = T + DELT
NSTART = 1

CALCULATE THE EXCESS PORE PRESSURE AT ALL THE POINTS FOR THE ICYCLE' TH CYCLE

DO 160 I = 1,LAYERS
NEND = NSTRAT(I) + NSTART

INTERNAL POINTS IN EACH LAYER

DO 155 J = NSTART,NEND
   IF ((I.EQ.1.AND.J.EQ.1)) GO TO 155
   IF (J.LT.NEND) U(J) = ALPHA(I)*(UINIT(J+1)+UINIT(J-1))
      + (1.0 - 2.0*ALPHA(I))*UINIT(J)
155 CONTINUE
NSTART = NEND
160 CONTINUE

INTER LAYER BOUNDARY POINTS
IMPOSE CONTINUITY OF FLOW ACROSS THE BOUNDARY

NFACE = 1
NL1 = LAYERS - 1
IF (NL1.LE.0) GO TO 181
DO 180 I = 1,NL1
   NFACE = NFACE + NSTRAT(I)
   U(NFACE) = U(NFACE + 1) - (U(NFACE + 1) - U(NFACE - 1))
      /((1.0 + (CV(I+1)/CV(I))*(DELZ(I)/DELZ(I+1)))
180 CONTINUE

CALCULATE THE PORE PRESSURE AT THE TOP AND BOTTOM OF THE SEQUENCE OF COMPRESSIBLE LAYERS. IT IS ASSUMED THAT UNDRAINED BOUNDARIES MAY BE SIMULATED WITH A MIRROR IMAGE OF THE EXISTING PORE PRESSURES ON THE OTHER SIDE OF THE BOUNDARY.

IF (RSTOP.EQ.0) U(1)=0.0
IF (RSTOP.EQ.1) U(1) = 2.0*ALPHA(1)*UINIT(2) + 1
   (1.0 - 2.0*ALPHA(1))*UINIT(1)

IF (RBOTTOM.EQ.0) U(NPTS) = 0.0
IF (RBOTTOM.EQ.1) U(NPTS) = 2.0*ALPHA(LAYERS)*UINIT(NPTS-1) + 1
   (1.0 - 2.0*ALPHA(LAYERS))*UINIT(NPTS)
CALCULATE THE PERCENT CONSOLIDATION

APRSNT = 0.0
IF (LAYERS.EQ.1) GO TO 186
DO 185 I = 1,LAYERS
   APRSNL(I) = 0.
185 CONTINUE
186 CONTINUE
NSTART = 1

CALCULATE THE PRESENT ISOCHRONE

DO 195 I = 1,LAYERS
NEND = NSTRAT(I) + NSTART
NEND1 = NEND - 1

DO 190 J = NSTART,NEND
   IF (J.NE.NSTART) GO TO 1181
   APRSNT = APRSNT + U(J)*DELZ(I)/2.0
   IF (LAYERS.EQ.2) APRSNL(I) = APRSNL(I) + U(J)*DELZ(I)/2.0
   GO TO 190
1181 CONTINUE
   IF (J.EQ.NEND1) GO TO 182
   APRSNT = APRSNT + U(J)*DELZ(I)
   IF (LAYERS.EQ.2) APRSNL(I) = APRSNL(I) + U(J)*DELZ(I)
   GO TO 190
182 CONTINUE
   IF (J.EQ.NEND) APRSNT = APRSNT + U(J)*DELZ(I)/2.0
   IF (J.EQ.NEND .AND. LAYERS.EQ.2)
      APRSNL(I) = APRSNL(I) + U(J)*DELZ(I)/2.0
190 CONTINUE
NSTART = NEND
195 CONTINUE
CNS = (1.0 - APRSNT/AINIT)*100.0
IF (LAYERS.EQ.1) GO TO 1196
DO 1195 I = 1,LAYERS
   CONSOL(I) = (1.0 -APRSNL(I)/AINTL(I))*100.
1195 CONTINUE
1196 CONTINUE

OUTPUT THE TIME, THE PERCENT CONSOLIDATION, AND THE EXCESS PORE PRESSURE AT EACH NODE

IF (IOUT.GT.0) GO TO 196
WRITE (6,145) T,CNS
GO TO 197
196 IF (IOUT.GT.1) GO TO 1197
WRITE (6,150) T,(U(J),J=1,NPTS)
WRITE (6,165) CNS
GO TO 197
1197 WRITE (6,1200) T,(CONSOL(I),I=1,LAYERS)
C
C CHECK TERMINATION LIMITS
C
197 IF (CNS.GT.CMAX) GO TO 300
IF (ICYCLE.EQ.CYCLES) GO TO 300
C
RESET THE EXCESS PORE PRESSURES AT ALL THE POINTS
FOR THE NEXT TIME CYCLE
C
DO 199 I = 1,NPTS
UINIT(I) = U(I)
199 CONTINUE
200 CONTINUE
C
300 CONTINUE
C
FORMATS
C
1 FORMAT (IH1,30(/),T50,'TIME RATE OF SETTLEMENT PROGRAM',/,
1 T52,'PROGRAMMED BY MARTY GOODMAN',//)
2 FORMAT (IH1,20(/))
55 FORMAT (T22,'LAYER ',T31,'THICKNESS',T43,'# OF STRATA',T58,
1 'STRATA THICKNESS',T78,'COEFF. OF CONSOLIDATION',T105,
2 'ALPHA',//)
60 FORMAT (T23,I2,T33,F5.1,T47,I2,T64,F5.2,T85,F8.5,T104,F6.4)
75 FORMAT (' LAYER','I2,' HAS A VALUE OF ALPHA MORE THAN'
1 '0.5',/',,' EITHER REDUCE THE TIME INCREMENT DELT',/',T20
2 'OR',/',
3 'REDUCE THE # OF STRATA IN THE LAYERS WITH CALCULATED'
4 'VALUES OF ALPHA GREATER THAN 0.5',/',
5 'THIS MAY BE ACCOMPLISHED BY INCREAIN6 DELZ')
85 FORMAT (IH1,T5,'LAYER',T20,'INITIAL EXCESS PORE PRESSURES',//)
90 FORMAT (5X,I2,5X,18(F5.0,1X))
98 FORMAT (10X,18(1X,F6.0))
99 FORMAT (2(/))
117 FORMAT (' THE TOP OF AND BOTTOM BOUNDARIES ARE UNDRAINED',//,
1 ' NO CONSOLIDATION WILL OCCUER')
130 FORMAT (IH1,5(/),T43,'TIME (DAYS)',T67,'PERCENT CONSOLIDATION',//
140 FORMAT (IH1,5(/),T2,'TIME (DAYS)',T50,'EXCESS PORE PRESSURE (PSF)'//
1 T120,'CONSOLIDATION',/',T59,'Z (FEET)'//)
145 FORMAT (T40,F10.1,20X,F10.2)
150 FORMAT (F8.1,2X,15(1X,F6.0))
165 FORMAT (IH+,T119,F10.2)
339 FORMAT(1H1,///,T11,'TIME',T50,'LAYERS',///,T25,10(I4,6X)//)
1200 FORMAT(T5,F10.1,8X,10(F8.1,2X))
END