Dynamic Characteristics of Twin-Piece Vane Compressors

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DYNAMIC CHARACTERISTICS OF TWIN-PIECE VANE COMPRESSORS

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ABSTRACT

This paper describes a twin-piece vane compressor which has freely movable double vanes assembled in each slot machined in the rotor with double seal lines between the vane tip and the inner surface of the cylinder. Analysis of the motion, forces, friction, wear and stresses in the vanes with a dynamic model demonstrates that the twin-piece vane compressor has decreased the binding forces, friction losses and wear on the vane tip, but increased vane stresses. The results also indicate that most of the friction loss occurs at the vane tip with additional friction loss on the vane sides. The viscous loss caused by the oil film between the double vanes is so small that it can be neglected in engineering calculations. The ratio of the upper vane thickness to the lower vane thickness is one of the most important parameters for the twin-piece vane compressor. Analysis of the influence of this ratio on the forces, friction, wear and strength of the vanes was used to optimize the thickness ratio for the twin-piece vane compressor.

NOMENCLATURE

\(a_v\) — vane acceleration relative to the slot
\(B\) — vane thickness (m)
\(E\) — Young’s modulus of elasticity (Nm\(^{-2}\))
\(e\) — difference between maximum and minimum cylinder profile radii (m)
\(F_t\) — vane tip binding force (N)
\(F_{t1}\) — friction on vane tip (N)
\(f_{j1}\) — friction factor between cylinder and vane
\(f_{j2}\) — friction factor between rotor slot and vane
\(H\) — vane axial length (m)
\(h\) — vane radial height (m)
\(L\) — vane extension out of rotor (m)
\(m\) — vane mass (kg)
\(M_0\) — binding moment between F-vane and R-vane (Nm)
\(p\) — pressure (Nm\(^{-2}\))
\(p_a\) — fluid pressure in the little chamber between vane tips (Nm\(^{-2}\))
\(p_b\) — fluid pressure acting on vane base (Nm\(^{-2}\))
\(p_h\) — pressure acting on vane trailing edge (Nm\(^{-2}\))
\(p_q\) — pressure acting on vane leading edge (Nm\(^{-2}\))
\(r\) — rotor radius (m)
\(R_1\) — binding force at outer support between rotor and vane (N)
\(R_2\) — binding force at inner support between rotor and vane (N)
\(R_{11}\) — friction at outer support between rotor and vane (N)
\(R_{12}\) — friction at inner support between rotor and vane (N)
\(R_G\) — distance between vane mass center and rotation center (m)
\(R_0\) — binding force between F-vane and R-vane (N)
\(R_{p1}\) — radius of vane tip arc (m)
\(R_p\) — cylinder profile curvature (m)
\(v_v\) — vane velocity relative to the slot
\(\alpha\) — vane inclination (rad)
\(\mu\) — Poisson’s ratio
\(\rho(\phi)\) — cylinder profile radius vector (m)
\(\omega\) — rotor angular speed (rad/s)
\(\phi\) — rotation angle or position angle (rad)
\(\tau\) — shear stress (MNm\(^{-2}\))

Subscripts
\(c\) — cylinder
\(d\) — one-piece vane
\(F\) — F-vane
\(R\) — R-vane
\(v\) — vane
1 INTRODUCTION

A sliding vane compressor has many advantages such as simple construction, no eccentric rotary parts, small size, and high volumetric efficiency, so it is widely used in refrigeration, automotive air conditioning and gas compression installations. Its main disadvantage is the large friction losses and wear between the vane tips and the inner surface of the cylinder. This can sharply decrease the service life and mechanical efficiency of the compressor, but special vane designs can reduce the problem. In the rotary compressor invented by Robert Groll in 1987, friction between the vane tips and the casing is eliminated by setting a small clearance between the vanes and the stator. The leakage through this clearance can be made equal to, or even less than, that across the equivalent line of contact for a free sliding vane by making the vane much thicker to create a very high resistance flow path. The vane assembly functions within a controlled rotary vane compressor in such a way that internal rubbing friction is virtually eliminated because the vane tips maintain a small clearance from the inside of the substantially elliptical contour of the interior of the stator housing. This occurs as the axle-roller assemblies fixed to the vanes follow the internal cam paths built into the stator end plates. Internal sealing is maintained by the viscoinertial effects of the lubricant within the machine. In through-vane compressors, the vanes are assembled in the through slot machined in the rotor. The two tips of each vane are always in contact with the inner surface of the stator as the rotor rotates. This can appreciably reduce friction between the vane tips and the inner surface of the stator. The friction and leakage between the vane tips and the inner surface of the cylinder can be reduced further with the twin-piece vane compressors.

