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MOTION OF THE SPRUNG MASS OF A RECIPROCATING HERMETIC COMPRESSOR DURING STARTUP

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ABSTRACT

A mathematical analysis is made of compressor crankcase motion within the housing during startup. Dynamic models are presented for a two-cylinder reciprocating compressor, the six-degree of freedom crankcase, and single and three-phase induction motors. Pressure, inertial, unbalanced and spring forces are included in the dynamic models. Both single and three-phase induction motors are modeled using reference frame theory. Solution of the combined models yields the linear and angular time displacements of the crankcase.

INTRODUCTION

The suspension system of a reciprocating hermetic compressor must be designed for long life and quiet operation. It must be soft enough for low noise transmission, yet stiff enough to survive thousands of start-stop cycles in addition to the stresses induced during compressor shipment. The discharge tube often adds considerable stiffness to the suspension system and must be designed in conjunction with the mounting springs and brackets. A drawing of a typical two-cylinder compressor and its suspension system is shown in Figure 1. This particular compressor is mounted on three springs that are bracketed to the housing. As shown, the discharge tube is connected to the discharge muffler, routed under the crankcase and exits the compressor through the steel housing.

The impetus for this study was suspension system fatigue failures in a 2 KW compressor in which either single or three-phase motors were used depending upon the application. The starting characteristics of the three-phase motor induced much greater stresses in the springs and discharge tube than the single-phase motor. The speed-torque curves of the single and three-phase motors shown in Figure 2 indicate the relative strengths of the motors used in this compressor. A tool to predict crankcase motion during the starting transient was needed to aid suspension system redesign.

Analytical attempts were made as far back as 1972 [1] to combine slider-crank dynamics, pressure forces and motor torque to develop a model descriptive of the six-degree-of-freedom motion of the compressor crankcase within the housing. At that time, computer capacity was limited and induction motor modeling still in its infancy, so a full transient system model was not practical. However, analytical and experimental results were obtained for the steady periodic movement of a small single-cylinder compressor crankcase. This paper extends that original model to a dual-cylinder compressor and adds single and three-phase transient induction motor models so crankcase movement during both starting and running conditions can be calculated.

MATHEMATICAL MODELS

Slider-Crank Mechanism

The analysis of the slider-crank mechanism proceeds in the same manner as in [1]. In the interest of brevity, only the technique and the results of that analysis are repeated here. Referring to the original paper and Figure 3, free body diagrams of the piston, connecting rod and crankshaft yield dynamic relationships between the geometry of the mechanism, forces between elements and the applied motor torque. For a single-cylinder compressor, the equations of motion plus the kinematic relationships between slider-crank elements yield five simultaneous equations in terms of the forces \( F_{3x}, F_{3y}, F_{23x}, F_{23y} \) and crankshaft angular acceleration \( \alpha_2 \), given known values of pressure force \( P \), motor torque \( T_m \), crank angular displacement \( \theta \) and initial angular velocity \( \omega_2 \). A simple extension to two cylinders yields nine equations in terms of two sets of slider-crank forces and crank angular acceleration. In this model, pressure forces for each cylinder are derived at each crank angle assuming polytropic compression and expansion processes. It is also assumed that compressor discharge and suction pressures are equalized at startup. The pressures are slowly ramped to operating values over an arbitrarily selected time interval that can be from several seconds to several minutes in length depending upon the system characteristics.
As detailed in [1], the equations for the slider-crank mechanism are solved assuming constant angular acceleration during a small angular displacement, \( \Delta \theta \). This technique permits computation of both the time increment for the displacement and the angular velocity at the end of the time increment. From [1], these are:

\[
\Delta t = \frac{-w_{21} + \sqrt{w_{21}^2 + 2A \Delta \theta A_{21}}}{A_{21}} \quad \text{for} \quad A_{21} \neq 0
\]
\[
\Delta t = \frac{\Delta \theta}{w_{21}} \quad \text{for} \quad A_{21} = 0
\]

(1)

The forces associated with unbalanced sections of the crankshaft are calculated once the crankshaft angular acceleration and velocity are known as a function of angular displacement. The then known slider-crank and unbalanced forces allow calculation of the \( x \) and \( y \)-forces on the compressor main and outboard bearings at the end of the time increment. The bearing forces, piston sidewall forces, mechanism friction forces and torques (both of which may be included in the model), pressure forces and motor stator reaction torque then act as applied forcing functions to the six-degree-of-freedom crankcase.

