A STUDY OF PAVEMENT STRAINS
AND DEFLECTIONS PRODUCED BY
DYNAMIC TIRE FORCES

JULY 1969 - NUMBER 21

BY
SVEIN O. ENGENES

JHRP
JOINT HIGHWAY RESEARCH PROJECT
PURDUE UNIVERSITY AND
INDIANA STATE HIGHWAY COMMISSION
Progress Report

A STUDY OF PAVEMENT STRAINS AND DEFLECTIONS PRODUCED BY DYNAMIC TIRE FORCES

To: J. F. McLaughlin, Director
Joint Highway Research Project

From: R. L. Michael, Associate Director
Joint Highway Research Project

July 25, 1969
File: 6-20-6
Project: C-36-527

Attached is a Progress Report on the JHR, Part II, research project titled "Stresses and Deflections." The title of the research report is "A Study of Pavement Strains and Deflections Produced by Dynamic Tire Forces." It has been authored by Mr. Svein O. Eagenes under the direction of Professor B. E. Quinn. The research was conducted in the School of Mechanical Engineering.

This report is of the activity conducted under Phase II of this research approved for the first year in late 1967. The study was to investigate the factors influencing the relationship between dynamic tire force and pavement strain. The non-availability of personnel resulted in little progress during FY 68 and a continuance of the project into FY 69. Weather conditions prevented the taking of sufficient field data until the spring of 1969 and the original first year's work was not completed until July 1969.

The report is presented to the Board for acceptance as fulfilling the tire force-strain objective of the study proposed and approved in 1967. Copies will also be sent to the ESHC and the HR for their review, comment and acceptance.

Respectfully submitted,

Harold L. Michael
Associate Director

HLM/56

cc: F. L. Ashbaucher  R. H. Merrall  C. F. Scholer
    W. L. Dolch  J. A. Fevers  M. B. Scott
    W. H. Goetz  V. E. Harvey  W. T. Spencer
    W. L. Grecco  G. A. Leonards  H. R. J. Walsh
    G. K. Hallock  F. B. Mendenhall  K. B. Woods
    M. E. Harr  R. D. Miller  E. J. Yoder
Progress Report

A STUDY OF PAVEMENT STRAINS AND DEFLECTIONS PRODUCED BY DYNAMIC TIRE FORCES

by

Svein O. Engen
Graduate Assistant in Research

Joint Highway Research Project
File No: 6-20-6
Project No: 3-36-529

Prepared as Part of an Investigative Study Continued by

Joint Highway Research Project Engineering Experiment Station Purdue University
in cooperation with the
Indiana State Highway Commission
and the
U.S. Department of Transportation Federal Highway Administration
Bureau of Public Roads

The opinions, findings, and conclusions expressed in this publication are those of the authors and necessarily those of the Bureau of Public Roads.

Not Released for Publication
Subject to Change

Not Reviewed By
Indiana State Highway Commission or the
Bureau of Public Roads

Purdue University
Lafayette, Indiana
July 15, 1969
ACKNOWLEDGMENTS

This investigation was made possible through support granted by the Bureau of Public Roads and Joint Highway Research Project, Civil Engineering School, Purdue University.

Special appreciation is expressed to Dr. Bayard E. Quinn, Professor of Mechanical Engineering, who as project director and major professor provided guidance, advice, and encouragement. His help in the preparation of this thesis is particularly appreciated.

Thanks are expressed to Mr. Walsh for providing a test site and for permitting the use of the facilities of the Indiana State Highway Commission Research and Training Center, West Lafayette, Indiana.

Suggestions and technical advice given by E. J. Yoder, Professor of Civil Engineering, are gratefully acknowledged.

The help and suggestions of fellow members of the Vehicle Dynamics Research Group, Jack Zable and Ali Sattaripour to the solution of some knotty problems encountered has been greatly appreciated.

Thanks are also given to Steven Hildebrand for his help in obtaining data.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. THEORETICAL PROCEDURES FOR PREDICTING TIRE FORCES AND PAVEMENT STRAIN</td>
<td>11</td>
</tr>
<tr>
<td>3. EXPERIMENTAL PROCEDURE TO STUDY THE EFFECT OF DYNAMICAL TIRE FORCE</td>
<td>41</td>
</tr>
<tr>
<td>4. PRESENTATION AND DISCUSSION OF RESULTS</td>
<td>54</td>
</tr>
<tr>
<td>5. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>69</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>72</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Illustration of the Relationship between Static and Dynamic Tire Forces</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Root Mean Square Value of the Dynamic Tire Force versus Vehicle Velocity</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Dynamic Tire Force versus Inflation Pressure</td>
<td>6</td>
</tr>
<tr>
<td>2.1</td>
<td>Components of Two Types of Pavement</td>
<td>13</td>
</tr>
<tr>
<td>2.2</td>
<td>Longitudinal Strain in an AC Pavement</td>
<td>15</td>
</tr>
<tr>
<td>2.3</td>
<td>Loadings of Beams on an Elastic Foundation</td>
<td>20</td>
</tr>
<tr>
<td>2.4</td>
<td>Force Power Spectra</td>
<td>27</td>
</tr>
<tr>
<td>2.5</td>
<td>Tire Force Probability Density Curve</td>
<td>29</td>
</tr>
<tr>
<td>2.6</td>
<td>Schematic Presentation of the Suspension System of a Car</td>
<td>31</td>
</tr>
<tr>
<td>2.7</td>
<td>Quarter Vehicle Model</td>
<td>32</td>
</tr>
<tr>
<td>2.8</td>
<td>Tire Enveloping Formulation</td>
<td>34</td>
</tr>
<tr>
<td>2.9</td>
<td>Vehicle Model with Tire Enveloping Properties</td>
<td>35</td>
</tr>
<tr>
<td>2.10</td>
<td>Input Bump to the Simulated Quarter Vehicle</td>
<td>37</td>
</tr>
<tr>
<td>2.11</td>
<td>Maximum Tire Force vs Velocity for Simulated Vehicle</td>
<td>38</td>
</tr>
<tr>
<td>2.12</td>
<td>Maximum Tire Force Experienced by a Vehicle for Different Tire Pressures and for an Impulse Type of Input to the Tire</td>
<td>39</td>
</tr>
<tr>
<td>3.1</td>
<td>Overall Pavement Modulus of Elasticity vs Foundation Thickness</td>
<td>42</td>
</tr>
<tr>
<td>3.2</td>
<td>Statical Strain Data for Different Gage Locations</td>
<td>45</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic Representation of the Pavement Model</td>
<td>46</td>
</tr>
<tr>
<td>3.4</td>
<td>Gage Location on the Plate</td>
<td>49</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.5</td>
<td>Arrangement and Connection of Strain Gages in a Wheatstone Bridge</td>
<td>50</td>
</tr>
<tr>
<td>4.1</td>
<td>A Typical Strain Output from the Pavement Model</td>
<td>55</td>
</tr>
<tr>
<td>4.2</td>
<td>Calibration Strain Curve for Pavement Model</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>Maximum Longitudinal Strain as a Function of Vehicle Velocity</td>
<td>59</td>
</tr>
<tr>
<td>4.4</td>
<td>Relationship Between Tire Force and Pavement Strain</td>
<td>60</td>
</tr>
<tr>
<td>4.5</td>
<td>Longitudinal Strain for Constant Vehicle Velocity vs. Tire Pressure</td>
<td>63</td>
</tr>
<tr>
<td>4.6</td>
<td>Illustration of Bump Location on Pavement Model</td>
<td>65</td>
</tr>
<tr>
<td>4.7</td>
<td>Strain Record from Bump Test</td>
<td>66</td>
</tr>
<tr>
<td>4.8</td>
<td>Strain Record from Bump Test</td>
<td>68</td>
</tr>
</tbody>
</table>
ABSTRACT

Engenes, Svein Olaf. MS.ME, Purdue University, August 1969. A Study of Pavement Strains and Deflections Produced by Dynamic Tire Forces. Major Professor: Dr. Bayard E. Quinn.

Irregularities in the highway profile cause extra forces and stresses in the pavement due to the induced vibratory motion of the vehicle.