Fig. 1 shows the twin-piece vane compressor in which freely movable dual vanes are assembled in each slot machined in the rotor. When the rotor is rotated by a prime mover, the vanes are pushed against the inner surface of the cylinder by centrifugal force. The round tips of the two vanes in contact with the inner surface provide double seal lines between the vane tips and the inner surface in the cylinder. A little chamber formed between the tips of the dual vanes collects lubricating oil from leakage through the clearance between the two vanes and the vane tips scraping on the inner surface of the cylinder. The oil stored in the chamber greatly reduces refrigerant leakage through the vane tip clearances from the high pressure side to the low pressure side. At the same time, since the binding force of the vane acting on the inner surface of the cylinder is borne by the two vanes, the maximum force can be decreased and the friction losses and wear on the vane can be reduced.

The leading vane in the twin-piece vane compressor relative to the rotation direction is referred to as the forward vane (abbreviated F-vane) while the trailing vane is called the rear vane (abbreviated R-vane). The ratio of the F-vane thickness to the R-vane thickness is a key design parameter for the twin-piece vane compressor. This paper analyzes the motion, forces, friction losses, wear and stresses of the two vanes with a dynamic model. The thickness ratio of twin-piece vane is optimized by analyzing the influence of the thickness ratio on the forces, friction losses, wear and strength of the vanes.

2 FORCES AND FRICTION LOSSES

2.1 Forces Acting on Vanes

A free body diagram of the forces acting on the vanes in the twin-piece vane compressor is shown in Fig. 2. Since the F-vane contacts with the R-vane on a surface, the binding forces between them can be simplified as a force, \( R_0 \), and a moment, \( M_0 \), acting at the mass center of each vane. If the directions of the forces and moment are consistent with the directions shown in Fig. 2, their values are positive. The sample compressor has a simple harmonic cylinder profile listed in Table 1. Taking the intersection point between the line dividing the two vanes and the inner surface of the cylinder as a reference point.
According to Coulomb’s friction law, the friction force between the two bodies equals the binding force times the friction factor between the two bodies. The viscous resistance caused by the fluid between the vanes and the slot surfaces is of the same order of magnitude as the viscous resistance between vanes, which the following analysis shows to be negligible, and it can be neglected. Using Darlambe’s theory, the force balance equations in the +X and +Y directions, and the moment balance equation about point G (the vane mass center) can be written as:

\[
\begin{align*}
 F_{\text{vane}} \cos \alpha_F - f_1 F_{\text{vane}} \sin \alpha_F + F_{\text{exp}} - F_{\text{fr}} &- f_2 R_1 + F_{\text{bf}} - F_{\text{df}} = 0 \\
 F_{\text{vane}} \sin \alpha_F - f_1 F_{\text{vane}} \cos \alpha_F - F_{\text{exp}} - F_{\text{fr}} + R_0 - R_2 - F_{\text{vf}} &= 0 \\
 \left(F_{\text{vane}} \cos \alpha_F - f_1 F_{\text{vane}} \sin \alpha_F\right) \frac{h}{2} + M_0 + R_2 \frac{h}{2} - F_{\text{fr}} \frac{h-L_F}{2} &= 0 \\
 - F_{\text{vane}} \cos \alpha_R - f_1 F_{\text{vane}} \sin \alpha_R + F_{\text{exp}} - F_{\text{fr}} - f_2 R_1 + F_{\text{br}} - F_{\text{dr}} &= 0 \\
 F_{\text{vane}} \sin \alpha_R - f_1 F_{\text{vane}} \cos \alpha_R - F_{\text{exp}} - F_{\text{fr}} + R_1 + R_0 + F_{\text{fr}} &= 0 \\
 \left(F_{\text{vane}} \sin \alpha_R - f_1 F_{\text{vane}} \sin \alpha_R\right) \frac{h}{2} + R_1 \left(\frac{h}{2} - L_R\right) - M_0 + F_{\text{fr}} \frac{h-L_R}{2} &= 0
\end{align*}
\]