**Sprung-Mass System**

The time-varying forces on the crankcase resulting from the mechanism and cylinder pressures are shown in Figure 4 and the geometric relationship of the forces exerted by the suspension system are shown in Figure 5. For small displacements of the crankcase, the spring forces are:

\[
F_{s1x} = -K_{s1x} (X + Y_{s1} \theta_z - ZS \theta_y)
\]
\[
F_{s1y} = -K_{s1y} (Y + X_{s1} \theta_z + ZS \theta_x)
\]
\[
F_{s1z} = -K_{s1z} (Z - Y_{s1} \theta_x - X_{s1} \theta_y)
\]
\[
F_{s2x} = -K_{s2x} (X - Y_{s2} \theta_z - ZS \theta_y)
\]
\[
F_{s2y} = -K_{s2y} (Y + X_{s2} \theta_z + ZS \theta_x)
\]
\[
F_{s2z} = -K_{s2z} (Z + Y_{s2} \theta_x - X_{s2} \theta_y)
\]

(2)

Three displacement and three rotation equations result from the application of Newton’s Second Law about the center of gravity of the crankcase. Equation 3 is slightly different from its counterpart in reference [1], Equations 7-12. Terms for the 2\( \text{nd} \) piston have been added, signs for the piston sidewall forces \( (F_{wp}) \) corrected, and velocity pressure product cross-coupling terms [3] added for the rotational axes.

\[
M \frac{d^2 X}{dt^2} = -F_{MBX} - F_{CBX} + F_{PRESS1} - F_{PRESS2} - F_{WP1} - F_{WP2} + F_{s1x} + F_{s1y} + F_{s1z} + F_{s2x} + F_{s2y} + F_{s2z}
\]
\[
M \frac{d^2 Y}{dt^2} = -F_{MBY} - F_{CBY} - F_{SP1} - F_{SP2} + F_{s1y} + F_{s2y} + F_{SLY}
\]
\[
M \frac{d^2 Z}{dt^2} = F_{s1z} + F_{s2z} + F_{SLZ}
\]

\[
I_{XX} \frac{d^2 \theta_x}{dt^2} = -F_{MBX} Z_{MB} - F_{SPY} X_{C1} - F_{SPY} Z_{C1} - F_{CBY} Z_{CB} - F_{BIX} Y_{BIX} - F_{s1z} Y_{s1} + F_{s1y} Y_{s2} + F_{s1y} ZS
\]
\[
+ F_{s2y} ZS + F_{SLZ} Y_{SL} - F_{SLY} Z_{SL} - (I_{zz} - I_{YY}) \left( \frac{d \theta_y}{dt} \right) \left( \frac{d \theta_y}{dt} \right) - I_{CS} \left( \frac{d \theta_x}{dt} \right) \left( \frac{d \theta_x}{dt} \right)
\]

(3)

\[
I_{YY} \frac{d^2 \theta_y}{dt^2} = F_{MBY} Z_{MB} + F_{CBY} X_{CB} - F_{PRESS1} Z_{C1} - F_{PRESS2} Z_{C2} - F_{WP1} Z_{C1} + F_{WP2} Z_{C2}
\]
\[
- F_{s1x} X_{s1} - F_{s1y} X_{s2} + F_{s2x} X_{s3} - F_{s1y} Z_{s3} - F_{s2z} ZS - F_{s3y} ZS - F_{SLY} Z_{SL}
\]
\[
+ F_{SLX} Z_{SL} - (I_{XX} - I_{ZZ}) \left( \frac{d \theta_x}{dt} \right) \left( \frac{d \theta_x}{dt} \right) + I_{CS} \left( \frac{d \theta_y}{dt} \right) \left( \frac{d \theta_y}{dt} \right)
\]