The load exerted on a highway by a vehicle consists of a constant static load and a fluctuating dynamic load. The measurement of the static weight of a vehicle creates no problems. The dynamic load which is dependent on highway conditions, vehicle characteristics and vehicle operating conditions is more difficult to determine.

A quarter vehicle simulation was used in order to analyze these factors theoretically. The results of this investigation indicated the conditions under which high dynamic tire forces could be expected.

In order to examine experimentally the effect of dynamical tire force on pavement deflections and pavement strain, an experimental pavement model was formulated. The major design criterion for the model was the simulation of the deflection curve of the pavement surface. Under identical loading conditions, the pavement model and the actual highway should have the same deflection curves.

A simple analysis proved the feasibility of such a model and it was therefore built and tested.

The significance of such factors as vehicle velocity, tire pressure,
and road roughness on pavement strain were then investigated.
CHAPTER 1

INTRODUCTION

A knowledge of the forces which are induced on a highway by a moving vehicle may be of considerable importance in the design and maintenance of highways. These pavements are stressed by vertical and horizontal forces which are transmitted by the wheels of the moving vehicle. These forces may be important causes of road damage.

Vertical loads may be considered to consist of essentially two components with respect to each wheel. When the vehicle is sitting motionless on a level pavement, the forces which interact between the vehicle and the pavement are due only to the static weight of the vehicle.

When the vehicle is moving at normal operating speed, irregularities in the highway profile will induce vibrations in the sprung and unsprung masses of the vehicle, and these motions will cause a fluctuating dynamic load to be superimposed on the static weight of the vehicle. These vertical forces are called dynamic tire forces (or just dynamic forces) and they are defined with respect to the static force. An illustration of the relationship between static and dynamic forces is shown in Figure 1.1. It should be noted that the total vertical force exerted on the pavement is the sum of the static and the dynamic tire forces.

Forces which are largely horizontal in direction may be generated by acceleration or braking of the vehicle, steering action of the driver
FIG. 1.1 ILLUSTRATION OF THE RELATIONSHIP BETWEEN STATIC AND DYNAMIC TIRE FORCES
and wind loads on the vehicle. These forces are not of great concern in this study.

Several factors are influencing the dynamic tire force. Of most concern is the road and its condition. When a vehicle is travelling on a relatively smooth pavement hardly any motion is induced to the vehicle body. However, when the car is moving along on a rough road, pitching and tilting motions are introduced to the vehicle body. These dynamic effects will cause inertia forces which may increase the total wheel loads considerably. Experimental tests have proved this. Tire pressure measurements made by Wilson (10) indicate that tire forces are higher on rougher roads.

This has been the subject of considerable study. An experienced driver knows that dips and waves in the highway profile, which may not be noticeable at slow speeds, may become hazardous at high velocities. However, does this mean that the tire force will increase with increasing speed? An investigation in which a test vehicle was operated over the same length of pavement at different velocities and for which the tire force measurements were recorded, indicated increased tire forces from increased operating speed (10). The resulting RMS value of the tire force from this study is shown in Figure 1.2.

Unfortunately, the tire force is influenced by other factors than pavement roughness and vehicle velocity. The suspension characteristics of the vehicle are significant as well as the tire inflation pressure. It has been shown that when a vehicle is operated over the same length of road at the same velocity (10), different tire forces will be present when the tire pressure is varied. The RMS values from such a test are
FIG. 1.2 ROOT MEAN SQUARE VALUE OF DYNAMIC TIRE FORCE VERSUS VEHICLE VELOCITY
shown in Figure 1.3.

In many areas weather conditions may be of significance to the tire force. Ice and snow covering the pavement surface may induce vehicle motions which will result in larger tire forces. Under such conditions a smooth road may experience relative large tire forces.

Evidently, if a comprehensive study is to be made of the dynamic tire force, the following factors must be considered:

1. Pavement profile
2. Vehicle velocity
3. Vehicle characteristics
4. Tire pressure
5. Weather conditions

Conversely, if only the effect of pavement condition on tire force is to be studied, then tests must be made with the same vehicle at the same speed and under the same operating condition over different pavement sections.

In studying the presence of dynamic tire forces on a pavement, both experimental and theoretical approaches exist.

Of the experimental approaches, two methods can be easily used. The first one involves the use of the pressure transducer previously mentioned (10). The second method is utilized by measuring the inertia force and the shear force on one of the vehicle wheels and the corresponding axle housing. Both methods have proved to give reliable results.

In order to theoretically predict the tire force two other methods are most commonly used.

In the first procedure, a highway elevation power spectral density
FIG. 1.3 DYNAMIC TIRE FORCE VS TIRE INFLATION PRESSURE

INDIANA S.R. 26
SECTIONS 17-20
VEHICLE VELOCITY = 53.3 MPH
analysis (6) is obtained that gives the presence of power in different wave bands. If these values are multiplied by the square of the vehicle characteristic transfer function (tire force divided by the input displacement to the wheel), the force power spectrum of the dynamic tire force can be obtained. The square root of the area under the force power spectrum curve is the RMS value of the dynamic tire force for a specific vehicle on a specific road. Engja (14) designed an instrument by which the highway power spectrum could be obtained directly.

Another method permits the prediction of the tire force in the time domain. The highway elevation is given as an input to the vehicle characteristics in the time domain. The output force is obtained by the use of an analog computer. In this procedure it is easier to deal with system nonlinearities if sufficient computer capacity is available.

In this theoretical study a simple simulation of a quarter vehicle was made (Chapter 2). The input to the tire was chosen to be a displaced cosine curve that represented the pavement profile. This was used since the major object of the study was to investigate the influence of factors such as vehicle velocity and tire pressure on the maximum tire force. The reason for this type of study was to detect theoretically the conditions under which high tire forces could be expected.

Many approaches have been used to find the actual deflection of the pavement under the tire force. Tests have indicated that relative high tire forces on the highway may create small deflections in the pavement layers.

However, although only small deformations may occur in the pavement, especially at moderate distances from the point of application of a dynamic force, this force may not be insignificant as far as damage to the
highway is concerned. High forces may result in high contact stresses, and rapid surface deterioration may occur even though the interior structure of the pavement may not be adversely affected by such forces. Therefore it is evident that the relationship between dynamic tire force and highway response is an important area of study.

To conduct this study a simulated section of highway, easy to build and easy to examine analytically, was built. The design criterion for this pavement model was the surface deflection curve. This deflection curve should be identical for the model and the actual pavement. A model consisting of a steel beam resting on an elastic foundation was found to fulfill this criterion.

When such a tool was obtained simple tests which otherwise would require a considerable amount of effort, could easily be performed. Surface strain in the pavement model could be detected from strain gages. Different types of highways could be analoged by changing the parameters of the model. Thus the significance of such factors as vehicle velocity, pavement roughness, and tire pressure on the strain in the highway surface could be examined.

Originally it was intended to use a truck as the test vehicle and to simultaneously obtain measurements of the truck tire force and the corresponding pavement strain. The tire force measuring system would consist of strain gages bonded to the truck's axle housing and accelerometers mounted on the wheel. These elements would respectively pick up the axle housing shear force and the inertia force of the wheel. This method was tested on a Chevrolet Biscayne and the results were good. Unfortunately, at that time, no truck was readily available. Hence the intended tests
could not be conducted.

However, in view of the extensive work already done in measuring tire force by the tire pressure measurement approach, and the reliable results that already have been obtained by this method, it was felt that more emphasis should be put on the development of the simulated pavement model and the study of the strains and deflections in this model.

Some unexpected problems were encountered in the development of the pavement model and the development was more time consuming than anticipated.

As a result, simultaneous readings of the tire force and pavement strain were not made as originally planned.

However, this could not be considered as a serious loss in this investigation. Due to previous experience in measuring tire forces, an approximate idea of what forces would have been experienced was available. Tests made under similar circumstances on another research project (10) permitted the prediction of the approximate magnitudes which should be present in the type of tests that were planned.