where \( F_{\text{ex}} = m_F \omega^2 \left(R_{GR}^2 - r^2 \sin^2 \theta\right)^{\frac{1}{2}} \); \( F_{\text{exR}} = m_R \omega^2 \left(R_{GR}^2 - r^2 \sin^2 \theta\right)^{\frac{1}{2}} \); \( F_{\text{ey}} = -m_F \omega^2 \sin \theta \); \( F_{\text{eyR}} = -m_R \omega^2 \sin \theta \); \( F_{\text{fr}} = m_F \omega^2 \sin \theta \); \( F_{\text{fr}} = m_R \omega^2 \sin \theta \); \( F_{\text{bf}} = -2m_F \omega \psi_F \); \( F_{\text{bfR}} = -2m_R \omega \psi_R \); \( F_{\text{fr}} = p_b B \); \( F_{\text{fr}} = p_b B \); \( F_{\text{fr}} = (p_q+p_a) B \); \( F_{\text{fr}} = (p_q+p_a) B \); \( F_{\text{fr}} = p_q L_B \); \( F_{\text{fr}} = p_b L_B \); \( \alpha_F = \psi_F + \sin^{-1}\left[r \sin \theta / \rho(\phi_F)\right] \); \( \alpha_R = \psi_R + \sin^{-1}\left[r \sin \theta / \rho(\phi_R)\right] \); \( \psi_F = \tan^{-1}(\rho \omega \sin \phi_F / \rho(\phi_F)) \); \( \psi_R = \tan^{-1}(\rho \omega \sin \phi_R / \rho(\phi_R)) \).

Ma \(^3\) presented a detailed analysis of the velocity \( v \), acceleration \( a \), inertia and gas forces on the vanes. Equation (1) can be solved by Gauss Elimination to give the binding forces acting on the vane tips and sides and between the vanes.

2.2 Comparison of Friction Losses

The friction loss on the vane tips can be defined as

\[
L_t = f_1 \left[F_{\text{vane}} \rho(\phi_F) + F_{\text{fr}} \rho(\phi_R)\right] \rho
\]

(2)

The friction loss on the vane sides can be defined as

\[
L_s = f_2 (R_1 v_{\text{fr}} + R_2 v_{\text{fr}})
\]

(3)

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The clearance in the contact section between the vanes provides a leakage path for lubricating oil to leak from the high pressure region at the base of the vanes through the clearance to the vane tips when the compressor is in running. The lubricant then leaks to the working elements. The relative motion between the vanes can cause viscous loss in the clearance. Assuming that the vanes cannot incline when moving, the leakage flow between the vanes is approximated as fully-developed laminar flow through the clearance between two infinite parallel plates, and the viscous loss can be calculated as follows:

Table I Main parameters of the sample compressor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>36.64 cm³</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>28 mm</td>
</tr>
<tr>
<td>Cylinder profile</td>
<td>28+8.5sin²φ mm</td>
</tr>
<tr>
<td>Vane radial height</td>
<td>17 mm</td>
</tr>
<tr>
<td>Vane axial length</td>
<td>44 mm</td>
</tr>
<tr>
<td>Rotor slot width</td>
<td>4 mm</td>
</tr>
<tr>
<td>Vane number</td>
<td>5</td>
</tr>
<tr>
<td>Vane tip radius</td>
<td>5 mm</td>
</tr>
</tbody>
</table>

The losses for the sample compressor whose specifications are listed in Table 1 were calculated. The friction loss on the vane tips is the largest, 87.1 percent of the total loss. The friction loss on the vane sides accounts for another, 12.6 percent. The viscous loss caused by the oil film between the vanes is only 0.3 percent, and it can be neglected in engineering calculations.

3 INFLUENCE OF THICKNESS RATIO ON FRICTION LOSSES AND WEAR

3.1 Influence of Thickness Ratio on Friction Loss and Wear at Vane Tips

For constant rotor slot width, the binding forces acting on the vane tip of the twin-piece vane compressor are compared for thickness ratios of 1:3, 2:3 and 3:3 with the forces on a one-piece vane compressor in Figs 3a-3c. \( F_{nr} \), the bending force on the F-vane tip and \( F_{nr} \), the bending force on the R-vane tip, are much less than \( F_n \) the bending force on the tip of one-piece vane. However the variation of \( F_{nr} \) is the same as \( F_n \) + \( F_{nr} \), shown by the dashed line in Figs 3a-3c, is nearly equal to \( F_n \) for rotation angles \( \phi \) between 0 and 90 degrees, but is less than \( F_n \) for rotation angles \( \phi \) between 90 to 180 degrees. The maximum value of \( F_{nr} \) + \( F_{nr} \) is 15 percent less than that of \( F_n \) demonstrating that the binding force and friction on the vane tips is clearly reduced by the twin-piece vane compressor which will reduce the friction losses and the wear on the vane tips.