\[
I_{ZZ} \frac{d^2 \theta_z}{dt^2} = F_{MBY} X_{MB} + F_{CBY} X_{CB} - F_{SPY} X_{C1} - F_{SPY} X_{C2} + F_{SPY} X_{C3} + F_{SPY} X_{S1} + F_{SPY} X_{S2} - F_{s1x} X_{s3}
\]
\[
+ F_{s1x} Y_{s1} - F_{s1y} Y_{s2} + F_{s2x} Y_{s3} - F_{s1y} X_{s3} - F_{s1y} Y_{s2} - F_{s1y} X_{s3}
\]
\[
+ F_{s1y} Y_{s2} + F_{s1y} X_{s3} - F_{s1y} X_{s3} - F_{s1y} Y_{s2} - F_{s1y} X_{s3}
\]
\[
+ F_{s1y} Y_{s2} + F_{s1y} X_{s3}
\]
Induction Motor Models

Speed-torque curves of induction motors show only the average component of torque. Single-phase motors, even during "steady-state" operation, have a significant double line frequency torque component. This alternating component is often as large or larger than the average component, depending upon the motor design and load. Motor torque during starting may have components that vary from line frequency to twice line frequency depending on motor load and at what point in the electrical cycle the motor is energized. Even three-phase motors have large line frequency torque components during acceleration, although the "steady-state" torque is constant under steady load. Dynamic models for both single and three-phase motors have been developed using reference frame theory. This theory involves a transformation of coordinates that, in most cases, simplifies the motor differential equations.

Single-Phase Motor

Transformation of the machine vector equations for a single-phase split-capacitor motor yield five simultaneous equations describing motor currents in terms of applied voltage. From [2], these equations are:

\[
\begin{align*}
\frac{dx_m}{dt} &= \frac{-awr(X_2 + X_m)}{w_b} \\
\frac{di}{dt} &= -\frac{C}{awr(X_2 + X_m)} \\
\frac{di'}{dt} &= 0 \\
\end{align*}
\]

Integration of Equation 4 yields instantaneous values of the stator and referred rotor currents. Once the currents are available, the instantaneous electromagnetic torque output of the single-phase motor is expressed as:

\[ T_m = \left( \begin{array}{c} P \end{array} \right) \left( \begin{array}{c} \frac{x_m}{w_b} \end{array} \right) \left( \begin{array}{c} -i_{qs} - i_{ds} \end{array} \right) \]

Three-Phase Motor

In a manner similar to the single-phase motor, the dynamic equation for the three-phase motor in the synchronously rotating reference frame can be expressed [2]:

\[
\begin{align*}
\frac{dx_m}{dt} &= \frac{-awr(X_2 + X_m)}{w_b} \\
\frac{di}{dt} &= -\frac{C}{awr(X_2 + X_m)} \\
\frac{di'}{dt} &= 0 \\
\end{align*}
\]
The electromagnetic torque output of the three-phase motor is expressed as:

\[ T_m = \frac{1}{2} \left( \frac{p}{2} \right) \left( \frac{x_m}{b} \right) \left( i_{q_0} i_{d_0} - i_{q_0} i_{d_0} \right) \] (7)

**SOLUTION METHOD**

Initial thoughts were to recast the slider-crank equations into a form compatible with the sprung-mass and induction motor equations. This would allow combining, or matrix coupling, the equations into a single system equation that could be solved using conventional Newmark or Runge-Kutta techniques. However, this approach was not pursued for two reasons. First, existing slider-crank routines for both single and multi-cylinder compressors had been around many years and were well proved. Second, some modularity for including different motor modules in the system of equations would be sacrificed by full matrix coupling.