Of considerable interest was the finding, early in this study, that different values of pavement strain can exist for nearly the same tire force, depending on the vehicle and pavement characteristics. This observation resulted in a large amount of research effort being directed toward a study of the factors that are significant in such a situation.

This together with the fact that the final location of the pavement model did not permit vehicle speeds high enough to generate large dynamic forces, was the basis for the decision to omit simultaneous measurements of tire force and pavement strain.
However, the final pavement model permitted tests which added confidence to tire force measurements already obtained.
CHAPTER 2

THEORETICAL PROCEDURES FOR PREDICTING TIRE FORCES AND PAVEMENT STRAIN

The Simulated Highway

The determination of strains in the pavement just underneath the tire is at the present time accomplished by using influence charts. This method gives a good indication of the deflections and the total strain present. However, if the strain distribution at a distance from the point of load application is desired, the task is more difficult because no good method has as yet been developed. The complexity of a highway and the fact that individual pavements have different properties, makes it difficult to set up general mathematical equations for strains and deflections. The many different layers in the pavement and the interaction between these layers are the factors which cause the problem.

However, experimental measurements of strain and deflection can be obtained. Strain gages, mounted directly on or in the pavement, give a good indication of the strains present. The deflections can be found by using linear, variable, differential, transformer gage installations, and a Benkelman beam. This has up to now proved to be one of the better methods for finding strains and deflections in the pavement.

However, these tests are inconvenient and require considerable time and effort. They also only give the characteristics of the specific highway where the experiment is conducted. In other words,
if another road is to be investigated, the whole set-up has to be moved to the new location and the tests have to be repeated.

For obvious reasons it would be of great interest if a simple highway model could be developed mathematically in which computed strains and deflections would be similar to those in an actual pavement. In order to pursue this any further a general knowledge of the design of different types of pavements is important.

In general, pavements can be divided into two categories (2). These are shown in Figure 2.1. A flexible pavement consists of a relatively thin wearing surface built over a base course and a sub-base course which rest on the compacted subgrade. The thickness of such a pavement includes all the components above the subgrade, and these are considered to be the structural components of the pavement.

In contrast, a rigid pavement is made up of Portland cement concrete. It may or may not have a base course between the pavement and the subgrade. The concrete exclusively is referred to as the pavement.

The essential difference between the two types of pavement is the manner in which they distribute the load over the subgrade. The rigid pavement tends to distribute the load over a wide area of soil. Thus a major portion of the structural capacity is supplied by the slab itself.

The load-carrying capacity of the flexible pavement is brought about by the load-distributing characteristics of the layered system. In these pavements the highest quality material is at the surface. The strength is a result of building up thick layers and thereby distributing the load over the subgrade rather than through the bending action of a slab as in the case of a rigid pavement.
BINDER COURSE  >  SURFACE COAT  >  SEAL COAT  >  TACK COAT  >  PRIME

BASE COURSE
SUBBASE COURSE
COMPACTED SUBGRADE
NATURAL SUBGRADE

FIG. 2.1a COMPONENTS OF A FLEXIBLE PAVEMENT

PORTLAND-CEMENT CONCRETE
BASE COURSE MAY OR MAY NOT BE USED
SUBGRADE

FIG. 2.1b COMPONENTS OF A RIGID PAVEMENT
From this brief survey it may be concluded that no simple model could represent the two types of pavements.

In order to achieve its purposes, a model should fulfill one of two following conditions. It should have either the same deflection characteristics as an actual pavement, or it should have the same strain distribution. Strain data obtained by a study group at California Division of Highways (3), are shown in Figure 2.2. The test was performed on an AC (asphalt cement) surfaced road by a 12,000-lb single axle load flotation tire. The solid line represents longitudinal strain measurements at the bottom of the asphalt surfacing. The dotted line represents the strain at the top of the surfacing. Since the strain gages would be destroyed if the tire travelled directly over them, data are not available at the surface directly under the tire.

In order to reproduce in a model the strain curves similar to those in Figure 2.2, the model must have an elastic foundation. The reversal of strain from positive to negative that is experienced in most pavements, is the base for this conclusion.

If an exact mathematical formulation of a pavement system is desired, the resulting model is quite complicated. Of great complexity are the dynamical effects which will be present in the uppermost layers of the model. These vertical motions are caused by the vibratory behaviour of the exciting vehicle.

A relatively simple model can be devised consisting of a steel plate resting on an elastic foundation. The foundation can simply be rubber plates or felt. The mathematical equations representing the strain in such a model when plotted, exhibit the characteristic reversal in strain
FIG. 2.2 LONGITUDINAL STRAIN IN AN AC PAVEMENT
In designing a simple model, a study of the size of the steel plate was made. If the plate was given the same dimensions area-wise as an actual concrete slab in a rigid pavement, it could be assumed to be an infinite plate. This would simplify the mathematics, but the loading would be more complicated. If the plate was made smaller, the boundary conditions would be a problem. However, if the steel plate was made small enough to be considered as an infinitely long beam, the mathematical analysis would be greatly simplified. Such a model has a form far different from that of an actual pavement, but for the purposes of this study which are to investigate surface strains at different vehicle velocities and vehicle tire pressures, such a model can be justified.

The transverse strains will of course be different (equal to zero). The longitudinal strains will be of higher magnitudes, but they should take the same general form as if the model had been a plate. Thus simply by increasing or decreasing the stiffness of the elastic foundation, longitudinal strain curves similar to those found in actual pavements can be realized.

The problem caused by dynamical effects within the model itself can now be neglected. The mass of the plate will be drastically reduced, and the forces created will, in comparison to those of the vehicle, be negligible. In addition to the fact that this model can easily be described mathematically, it has other advantages. It can cheaply be built and experimentally tested. (This will be discussed in Chapter 3). By having a relatively thick and stiff beam resting on a soft foundation, the model can represent a rigid highway. One would then get the long
damped sinusoidal waveform experienced in this type of pavement. On the contrary, if the steel plate is made thinner and more flexible and at the same time the stiffness of the foundation is increased, the strain picture would approximate that of a flexible pavement.

In formulating the deflection curve of a pavement model consisting of a steel beam on an elastic foundation, the following differential equation can be used: (4)

\[ EI \frac{d^4 Y}{dx^4} = q \]  \hspace{1cm} (2.1)

where

\[ E = \text{Modulus of elasticity of the beam} \]
\[ I_z = \text{Moment of inertia of the beam around its transverse axis} \]
\[ Y = \text{Vertical deflection} \]
\[ x = \text{Longitudinal distance in the direction of vehicle travel} \]
\[ q = \text{Intensity of load acting on the beam} \]

For an unloaded portion, the only force on the beam is the continuously distributed reaction from the foundation. This has the intensity \( kY \).

Hence

\[ q = -kY \]  \hspace{1cm} (2.2)

where

\[ k = \text{Modulus of the foundation} \]

Thus:

\[ EI \frac{d^4 Y}{dx^4} = -kY \]  \hspace{1cm} (2.3)

Using the notation

\[ \beta = \sqrt{\frac{k}{4EI_z}} \]
the general solution to the differential equation can be expressed as follows:

\[ Y = e^{\beta x}(A \cos \beta x + B \sin \beta x) + e^{-\beta x}(C \cos \beta x + D \sin \beta x) \]  \hspace{1cm} \text{(2.4)}

This can easily be verified by substituting the values from eqn. (2.4) into eqn. (2.3). The constants of integration \(A, B, C\) and \(D\) must be determined from known conditions at certain locations.