The difference between \( F_n \) and \( F_{nr} + F_{nr} \) results from the different motions in the two systems. When the vanes are pushed radially outward as the rotor rotates from 0 to 90 degrees, the cavity at the vane base gradually enlarges and lubricating oil is sucked into the cavity. The leakage through the clearance between the two vanes is very small because of the small pressure difference between the base cavity and the tip chamber. The slightly lower pressure in the cavity caused by the reduced leakage results in \( F_{nr} + F_{nr} \) being slightly less...
than \( F_n \). When the vanes are drawn back into the cavity as the rotor rotates from 90 to 180 degrees, the base cavity gradually shrinks and the lubricating oil is discharged from the cavity. The oil pressure in the cavity increases during the discharge process, but the pressure increase in the cavity is less than that of the one-piece vane because of the leakage through the clearance between the vanes. Thus \( F_{nr} + F_{nR} \) is less than \( F_n \). When the rotation angle approaches 180 degrees, the extrusion of the oil by the vane through the clearance between the vanes is very small because the vane velocities are very low. Therefore the difference between \( F_{nr} + F_{nR} \) and \( F_n \) gradually lessens and finally becomes zero.

As the vane thickness ratio increases from 1:3 to 3:3, \( F_{nr} \) gradually increases and \( F_{nR} \) gradually decreases. However, \( F_{nr} + F_{nR} \) is nearly constant and the friction loss on the vane tips increases slightly. The binding forces and friction losses in the twin-piece vane compressor for normal operating conditions are listed in Table 2. The results demonstrate that the thickness ratio has little effect on the forces, friction losses and wear at the vane tips. Though the forces, friction losses and wear at the vane tip decrease as the ratio decreases, the change is within the calculation error.

### 3.2 Influence of Thickness Ratio on Friction Losses and Wear at Vane Sides

For constant rotor slot width, the binding forces acting on the vane sides of the twin-piece vane compressor are compared for the thickness ratios of 1:3, 2:3 and 3:3 with the forces on a one-piece vane compressor in Figs 4a~4c. The variation of the binding force \( R_2 \) acting on the inner point between the vane and the slot is similar to that of the one-piece vane compressor. But the absolute value of the binding force \( R_1 \) acting on the outer point between the vane and the slot is less than that of the one-piece vane compressor, with the maximum for the twin-piece vane much smaller than that of the one-piece vane. Thus, the binding forces and the friction on the vane sides are reduced in twin-piece vane compressor, so the friction losses and the wear on the vane sides will be lessened. The binding forces are reduced because \( F_{nr} + F_{nR} \) is always less than \( F_n \), so the friction on the vane tips of the twin-piece vane compressor is less than that of the one-piece vane compressor. Since \( R_1 \) acts in the opposite direction of the friction, the variation of the absolute value of \( R_1 \) has the same tendency as that of the friction, so \( R_1 \) is less than \( R_{1d} \).

As the vane thickness ratio is increased from 1:3 to 3:3, \( R_1, R_2 \) and the friction losses on the vane sides increase slightly (see Table 2). For example, the maximum values of \( R_1 \) and \( R_2 \) and the average friction loss on the vane sides for a thickness ratio 1:3 are 2.7%, 4.1% and 1.7% larger than those of the compressor with a thickness ratio of 3:3. So the forces, friction losses and wear on the vane sides can be lessened by reducing the thickness ratio.

In addition, the locations where the binding forces act on the vane tips and sides differ for the different
thickness ratios which also causes slight differences in the forces, friction losses and wear.

Table 2 Comparison of forces and friction losses

<table>
<thead>
<tr>
<th>thickness ratio</th>
<th>( (F_{nc} + F_{nr})<em>{\text{max}} ) or ( (F_n)</em>{\text{max}} ) (N)</th>
<th>friction losses on vane tip (w)</th>
<th>( (R_1)_{\text{max}} ) (N)</th>
<th>( (R_2)_{\text{max}} ) (N)</th>
<th>friction losses on vane sides (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 3</td>
<td>241.38</td>
<td>123.68</td>
<td>519.54</td>
<td>127.86</td>
<td>17.75</td>
</tr>
<tr>
<td>2 : 3</td>
<td>241.36</td>
<td>123.84</td>
<td>527.97</td>
<td>130.99</td>
<td>17.93</td>
</tr>
<tr>
<td>3 : 3</td>
<td>241.36</td>
<td>123.95</td>
<td>533.61</td>
<td>133.16</td>
<td>18.05</td>
</tr>
<tr>
<td>one-piece</td>
<td>277.40</td>
<td>134.86</td>
<td>578.38</td>
<td>134.23</td>
<td>19.86</td>
</tr>
</tbody>
</table>