Thus, the procedure for solving for the crankcase motion consisted first of calculating a time increment for the slider-crank model given a small initial angular crankshaft velocity at starting. The resultant time increment and angular velocity along with the required mechanism torque were then passed to a Runge-Kutta routine to compute motor currents at the end of the time interval. Either Equation 4 or 6 was used depending upon the type of motor. Motor torque was then calculated using Equation 5 or 7. The torque was then passed back to the slider-crank model to obtain updated mechanism outputs. Although the motor torque applied to the crankshaft is one time increment behind in the solution, crankshaft angular displacements are controlled such that the variables change slowly. This precludes the need for iteration between the slider-crank and motor models. Outputs from both the slider-crank and motor models were then used as inputs to a Runge-Kutta routine to calculate crankcase rotations and displacements.

**RESULTS**

The crankcase movements as given by the solution of Equation 3 oscillate at the resonance frequencies of the suspension system and steady-state conditions would never be reached if the system were without damping. Thus, a small amount of Coulomb damping was introduced into each section of Equation 3 to obtain the steady-state crankcase movements. This damping was sufficient to dampen transients, but did not significantly affect maximum displacements or rotations of the crankcase. A version of Equation 3 was also derived that included the gyroscopic effects of the crank and rotor, but the gyroscopic effects on crankcase motion proved insignificant for this particular compressor.

Figure 6 is typical of the model output and shows the rotation of the crankcase about the vertical axis for the first one-half second after power is applied to the compressor. Traces for both the single and three-phase compressor models are shown.

**CONCLUSIONS**

Models of the slider-crank mechanism and six-degree-of-freedom crankcase from previous work are revisited. Dynamic models of single and three-phase induction motors are then presented. The models are combined to yield the time dependent solution to the motion of a hermetic compressor crankcase during starting. The known displacements and rotations of the crankcase center-of-mass can then be used to calculate the displacements and rotations of arbitrary points on the crankcase body. Future work will seek experimental verification of the analytical results.
LIST OF SYMBOLS