For simplicity, consider the case of a concentrated load \(P\), acting on the midpoint of the beam. Using the conditions:

\[
Y = 0 \quad \text{at} \quad x = \pm \infty
\]

\[
\frac{dY}{dx} = 0 \quad \text{at} \quad x = 0
\]

\[
V = \frac{d^2Y}{dx^2} = -EI_z \left( \frac{d^3y}{dx^3} \right) = -\frac{P}{2} \quad \text{at} \quad x = 0
\]

where

\[ V = \text{Shear force} \]

\[ M = \text{Bending moment} \]

one arrives at the following equation:

\[
Y = \frac{P}{8\beta^3EI_z} e^{\beta x}(\cos \beta x + \sin \beta x) \]  \hspace{1cm} \text{(2.5)}

\[
= \frac{PB}{k} e^{\beta x}(\cos \beta x + \sin \beta x) \]  \hspace{1cm} \text{(2.6)}

In order to obtain an expression for the longitudinal strain at either the top or bottom surfaces of the beam, the following relations can be used:

\[
\varepsilon = \frac{M C}{EI_z} \]  \hspace{1cm} \text{(2.7)}
where

\[ C = \text{One half the thickness of the plate} \]

\[ \varepsilon = \text{Strain} \]

and

\[ M = - EI_z \frac{d^2 Y}{dx^2} \]

\[ = - \frac{P}{4\beta} e^{-\beta x}(\sin \beta x - \cos \beta x) \]

Thus,

\[ \varepsilon = - \frac{PC}{4\beta EI_z} e^{-\beta x}(\sin \beta x - \cos \beta x) \]

In the practical situation; the load exerted by a wheel of a vehicle on the beam can hardly be approximated as a point load. The print of the tread of the tire usually takes an elliptical form. However, it is satisfactory to consider this print to be a rectangular one having the load uniformly distributed over a length \( L \) (See Figure 2.3a). The procedure for determining the deflection at any point \( A \) on the beam will now be developed.

If the load produced by an element \( dx \) is substituted for \( P \) in eqn. (2.6) one will obtain the following:

\[ \frac{dY}{dx} = \frac{q}{8\beta^3 EI_z} e^{-\beta x}(\cos \beta x + \sin \beta x) \]

or

\[ \frac{dY}{dx} = \frac{q}{8\beta^3 EI_z} e^{-\beta x}(\cos \beta x + \sin \beta x) \]

The deflection produced at \( A \) by the loading distributed over the length \( L \), if \( A \) is inside the loaded region (Figure 2.3a) then becomes:
CASE #1. POINT IN LOADED REGION

\[ q = \text{INTENSITY OF LOAD} \]

\[ \epsilon_A = \frac{q C}{4 \beta^2 EI_z} [e^{-\beta c} \sin \beta c + e^{-\beta b} \sin \beta b] \]

BEAM ON ELASTIC FOUNDATION UNIFORMLY LOADED OVER A DISTANCE L

FIG. 2.3a

CASE #2. POINT OUTSIDE LOADED REGION

\[ \epsilon_A = \frac{q C}{4 \beta^2 EI_z} [e^{-\beta b} \sin \beta b - e^{-\beta c} \sin \beta c] \]

BEAM ON ELASTIC FOUNDATION UNIFORMLY LOADED OVER A DISTANCE L

FIG. 2.3b
\[
Y = \int_{0}^{b} \frac{-q}{8B^3EI_z} e^{-Bx}(\cos Bx + \sin Bx) \, dx + \int_{0}^{c} \frac{q}{8B^3EI_z} e^{-Bx}(\cos Bx + \sin Bx) \, dx
\]  \hspace{1cm} (2.13)

This becomes
\[
Y = \frac{q}{2k} (2 - e^{-Bb} \cos Bb - e^{-Bc} \cos Bc)
\]  \hspace{1cm} (2.14)

If the test point A is outside the loaded region (Figure 2.3b) the deflection becomes
\[
Y = \frac{q}{2k} (e^{-Bc} \cos Bc - e^{-Bb} \cos Bb)
\]  \hspace{1cm} (2.15)

By using equations 2.7, 2.8, and 2.12 the strain at point A can be obtained. If this point is inside the loaded region one will arrive at the following expression
\[
\varepsilon = \frac{qC}{4B^2EI_z} (e^{-Bb} \sin Bb + e^{-Bc} \sin Bc)
\]  \hspace{1cm} (2.16)

When the point is outside the loaded region one will obtain
\[
\varepsilon = \frac{qC}{4B^2EI_z} (e^{-Bb} \sin Bb - e^{-Bc} \sin Bc)
\]  \hspace{1cm} (2.17)

By using equations 2.16 and 2.17 the longitudinal strain in the simulated pavement at either the top or bottom surface of the plate can be easily calculated and compared to any experimentally obtained results.

Moreover, by using this model it is also possible to estimate the strain in the pavement at any selected distance from the wheel. The magnitude and location of strain reversals such as those shown in Figure 2.2
can therefore be computed.

Highway Power Spectrum

In order to properly predict the expected strain in the formulated model, the tire forces created by the vehicle must be known. Several types of procedures have been developed to achieve this. By using a pressure transducer (10) one will be able to obtain a time record of the force directly. However, in some cases the nature of the highway is such that the description of the road lends itself to statistical procedures. The power spectral density analysis of the dynamic force under the wheel is one characterization which has been found to be useful in investigations of highway roughness problems. The root-mean-square value of the tire force fluctuations about the static load is the part of the dynamic force which may provide useful information.

The power spectral density analysis has been applied previously to this type of operation (5, 6) however, a brief explanation of the nature of this analysis will be given to clarify the procedure. An essential step in the analysis is to characterize the highway in the form of a power spectral density analysis of the highway profile. The vehicle can be described as an equivalent linear system in the frequency domain. Now it is possible to predict the force power spectrum by using the following relationship (7)

\[ P_F(f) = P_H(f) \cdot |F/X(f)|^2 \]  \hspace{1cm} (2.18)

where:

- \( P_F(f) \) = power spectrum of the dynamic force as a function of frequency in cycles per second
- \( P_H(f) \) = power spectrum of the deviations of the highway profile elevations as a function of frequency in cycles per second
\[ F/X (f) = \text{steady-state sinusoidal relationship between F/X and frequency in cycles per second (vehicle characteristics) where:} \]

\[ F = \text{force in lbf (tire force)} \]

\[ X = \text{input displacement in inches to the tire} \]

The first essential step in computing the input power spectrum, (in this case the highway elevation power spectrum) is to determine the autocovariance function defined by the following equation:

\[ C(\tau) = \lim_{h \to \infty} \left[ \frac{1}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} h(t) h(t + \tau) \, dt \right] \tag{2.19} \]

where:

\[ h(t) = \text{highway elevation measurement at station t} \]

\[ h(t+\tau) = \text{highway elevation measurement at distance from station t} \]

\[ T = \text{total length of highway profile being analyzed} \]

\[ C(\tau) = \text{autocovariance function of h(t) in feet squared} \]

\[ \tau = \text{lag value in feet} \]

The autocovariance function indicates whether or not a highway profile can be considered as a random function. A well-behaved autocovariance function will approach zero as the lag values are increased. The power spectrum, which is one method to characterize a random function, is the Fourier transform of the autocovariance function. It is possible to obtain a power spectrum of highway elevation measurement if the autocovariance function is well-behaved. The relationship for the power spectrum is given by the following formula:

\[ P_H(w) = 2 \int_{-\infty}^{\infty} C(\tau) e^{-i2\pi w \tau} \, d\tau \tag{2.20} \]
where:

\[ P_H(\omega) = \text{power spectrum of the deviations of the highway elevation measurements in feet squared per cycle per foot} \]

\[ C(t) = \text{autocovariance function of } h(t) \text{ in feet squared} \]

\[ w = \text{frequency in cycles per foot} \]

\[ i = \sqrt{-1} \]

\[ \tau = \text{lag value} \]

The power spectral density function indicates how the total variance of a random function is distributed over the frequency domain. It describes the highway profile in terms of existing wave lengths and can be used to obtain the mean-squared-value of the profile deviations. Further, it shows the contribution to the mean-squared-value that is made by the various ranges of wave lengths.

The power spectrum can be used along with a suitable vehicle characteristic and vehicle velocity to predict the dynamic force which a vehicle will exert on a highway. The power spectral density analysis of highway elevation measurements will not contain the dimension time, since a highway is a geometric quantity. The selection of vehicle velocity is necessary in order to relate the wave lengths in the highway to the vibrational frequencies of the vehicle. Briefly the procedure for making an analysis is as follows:

1. Highway elevation profile measurements are made along the highway at one-foot intervals for a sufficient distance in the outer wheel path.