4 INFLUENCE OF THICKNESS RATIO ON VANE STRENGTH

4.1 Influence of Thickness Ratio on Vane Contact Strength

The vane tip contact with the inner cylinder surface is modeled as the contact between one cylinder inside another cylinder with parallel axes with a maximum contact stress between them of

\[
\sigma_H = \left[ \frac{F_n (r_p + r_r)}{2 \pi r_r r_p} \right] \left[ 1 - \frac{\mu_s^2}{E_s} \right]^{\frac{1}{2}} \frac{1}{\left( 1 + \frac{r_s}{r_p} \right)}
\]

Equation (4) can be rearranged as

\[
\bar{\sigma}_H = \frac{\sigma_H}{K_H} = \left[ \frac{F_n (r_p + r_r)}{2 \pi r_r r_p} \right] \left[ 1 - \frac{\mu_s^2}{E_s} \right]^{\frac{1}{2}} \frac{1}{\left( 1 + \frac{r_s}{r_p} \right)}
\]

where \( K_H = \left[ \frac{\pi (1 - \mu_s^2)}{E_s} \right] \) which is only related to the material properties of the bodies in contact with each other.

For constant \( K_H \), namely a fixed set of materials, \( \bar{\sigma}_H \) is directly proportional to \( \sigma_H \) and is called the equivalent contact stress.

The maximum \( \bar{\sigma}_H \) for the twin-piece vane design is always less slightly than that of the one-piece vane with the same vane slot width. The maximum R-vane's \( \bar{\sigma}_H \) for the twin-piece vane increases slightly with decreasing thickness ratio. The maximum \( \bar{\sigma}_H \) for the F-vane of the twin-piece vanes decreases with decreasing thickness ratio. For the thickness ratios of 1:3, 2:3 and 3:3, the maximum \( \bar{\sigma}_H \) for the F-vane are 1.25, 1.14, 1.06 times greater than that of the one-piece vane. The results show that the contact strength of the twin-piece vanes increases as the thickness ratio increases.

4.2 Influence of Thickness Ratio on Bending Strength and Shearing Strength

Since the relative displacement between the F-vane and the R-vane is very small as the rotor rotates in twin-piece vane compressor, the vanes are analyzed as being in surface contact throughout the motion. When the gas force formed by the pressure difference between the leading and trailing chambers of the vane acts on the part of the F-vane extended out of the rotor, the force transfers to the R-vane through the contact surface. The F-vane can not break unless the R-vane is broken. Therefore the bending strength and shearing strength of the twin-piece vane depend on the strength of the R-vane.

The bending moment and shearing force acting on R-vane are shown in Fig.2b. The bending stress and shearing stress on the weakest section A—A are

\[
\sigma_s = 6 \left[ \frac{F_{pr} - F_{ph}}{2} \right] \frac{L_R}{2} \left( \frac{F_{nr} \sin \alpha_R - F_{nr} \cos \alpha_R}{L_R} \right) \frac{1}{(H B_R^2)}
\]

\[
\tau_s = 3 \left[ \frac{F_{pr} - F_{ph}}{2} \right] \left( \frac{F_{nr} \sin \alpha_R - F_{nr} \cos \alpha_R}{2 H B_R} \right)
\]
For thickness ratio of 1:3, 2:3 and 3:3, the maximum bending stress for the twin-piece vane is 1.34, 2.1 and 3.0 times greater than that for the one-piece vane. The shearing stress is 1.09, 1.36 and 1.63 times greater than that of the one-piece vane. The results show that the bending stress and shearing stress for the twin-piece vane increase rapidly as the thickness ratio varies from 1:3 to 3:3.

The analysis indicates that the thickness ratio has little influence on the forces, friction losses and wear, but greatly influences the stresses of the twin-piece vane compressor. Therefore the compressor can use a smaller thickness ratio to strengthen the vanes. Since thinner vanes are more difficult to machine, the minimum vane thickness is restricted by the machining. Considering all the effects, the best twin-piece vane compressor design has a thickness ratio of 1:3.

5 CONCLUSIONS

The analysis of the twin-piece vane compressor shows that:

Most of the friction loss occurs at the vane tip, with some of the friction loss on the vane sides and a small amount of the viscous loss caused by the oil film between the F-vane and the R-vane, so the viscous loss can be neglected in engineering calculations.

The maximum force of the vane acting on the inner surface of the cylinder in the twin-piece vane compressor is less than that of the one-piece vane compressor with the same vane slot width. Though the friction losses and the wear on the vane are decreased, the stresses in the vane are increased in the twin-piece vane compressor.

In the twin-piece vane compressor, the thickness ratio has little influence on the forces, friction losses and wear, but greatly influences on the stresses in the vane. Considering the stresses and the machining of the vane, the best twin-piece vane compressor has a thickness ratio of 1:3.

6 REFERENCES