Mechanical symbols -

\( c_g \) Center of gravity
\( A_{CGX} \) Acceleration of the connecting rod \( c_g \) in the \( x \)-direction.
\( A_{CGY} \) Acceleration of the connecting rod \( c_g \) in the \( y \)-direction.
\( \alpha_p \) Acceleration of the piston in the \( x \)-direction.
\( \lambda_3 \) Crankshaft angular acceleration.
\( \lambda_0 \) Connecting rod angular acceleration.
\( \dot{\delta} \) Crankshaft rotation direction vector, clockwise = -1.
\( d/dt^n \) \( n \)-th derivative with respect to time.
\( F_{TXX} \) Force exerted on the connecting rod in the \( x \)-direction by the crankshaft.
\( F_{TYY} \) Force exerted on the rod in the \( y \)-direction by the crankshaft.
\( F_{TZZ} \) Force exerted on the piston by the comm rod in the \( z \)-dir.
\( F_{RXX} \) Resultant mechanism \( X \)-dir force on outboard bearing.
\( F_{RYY} \) Resultant mechanism \( Y \)-dir force on outboard bearing.
\( F_{RXX} \) Resultant mechanism \( X \)-dir force on main bearing.
\( F_{RYY} \) Resultant mechanism \( Y \)-dir force on main bearing.
\( F_{PNN} \) Pressure force at piston \( n \) on sprung mass.
\( F_{TAxx} \) Discharge tube \( X \)-direction spring force.
\( F_{TAyy} \) Discharge tube \( Y \)-direction spring force.
\( F_{TAzz} \) Discharge tube \( Z \)-direction spring force.
\( F_{Xxx} \) X-direction spring force at spring \( n \).
\( F_{Xsx} \) X-direction spring force at spring \( n \).
\( F_{Xsn} \) X-direction spring force at spring \( n \).
\( F_{Xpn} \) Friction force on piston \( n \).
\( F_{Xsp} \) Piston \( n \) side wall force.
\( I_{c1} \) Connecting rod moment of inertia about its \( c_g \).
\( I_{c2} \) Crankshaft and rotor moment of inertia about their \( c_g \) of rotation.
\( I_{x1} \) Moment of inertia of sprung mass about an \( x \)-axis through its \( c_g \) parallel to the \( x \)-direction.
\( I_{y1} \) Moment of inertia of sprung mass about an \( y \)-axis through its \( c_g \) parallel to the \( y \)-direction.
\( I_{z1} \) Moment of inertia of sprung mass about an \( z \)-axis through its \( c_g \) parallel to the \( z \)-direction.
\( K_{snr} \) Spring constant of spring \( n \) in the \( r \)-direction.
\( P \) Force exerted on the piston due to gas pressure diff.
\( T_n \)Torque exerted on the rotor by the motor.
\( \omega_2 \)Crankshaft angular velocity.
\( \omega_3 \)Crankshaft angular velocity at the start of a rotational increment.
\( \omega_3 \)Connecting rod angular acceleration.
\( X \)Linear displacement of sprung mass \( c_g \) in the \( x \)-dir.
\( XB \)X-distance of crankshaft from sprung mass \( c_g \).
\( X_{mn} \)Instantaneous \( X \)-distance of piston \( n \) from sprung mass center of gravity.
\( X_{mx} \)X-distance of spring \( n \) from sprung mass \( c_g \).
\( X_{my} \)X-distance of discharge tube from sprung mass \( c_g \).
\( Y \)Linear displacement of sprung mass \( c_g \) in the \( y \)-dir.
\( Y_{mn} \)Y-distance of spring \( n \) from sprung mass \( c_g \).
\( Y_{mx} \)Y-distance of discharge tube from sprung mass \( c_g \).
\( Z \)Linear displacement of sprung mass \( c_g \) in the \( z \)-dir.
\( Z_{cn} \)Z-distance of piston \( n \) from sprung mass \( c_g \).
\( Z_{cb} \)Z-distance of outboard bearing from sprung mass \( c_g \).
\( Z_{mb} \)Z-distance of main bearing from sprung mass \( c_g \).
\( ZS \)Z-distance of spring plane from sprung mass \( c_g \).
\( Z_{sn} \)Y-distance of spring \( n \) from sprung mass \( c_g \).
\( Z_{nx} \)Z-distance of discharge tube from sprung mass \( c_g \).
\( \theta \)Crankshaft rotation angle.
\( \phi \)Connecting rod rotation angle.
\( \Delta \theta \)Increment of crankshaft rotation.
\( \Delta t \)Time increment for crankshaft \( \Delta \theta \).
\( \theta_1 \)Rotational displacement of sprung mass center of gravity about \( x \)-axis.
\( \theta_2 \)Rotational displacement of sprung mass center of gravity about \( y \)-axis.
\( \theta_3 \)Rotational displacement of sprung mass center of gravity about \( z \)-axis.

Single-Phase Motor Electrical symbols -

\( d/dt^n \) \( n \)-th derivative with respect to time.
\( I_{s}, I_{a}, I_{e} \) Stator currents transformed to the quadrant and direct axes.
\( i_{s}, i_{a}, i_{e} \) Rotor currents referred to the main winding and transformed to the quadrant and direct axes.
\( P \) Number of poles in the motor.
\( q \) Capacitor charge.
\( f_1, f_2, f_3, f_e \) Main winding, auxiliary winding, referred rotor and capacitor resistances, respectively.
\( \omega_{s}, \omega_{a} \) Rotor electrical frequency and the inductances base frequency, respectively.
\( X_{s}, X_{a} \) Main winding, auxiliary winding, referred rotor, and magnetizing reactances, respectively.

Three-Phase Motor Electrical symbols -

\( d/dt^n \) \( n \)-th derivative with respect to time.
\( i_{s}, i_{a}, i_{e} \) Stator currents transformed to quadrant, direct and '0' axes.
\( i_{s}, i_{a}, i_{e} \) Referred and transformed rotor currents.
\( P \) Number of poles in the motor.
\( f_1, f_2 \) Stator phase and referred rotor phase resistances.
\( v_{na}, v_{sa}, v_{oa} \) Transformed stator voltages.
\( w_{e}, w_{a} \) Applied electrical, rotor electrical, and inductances base frequencies, respectively.
\( X_{s}, X_{a} \) Stator phase leakage, referred rotor phase leakage and 3/2 the stator magnetizing reactances.

REFERENCES


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