2. Deviations from selected base lines are obtained and the power spectrum analysis is made using these deviations.

As previously mentioned, if the vehicle velocity is used as a scaling factor, the power spectrum of the highway can be converted to the
frequency domain. The ordinate will then be changed from feet squared per cycle per foot to feet squared per cycle per second, and the abscissa from cycles per foot to cycles per second, since the following relationship exists:

\[ f_t \left( \frac{\text{cycles}}{\text{sec}} \right) = f_d \left( \frac{\text{cycles}}{\text{ft}} \right) v \left( \frac{\text{ft}}{\text{sec}} \right) \]

and

\[ P_H(f) \left( \frac{\text{ft}^2}{\text{cycles} \cdot \text{sec}} \right) = P_H(w) \left( \frac{\text{ft}^2}{\text{cycles} \cdot \text{ft}} \right) / v \left( \frac{\text{ft}}{\text{sec}} \right) \]

where:

- \( f_t \) = time-based cyclic frequency
- \( f_d \) = distance-based cyclic frequency
- \( P_H(f) \) = time-based power spectral density
- \( P_H(w) \) = distance-based power spectral density
- \( v \) = velocity of the vehicle

In order to use the highway elevation power spectrum to compute a tire force power spectrum, the vehicle characteristics in the frequency domain are needed. It is customary in obtaining a frequency description for the vehicle to select two variables. These two variables are usually defined as input and output. The input for a vehicle suspension system can therefore be considered to be the vertical displacement of the tire tread and is represented by \( X \). Since the highway profile gives rise to the dynamic force, it is reasonable to select the force of the tire against the highway as the output. The transfer function (F/X vs frequency) for any vehicle is best obtained experimentally. This is due to the fact that most of the elements in the suspension systems of cars are
nonlinear. However, once the transfer function is obtained, the force PSD function can readily be computed by using equation (2.18). As may well be expected, this function represents only the "power" experienced by one specific vehicle on one specific highway. In other words, if another car or another pavement had been tested the result would have been different.

A plot of a typical force PSD function is shown in Figure 2.4. The mean-square value of the dynamic force, equal to the area under the force power spectral density curve, is obtained by integration. The RMS (root-mean-square) value of the dynamic force, can be obtained easily from the mean-square value. The RMS value of the dynamic force represents a number that gives a convenient quantitative measure of the "spread about the mean". Since the theory of the PSD analysis is based on the assumption that the highway can be considered to be random, and since the number of elevation measurements is large, it can be assumed that the force at any instant is a random variable. If a normal distribution is assumed then the equation describing such a function is as follows (8):

$$p(P, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2} \left(\frac{P - \mu}{\sigma}\right)^2}$$  \hspace{1cm} (2.21)

where:

- $p(P, \mu, \sigma^2)$ = probability density value for the force $P$ given a specific mean and variance
- $\mu$ = mean
- $\sigma^2$ = variance
- $\sigma$ = root-mean-square

A plot of a typical normal probability density function is given on
TEST VEHICLE NO. 210
VEHICLE VELOCITY = 95.3 FT./SEC.
IND. HIGHWAY 45

\[ \sigma^2 = 9.209 \]

FIG. 2.4 FORCE POWER SPECTRA
Figure 2.5. The area under this curve is always equal to unity. It has been shown (9, 10, 5) that the dynamic tire force and hence the variance, increases as:

(a) the vehicle velocity increases
(b) the road gets rougher
(c) the tire pressure increases

On Figure 2.5 this means that the curve will be flattened out, due to the fact that the root-mean-square value will increase. This means that the probability of encountering larger tire forces will also increase.

When the tire force probability density curve is established, a corresponding characteristic for strain can be calculated. By examining equation 2.10, one will observe that the strain just underneath the tire tread is a linear function of the tire force. This equation becomes for \( x = 0 \):

\[
\varepsilon = \frac{C}{4\beta E_i^2} P
\]

or

\[
\varepsilon = C_c P
\]

where

\[ C_c = \text{constant} \]

For some materials such as concrete, the relationship between the stress level and the number of cycles that this stress can be applied until failure occurs (S-N curve) is known. It is therefore possible to determine the pavement stress levels at which early failure may be expected, and to estimate the probability of such failure if the stress probability density curve can be determined.
FIG. 2.5 TIRE FORCE PROBABILITY DENSITY CURVE
Further effort is needed to develop this procedure. This research project was directed toward the problem of determining pavement strain (hence stress) experienced under various conditions of vehicle operation.

**Prediction of Tire Forces From a Simulated Vehicle**

An automobile can be modeled in many ways. In this specific test, it is assumed that all four wheels have the same type of motion and that the car is moving straight forward on an even surface. Yaw action of the wheels and pitching motion of the body about its center of gravity are neglected. A typical schematic representation of the suspension system of a car is shown in Figure 2.6. If the assumption is made that all the coil springs, dashpots, tire spring rates, and tire damping rates have the same respective values, the car can be further simplified to a so-called quarter-vehicle model (Figure 2.7). In establishing the mathematical model or equations of motion for this system, assumptions were made as follows:

1. The springs and tires have linear spring rates
2. The damping in the shock absorbers is considered to be viscous and linear.
3. The tire has point contact with the supporting surface.

The decision to consider all of the elements in the model to be linear was made to simplify the mathematics. This has been proved to be a valid assumption for the purposes of this investigation. In making the third assumption an error is introduced. It has been shown that the computed loads on the pavement are consistently too high if tire enveloping effects are ignored (11). However, a method to simulate tire enveloping effects by generating road profiles as suggested by
FIG. 2.6 SCHEMATIC REPRESENTATION OF THE SUSPENSION SYSTEM OF A CAR
FIG. 2.7 QUARTER VEHICLE MODEL
experimental evidence has been developed. (12) Vertical forces produced by a slowly moving vehicle over a one-inch wide cleat will take a wave-form as shown in Figure 2.8a. A relative linear relationship between force and the height of the cleat exists. Hence the road profile can be thought of as a series of cleats of different amplitudes. (Figure 2.6b) Thus, in order to properly take the tire effects into consideration, a more representative model would be as shown in Figure 2.9. Unfortunately, lack of time and equipment prevented the use of this idea. However, since the purpose of this study is to determine the effect of tire pressure and vehicle velocity on tire forces, the model illustrated in Figure 2.7 will give useful information.

The equations of motion describing the behavior of this model are as follows:

\[ m_0 \ddot{x}_3 = k_2 (x_1 - x_0) + (\dot{x}_2 - \dot{x}_1) c_2 \]  
\[ m_1 \ddot{x}_1 = x_1 (k_1 + k_2) - k_1 x_0 - k_2 x_2 + \dot{x}_1 (c_1 + c_2) - c_1 \dot{x}_0 - c_2 \dot{x}_2 \]  

where:

- \( m_1 \) = unsprung mass of the quarter vehicle
- \( m_2 \) = sprung mass of the quarter vehicle
- \( k_1 \) = tire spring rate
- \( k_2 \) = coil spring rate
- \( c_1 \) = tire damping coefficient
- \( c_2 \) = shock absorber damping coefficient
- \( \ddot{x}_n \) = acceleration
FIG. 2.8a VERTICAL FORCE PRODUCED BY ONE INCH WIDE CLEAT

FIG. 2.8b ROAD PROFILE APPROXIMATED BY SERIES OF ONE INCH WIDE CLEATS
FIG. 2.9 VEHICLE MODEL WITH TIRE ENVELOPING PROPERTIES
\( x_n = \text{Displacement} \)
\( n = \text{The indexes 1, 2, or 0} \)

The output, the tire force, is given by the following equation:

\[
F_0 = k_1(x_0 - x_1) + c_1(\dot{x}_0 - \dot{x}_1) + \text{stat} \tag{2.23}
\]

where:

- \( F_0 = \text{Tire force} \)
- \( \text{Stat} = \text{The static weight of the quarter vehicle} \)

The input to the system is chosen to be of the form shown in Figure 2.10. The equation describing this input is as follows:

\[
X_0 = S(1 - \cos y) \tag{2.24}
\]

This can be written as:

\[
X_0 = S \left(1 - \cos \left( \frac{\nu t}{L} + 2\pi \right) \right) \tag{2.25}
\]

In order to get a good estimate of the types of irregularities that can be encountered in a highway, values of bump length and bump height from an actual highway elevation survey were chosen.

The differential equations 2.21 and 2.22 were solved on a digital computer. Figure 2.11 shows the effect of vehicle velocity on the tire force. It should be kept in mind that these forces are high since the tire enveloping effects are neglected. For every different velocity, the maximum tire force is plotted.

It was also possible to study the effect of various tire inflation pressures on the resulting tire force. This was possible because the relationships between tire stiffness \( k_1 \) and the tire damping ratio were known as a function of tire pressure. Figure 2.12 shows to what extent
FIG. 2.10 INPUT BUMP TO THE SIMULATED QUARTER VEHICLE
FIG. 2.11 MAXIMUM TIRE FORCE VS VELOCITY FOR SIMULATED VEHICLE
FIG. 2.12 MAXIMUM TIRE FORCE EXPERIENCED BY A VEHICLE FOR DIFFERENT TIRE PRESSURES AND FOR AN IMPULSE TYPE OF INPUT TO THE TIRE
tire pressure changes the force. Here too, the forces are higher than what the pavement actually experiences. In this test the velocity was kept constant.

The results obtained were used as a criterion for actual vehicle tests on the simulated pavement, described earlier in this chapter.
CHAPTER III

EXPERIMENTAL PROCEDURE TO STUDY THE EFFECT OF DYNAMICAL TIRE FORCE

In order to check the validity of the proposed simulated pavement model consisting of a steel beam resting on an elastic foundation, it was desirable to have it built and tested. As discussed previously in Chapter II, the major design criterion for this model was the simulation of the deflection curve of the pavement surface. The most significant parameter controlling this effect is the overall pavement foundation stiffness. This factor, which many authors call the modulus of the foundation, varies from 100 psi/in to over 500 psi/in depending upon the type of road that is considered.

When the first model was built, no attempt was made to simulate a specific pavement. Average values of stiffness were used to determine whether or not an acceptable deflection curve could be obtained. One inch thick rubber pads, 20 inches long, and 13 inches wide, were used as the elastic pavement foundation. Figure 3.1 shows how the modulus of the foundation varies with the number of pads used. Since the rubber was of high quality, it was assumed that:

1. No slippage would occur between two layers of rubber.
2. The material was homogeneous.

It was then necessary to choose the proper dimensions for the steel
FIG. 3.1 OVERALL PAVEMENT MODULUS OF ELASTICITY VS FOUNDATION THICKNESS

EACH PAD IS ONE INCH THICK
beam that would be placed on these pads. The following considerations were involved:

1. If the beam was too thick, very small deflections and strains would be present.

2. If on the other hand, the beam was too thin, permanent deformation might occur when the car passed over it.

3. The beam must be wider than the car tire so that it would be easy to drive along at high velocities.

To satisfy these conditions, a steel section one eighth inch thick, a hundred inches long, and seventeen inches wide was chosen to represent the beam. As will soon be seen, this represented a reasonable choice.

An advantage of being able to simulate a pavement with a steel plate is the fact that strain gage measurements can easily be made since SR-4 gages mounted on the plate will read the surface strain directly. When the gages are mounted in the proper manner, they will pick up longitudinal strain only. This strain should be the same for a plate and a beam if the cross sectional rigidities are equal.

The question as to where the gages should be mounted on the plate received some consideration. Two possibilities were present:

1. On the free surface facing the wheel.

2. On the surface facing the rubber pads.

If the loading is pure bending, the strains will be of equal magnitudes but of opposite sign on the two sides. However, the wheel tread will be in direct contact with the plate and thus some contact strain should be expected.

By conducting a simple test, the effect of contact strain was found to be negligible. This was investigated by placing the plate on a completely rigid foundation and by pushing the car slowly over it in such a
manner that the wheel came in direct contact with the gages. No determinable strain was detected and hence it was concluded that the major portion of the strain was due to bending. The gages were therefore allowed to face the foundation since in this position they were better protected from damage. An additional experiment was performed at the test site (Indiana State Highway Commission Research and Training Center, West Lafayette, Indiana) in which the test vehicle was moved slowly over the plate which had gages mounted on both sides. The results are shown in Figure 3.2. As can be seen, the strains, shown as functions of time, are of opposite sign, but the magnitudes are equal. The reason for the difference in the width just underneath the maximum peaks is due to different vehicle velocities.

As mentioned above, the tests involving a moving vehicle on the simulated pavement were all conducted at a selected test site. A hole of approximately the same size as the pavement model was dug in an asphalt coated flexible road. Into this hole the model was carefully placed so as to provide a smooth connection with the surrounding pavement in order to minimize undesired vehicle motions and tire forces. Figures 3.3a and b show respectively a side and an end view of the pavement model mounted in a section of existing pavement.

An eight inch thick concrete slab was poured into the bottom of the hole to give a rigid support to the entire pavement model. A layer of one inch thick plywood plates, shown in the same figures, was nailed together and could hence be considered as a rigid unit. Their purpose was mainly to provide additional space for a parameter study. If the stiffness of the foundation was to be changed, additional rubber pads would
FIG. 3.2 STATISTICAL STRAIN DATA FOR DIFFERENT GAGE LOCATIONS
FIG. 3.3a SIDE VIEW OF THE PAVEMENT MODEL

FIG. 3.3b END VIEW OF THE PAVEMENT MODEL
replace the plywood.

Six bolts located on the sides and extending from the top of the plate down into the concrete, were placed as shown to prevent the system from moving under the vehicle at high velocities. The nuts on the top were not tightened in such a manner as to put any stress in the plate.

This model can then be used to simulate either a rigid or a flexible pavement. A thick layer of rubber pads (which would result in a lower modulus for the foundation) together with a relative rigid steel plate on top, would characterize the rigid pavement. A higher modulus of foundation (two or three rubber pads) and a more flexible steel plate would represent the flexible pavement.

As previously mentioned, the strain can be measured by using bonded strain gages. In order to produce an output proportional to the change in resistance, a stable source of excitation is needed. Two circuit arrangements are usually used for this purpose.

1. The simple voltage-dividing potentiometer or ballast circuit.

2. The Wheatstone bridge.

The Wheatstone bridge is used to a great extent for connecting passive transducers to associated equipment in making a measuring system. A resistance bridge arrangement is particularly convenient for use with strain gages because it may be easily adjusted to null for zero strain. At the same time it provides means for effectively reducing or eliminating temperature effects.

The output voltage of a strain gage is usually from 10 to 1000 microvolts for common applications. In order to record the output signal, some form of electrical amplification is necessary. Thus, a stable
source of excitation and suitable amplification of the output signal are of primary importance in selecting the nature of the electrical instrumentation system which can be used satisfactorily with the resistance strain gage.

For this application an Ellis Bridge Amplifier, model BAM-1 was selected. One favorable characteristic of this instrument is that it is battery powered and transistorized for portability. The BAM-1 consists of a DC powered bridge circuit, DC transistor amplifier, static output meter and static and dynamic output connections. It has provisions for employing one, two, or four active strain gages in the bridge circuit, and is equipped with a calibrated balancing slide wire for accurate null-balance operation.

By applying a load of 1000 lbs (approximately one quarter of the weight of the test vehicle) and by using equation 2.10, the maximum strain underneath the tire tread was found to be in the range of 200 to 400 με (micro inches/inch) depending on the stiffness of the foundation. Strains of these magnitudes can easily be recorded by use of strain gages.

An optimum sensitive system was wanted in order to obtain high quality strain readings when the vehicle wheel was located some distance away from the gage locations.

Based on elementary strain gage theory it was decided initially to use four gages, set up in such a manner that all strains would be added and temperature compensation would be present. Figure 3.4 shows the gage locations and Figure 3.5 shows how the gages were connected in a Wheatstone bridge.

When actual tests were conducted, no single run indicated the same
FIG. 3.4. GAGE LOCATION ON THE PLATE
C & D ARE ACTIVE GAGES
A & B ARE "DUMMY" GAGES

FIG. 3.5 ARRANGEMENT AND CONNECTION OF STRAIN GAGES IN A WHEATSTONE BRIDGE
results under almost identical operating conditions. The reason for this was due to the fact that the system was too sensitive to strain in the transverse direction of the plate. If the wheel followed a path as little as one half inch away from the center line of the plate in the longitudinal direction, the strain reading would be very different from the strain measured when the vehicle followed the true center line. This difficulty was traced to gages A and B which measured the transverse strain as indicated in Figure 3.4. This was corrected by using two dummy gages, mounted to the same type of material as the steel plate and placed in the same environment as gages C and D. Under these conditions longitudinal strain is the only type of strain being measured and the assumption that the steel plate could be considered as a beam will be satisfied. At the same time, the system will be temperature compensated.

Under these conditions the system had less sensitivity but a higher gain amplifier was used to compensate for this loss. A Sanborn model 320 Dual channel DC Amplifier chart recorder, which had reliable sensitivity down to 1/4 millivolt was used together with the Ellis bridge. With this equipment a permanent record of the strain output could be obtained.

In order to determine the location of the wheel of the vehicle on the pavement model from the strain gage record, a timing switch was placed at the joint between the pavement model and the pavement. When the vehicle wheel reached the beginning of the pavement model, an instantaneous voltage was recorded on one of the recorder channels. Thus, knowing the vehicle velocity and the speed of the recording paper, the location of the wheel on the plate and the corresponding recorded strain could be referenced.

The calibration factor for any type of resistance transducer can be
obtained either by direct calibration such as by applying a known load or by computations from given conditions. For this installation the latter method was easier to apply. The gage factor and the resistance of the SR-4 gages supplied by the manufacturer are used in the calibration procedure by applying the following formula:

\[
CF = \frac{Gage\, resistance\, (gage\ C)}{Gage\, Factor} \times \frac{Calibration\, setting}{Number\, of\, working\, arms}\, (\text{micro}\, \text{in})
\]

Where in this case:

- Gage factor = 1.93
- Gage resistance = 120 Ohms
- Number of working arms = 2
- Calibration setting = 10

Inserting this into the equation above yields:

\[
CF = \frac{(120)}{(1.93)} \times \frac{(20)}{(2)} \times 10^{-6} = 303\, \text{micro\, inches/inch}
\]

When the Ellis bridge has been calibrated, strain can be read directly from the meter. When dynamical strains are to be recorded such as in this case, a chart recorder is more convenient. By applying a static load on the model and then simultaneously taking readings on both the Ellis Bridge and the chart recorder, one can obtain the relationship between these two values. The strain is read directly from the Ellis Bridge while at the same time a certain voltage output will appear on the recorder. These two values represent the same strain, hence one can say that:

\[
P\, \text{volts (recorder)} = Q\, \mu e(\text{Ellis Bridge})
\]

where:

- \(P\) = Voltage output from chart recorder
\[ Q = \text{Known strain magnitude in } \mu\varepsilon \text{ from the Ellis Bridge} \]

Thus:

\[ 1 \text{ volt} = \frac{Q}{P} \]

It is assumed that this relationship is valid for dynamic tests even though it was obtained statically.

Previous work performed by other research groups \((5, 10, 13)\) has shown that vehicle velocity, tire pressure, pavement roughness, and other factors influence pavement stresses. Therefore it is of interest to test the simulated pavement model to see if the same trends exist here as are present in the highway. In order to do this a series of tests were devised. These are discussed in the next chapter.
CHAPTER 4

PRESENTATION AND DISCUSSION OF RESULTS

In this investigation a study was made of the surface stresses and strains in a simulated highway pavement. Factors influencing surface strain are numerous and some of them can be controlled while others cannot. Vehicle velocity, which most people want to keep as high as possible, is considered as a controllable factor as well as the tire inflation pressure. In these tests vehicle velocity, tire inflation pressure and pavement roughness were varied and the resulting effects on pavement strain in the pavement model were measured.

The tests were conducted at the test site at the Indiana State Highway Commission Research and Training Center, West Lafayette, Indiana, and the cooperation of this organization greatly facilitated this effort.

Parameters for the pavement model were selected so as to provide a simulated pavement that would be relatively sensitive to changes in the dynamic tire force. The resulting simulation closely approximated the behavior of a flexible pavement. A typical strain output from the pavement model is shown in Figure 4.1. The short length of record and the strain reversal representing the tensile strain in the pavement model could characterize an asphalt cement coated road on a warm day.

This type of output was typical for all tests conducted. The tensile strain in front of and behind the wheel on the pavement model is
Fig. 4.1. A typical strain output from the pavement model (vehicle moves slowly over the plate).
due to the flexible foundation. Physically this may be considered as a wave of strain travelling through the pavement in front and behind the wheel and moving with the same velocity as the vehicle. By analyzing the differential equation describing the deflection of the model, one can see that its solution is in the form of a damped sinusoidal wave (equation 2.6).

The creation of the waveform shown in Figure 4.1 can be explained as follows. When the vehicle wheel first reaches the pavement model virtually no strain is detected. As the wheel moves closer to the gage locations, some of the characteristic tensile overshoot appears. At the time the wheel is about five to six inches away from the gages, the strain reverses and the compression, always present underneath the tire, will appear. When the wheel has passed the gages and is five to six inches away from them, tensile strain is again detected. The second image of the described event is created when the rear tire of the vehicle passes over the model.

It should be mentioned that the wheel base of the test vehicle was longer than the pavement model. Thus only one wheel could be on the model as the vehicle progresses over it.

A valuable characteristic that the pavement model possesses is the fact that the strain in front of and behind the point of load application can be determined when the model has been calibrated.

The calibration procedure was performed as follows. The vehicle wheel was placed at known distances from the gages on the pavement model and the corresponding strain was recorded. Figure 4.2 illustrates the results. As a comparison, a strain curve is plotted for a flotation tire having a 12,000 lbs single-axle load on an asphalt cement coated
Fig. 4.2 CALIBRATION STRAIN CURVES FOR PAVEMENT MODEL
flexible road. In this test (which was performed by a research group in California (3)) the gages were mounted directly on the pavement surface. The two strain curves have the same characteristic shape. Since the flotation tire carries six times the load and has a larger contact surface than the tires of the test vehicle used in this investigation, the wave form will be longer and the strain magnitudes higher. The third plot on the same figure represents a 1000 lbs concentrated load applied directly above the gages on the pavement model. This is a theoretically obtained curve, calculated by using the same stiffness as the model. In this case the compressive strain just underneath the point load is higher than that produced by the test vehicle and the wave form is shorter. This is due to the larger contact area (the area between the pavement model and the tire) of the test vehicle.

The encouraging results obtained by the calibration test proved the feasibility of the pavement model.

Of great interest is the relationship between the dynamic tire forces and the vehicle speed. Tests were conducted in which the vehicle was driven at different velocities across the pavement model. Figure 4.3 shows the results. Unfortunately, the nature of the test site was such that it only allowed a maximum speed of 35 mph and thus high dynamic tire forces could not be expected. However, Figure 4.3 shows that the strain in the pavement increased as the vehicle velocity increased.

If a linear relationship is assumed between dynamic tire force and the corresponding maximum compressive strain (Figure 4.4), a curve of dynamic tire force versus vehicle velocity can be constructed using Figure 4.3. The tire forces obtained by this method are less, however,
FIG. 4.3 MAXIMUM LONGITUDINAL STRAIN AS A FUNCTION OF VEHICLE VELOCITY

TIRE PRESSURE IS CONSTANT (25 PSIG)
FIG. 4.4 RELATIONSHIP BETWEEN TIRE FORCE AND PAVEMENT STRAIN
than those computed using the quarter vehicle model. Moreover, these forces are also less than corresponding forces measured by Wilson (10).

The results of the quarter vehicle simulation (Chapter 2) indicate an increase in maximum tire force by a factor of 1.8 in the velocity range from 0 to 35 mph and by a factor of 3.8 in the range from 0 to 80 mph. (Figure 2.11).

The experiment performed by Wilson at Purdue University (10) indicate that the dynamic tire force increased by a factor of 2.4 in the velocity range from 30 to 60 mph. In these tests the tire force was measured by the aid of a pressure transducer connected to the vehicle tire.

Even though the test site prevented tests in the velocity range where the relationship between tire force and vehicle velocity seems to increase more rapidly (from 35 mph and up) one will have to assume from the obtained data that the relationship between tire force and pavement strain is not linear. The quarter vehicle simulation and the pressure measurements indicate higher tire forces than those computed from pavement model strain values, when vehicle velocity is increased.

This may be due to tire enveloping effects. At higher tire forces the contact area between the tire tread and the pavement model will be larger than for lower forces. These tests indicate that the relationship between pavement strain and dynamic tire force is similar to that shown in Figure 4.4. This effect was clearly demonstrated by a simple experiment.

The test vehicle was run several times over the pavement model at constant velocity. The tire pressure was for each run varied in the
range of 20 to 55 psi. The results are shown in Figure 4.5. A reasonably linear relationship exists between pavement strain and tire pressure in the given pressure range.

A similar study was performed by using the simulated quarter vehicle model (Chapter 2). In this study the tire force was computed for different tire pressures using a constant vehicle velocity. The results shown in Figure 2.12 indicate that the tire force increases at a lower rate than does the corresponding pavement strain.

Records of tire force versus tire pressure, available from tire pressure measurement tests (6, 13) indicate the same trend. Dynamic tire forces seem to increase at a lower rate than pavement strain for increasing tire pressure.

However, the three different methods all indicate that higher tire pressures cause increased tire force and pavement strain.

Of interest also was the magnitude of the strain present in the pavement model when bumps were placed in the wheel path of the test vehicle. In such cases high dynamic forces should be present at the wheel and on the pavement.

Certain difficulties were encountered when tests of this type were conducted. The bumps were of such nature that they caused the vehicle wheel to separate from the pavement model and thus great problems were encountered in having the tire hit the pavement model at the point where the gages were mounted. A trial and error technique was employed in which the bump was moved forwards and backwards in the wheel path after successive tests. The position in which the vehicle created the greatest maximum strain was assumed to be the position which would give the
FIG. 4.5 LONGITUDINAL STRAIN FOR CONSTANT VEHICLE VELOCITY VS TIRE PRESSURE
maximum tire force on the pavement model at the gage location. Figure 4.6 is an illustration of the nature of the test.

Since the body and the wheel of the vehicle do not have identical motions, there is no guarantee that the maximum downward force will occur at the point where the wheel hits the pavement model. The direction of the interacting forces may be reversed to produce a tire force less than the maximum value at this point.

Figure 4.7 shows the results of the tests. The first tensile strain peak is obtained when the wheel hits the bump. Then for a short time the wheel will be in the air until it hits the pavement model just over the gage location. The high compressive strain illustrates this condition. The wheel then leaves the model (wheel separation) until it hits the model again. The second tensile strain peak indicates this happening.

When the rear wheel hits the bump, the strain record shows that the rear wheel in turn hits the pavement model some distance in front of the gage location. This causes the rear wheel to jump over the gages. The high tensile strains on each side of the relatively low compressive peak indicates this situation. Pitching motion of the vehicle body can also result in the absence of the high compressive strain for the rear wheel.

The maximum compressive strain in this experiment is 2.7 times greater than the static strain at the same point. Based on the observations made previously, this does not mean that the tire force is 2.7 times higher than the static force. It may likely be larger. However, it should be kept in mind that the tire pressure in this test was 55 psi. This will certainly create larger pavement strains than if the pressure
TIRE PRESSURE = 55 psi

TIRE VELOCITY = 35 MPH

PATH OF TIRE TREAD

LOCATION OF MAXIMUM DYNAMIC TIRE FORCE

BUMP

GAGES

STEEL PLATE

FIG. 4.6 ILLUSTRATION OF BUMP LOCATION ON THE PAVEMENT MODEL.
LEGEND:
TIRE PRESSURE = 55 PSI
VEHICLE VELOCITY = 35 MPH

FIG. 4.7 STRAIN RECORD FROM BUMP TEST (BUMP IS 25 INCHES AWAY FROM GAGES PLACED ON THE PAVEMENT MODEL)
had been 25 psi as it was in the calibration experiment.

A similar test was performed with the bump in the other wheel path. In order to get the maximum tire force just above the gage location, the bump had to be placed 5 inches in front of the gages. In this case the tilting action of the vehicle caused the excessive strain in the pavement model. The output is shown in Figure 4.8. The pavement strains are of lower magnitudes than those obtained when the bump was placed on the pavement model itself. In this case the rear wheel created a greater tire force than the front wheel. This may be due to pitching motion in the vehicle body, created when the first wheel hit the bump.

It would have been of great interest if tire force measurements could have been taken simultaneously with the strain readings when experiments were conducted with the bumps. However, time and resources did not make this possible. Such records would permit a more meaningful analysis of the bump experiments.
FIG. 4.8 STRAIN RECORDS OF BUMP TEST (BUMP 5 INCHES AWAY FROM GAGES, LOCATED IN THE WHEEL PATH WHICH DOES NOT PASS OVER THE PAVEMENT MODEL)
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

In this investigation the research effort was divided between two studies. The first study was to simulate a vehicle theoretically and to use this simulated vehicle model to predict the tire forces that could be expected under different operating conditions on the highway. The second study involved the experimental measurement of pavement strain under actual dynamic vehicle loads.

The conditions of most concern in the first study were vehicle velocity and tire pressure. A highway profile defined by a displaced cosine function was given to the tire of the vehicle. For each specific run, the tire force was computed as a function of time and the maximum tire force was observed. This investigation led to the following conclusions:

1. The dynamic tire force increases when the vehicle velocity is increased. The rate of increase for the conditions used in this simulation is higher for velocities larger than 35 mph than below this value.

2. If the vehicle velocity is kept constant, the dynamic tire force increases slightly as the tire pressure is increased.

3. A rougher highway profile to the simulated model causes higher tire forces.

It should be noted that the results from this investigation are not actual values that could be expected from the vehicle which was simulated
under actual pavement conditions. Several assumptions such as linear suspension elements and no tire enveloping effects restrict the accuracy of the calculations. However, the results are consistent in the sense that they are relative and therefore they indicate under which conditions high tire forces could be expected.

The second part of the study involved the experimental measurement of pavement strain which is of great concern in the design and maintenance of highways. As previously described, a pavement model was built and tested for such strains. A test vehicle (1967 model Chevrolet BelAir sedan) was driven across the pavement model under such conditions as to generate high tire forces. The results of this investigation permitted the following conclusions:

1. An effective highway pavement can be closely simulated by a steel beam on an elastic foundation. The pavement deflection curve can be the design criterion for such a model. The deflection curve can be controlled by the overall stiffness of the model. Thus a flexible pavement can be simulated by having a relative stiff foundation and a more flexible steel beam. A rigid pavement would be obtained by using a softer foundation and a more rigid steel beam. Such models simulated with reasonable accuracy the longitudinal strains present in an actual highway.

2. Pavement strain increases rapidly with increased tire pressure. This is due to the decreasing contact area between the tire and the pavement model.

3. Pavement strain increases as the roughness of the pavement in the wheel paths increases.

4. No linear relationship exists between tire force and pavement strain. The rate of increase of tire force to pavement strain is higher for higher values of strain than for lower values. This is concluded to be due to tire enveloping effects.

5. In general, tire force increases with vehicle velocity. However, for the maximum possible velocities in these tests no large increase in pavement strain was observed, even though this trend was indicated in the permitted speed range.
A further study by the use of the simulated pavement model is recommended. The use of a long runway to permit high vehicle velocities across the pavement model should be employed to obtain results for higher vehicle velocities.

Furthermore, simultaneous measurement of tire pressure and pavement strain is recommended, especially in bump tests. In such a way the relationship between tire force and pavement strain can be exactly determined for every desirable value of tire pressure.
BIBLIOGRAPHY
