Finite Element Calculation of Rotor Sidepull Forces in Single-Phase Induction Motors

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ABSTRACT

Knowledge of motor sidepull forces is important for cantilevered rotor designs used in refrigeration compressors as these forces determine shaft and bearing design to control deflection and minimize acoustic noise. The finite element method was used to calculate rotor sidepull forces for four single-phase motors at no-load, full load and locked rotor conditions. The solution method combines the time dependent, two-dimensional form of Maxwell's field equations with electric circuit equations to calculate both the magnetic fields and currents in the motor for a given applied voltage. Rotor sidepull forces were calculated from the resultant field solutions using the method of virtual work and compared with forces from classical solutions.

INTRODUCTION

Magnetic forces exist between any materials lying within a magnetic field. In motors, substantial radially directed forces exist between the rotor and stator, but cancel when the rotor is magnetically aligned within the stator. If the rotor is not aligned, or if mechanical or electrical factors exist that cause magnetic unbalance, significant sidepull forces occur. This paper calculates the unbalanced pull in single phase induction motors using finite element methods and compares the results with the analytical method presented by Covo [1] in his classic AIEE paper of 1954. Covo's analysis considered the no-load case only, neglecting the effects of rotor currents and skew, and was directed more toward three phase machines. This analysis calculates rotor sidepull forces for four single phase motors from 1/4 through 4 horsepower. No-load results are presented for the four motors and results for locked rotor and full load conditions for two of the four.

MAGNETIC FORCES

The force of attraction between two magnetic materials lying in a magnetic field is derived by considering the change in magnetic energy for a differential displacement. Magnetic energy can be expressed as:

\[ W_m = \int_V \left( \int_0^B HdB \right) dV \]  

(1)

Where: \( W_m \) = magnetic energy, newton-meters; \( H \) = magnetic force, amps/meter; \( B \) = magnetic flux density, webers/sq-meter and \( V \) = volume of magnetic circuit, cubic meters.

Taking a differential displacement and assuming constant flux density, no flux fringing, a linear material (air), steel permeability much greater than air, and parallel faces, the force of magnetic attraction is:

\[ F = \frac{B^2A}{2\mu_0} \]  

(2)

Where \( F \) = force, newtons, \( B \) = flux density, webers/sq-meter, \( A \) = area, sq-meter, and \( \mu_0 \) = permeability of free space. webers/amp-meter.

Covo uses the differential form of Equation (2) and computes the airgap flux density along the circumference of a radially displaced rotor assuming a sinusoidally distributed magnetomotive force in the stator. His final equation for rotor sidepull force in British units is:
\[ F = \frac{\pi Dw}{72.13} B_n^2 C \]  

(3)

Where \( F \) = sidepull force, pounds, \( D \) = airgap mean diameter, inches, \( w \) = motor stack length, inches, \( B_n \) = maximum no load flux density, kilolines/sq-in, and \( C \) = a constant depending on the angle of rotor displacement from the magnetic pole axis, the number of poles in the motor and the effective relative eccentricity of the rotor.

Using analytical and graphical methods, Covo developed a graph for the constant \( C \) as a function of effective rotor eccentricity, angle of rotor displacement from the magnetic axis of the poles, and the number of motor poles. Covo's analysis and graph were duplicated using Mathcad [2] and the resulting program used to calculate rotor sidepull forces against which those of the finite element method were compared.

**FINITE ELEMENT METHOD**

Marriott and Griner [3] presented an induction motor model that solved the time dependent two-dimensional form of Maxwell's equations coupled with electric circuit equations describing motor end effects. The forcing function for their model is the applied line voltage instead of current density as used in commercial finite element analysis (FEA) packages. The other main feature of their approach is that the rotor can rotate in the stator. Both the magnetic fields and all currents in the motor are calculated under running conditions as a function of time. This motor model was used to calculate magnetic fields for two single-phase induction motors at locked rotor, no-load and under full load conditions and for two other motors at no-load. Maximum rotor sidepull forces were calculated for the motors using the finite element virtual work method introduced by Coulomb and Meunier [4]. The magnetic energy expression is differentiated with respect to a displacement and then integrated over the solution space. This results in the following expressions for the force contribution per unit depth of each virtually distorted element in the airgap:

\[ F_x = -v_o \left[ B_x \frac{d}{dx} \left( \sum_{i=1}^{n} \frac{dN_i}{dY} A_i \right) - B_y \frac{d}{dy} \left( \sum_{i=1}^{n} \frac{dN_i}{dx} A_i \right) \right] A_e - \frac{v_o B_x^2}{2} \frac{1}{|J|} \frac{d|J|}{dx} A_e \]  

(4)

\[ F_y = v_o \left[ B_x \frac{d}{dy} \left( \sum_{i=1}^{n} \frac{dN_i}{dy} A_i \right) - B_y \frac{d}{dx} \left( \sum_{i=1}^{n} \frac{dN_i}{dx} A_i \right) \right] A_e - \frac{v_o B_y^2}{2} \frac{1}{|J|} \frac{d|J|}{dy} A_e \]  

(5)

Where \( F_x, F_y \) = element x and y force components, \( v_o \) = reluctivity of air, \( B_x \) and \( B_y \) = x and y flux densities, \( B_x^2 \) = flux density squared, \( A_i \) = nodal vector potential, \( A_e \) = element area, \( \frac{dN_i}{dx}, \frac{dN_i}{dy} \) = derivatives of the shape functions \( N_i \), \( n \) = number of element nodes and \( J \) = the element Jacobian matrix. Element forces are summed to obtain the total force acting on the magnetic structure.

**Airgap Flux Density**

Figure 1 shows the airgap flux density of a 2-pole, one horsepower, permanent split capacitor (PSC) motor calculated using the finite element method. The flux distribution is shown for no-load conditions, i.e., with no rotor current, with voltage applied only to the main winding and with a magnetically centered rotor. The waveform clearly shows (1), the stairstep nature of the discrete stator windings, (2), the nearly sinusoidal airgap flux density distribution, and (3), the reduced flux densities at the stator slot openings. Figure 2 shows the flux density distribution when the rotor is displaced at an eccentricity of about 80% of the airgap at 90 electrical degrees to the magnetic pole axis. A similar plot at zero degrees from the magnetic pole axis would show a peaked flux density distribution near one pole center and a flattened flux density distribution at the other pole. It is the asymmetrical flux density distributions which create unbalanced sidepull forces in motors of any type.
Rotor Sidepull Forces

Peak rotor sidepull forces were calculated for four single phase PSC motors of 1/4, 1, 2.7 and 4 horsepower sizes at no-load conditions for various rotor eccentricities using Equation (3) and the finite element method. Rotor eccentricity in this case is defined as the rotor radial displacement divided by the nominal airgap. For the FEA no-load calculations, motor magnetizing current was calculated for the main winding only and a magnetostatic form of the Marriott and Griner model used for the magnetic field solution. Equation (4) was then used to calculate sidepull forces. It is noted that only a magnetostatic FEA solution is required for the no-load cases, and hence any commercial electromagnetic FEA package could be used for no-load results. Peak sidepull forces for the locked rotor and full load conditions for the 1/4 and one horsepower motors were calculated at a single rotor eccentricity using the complete FEA induction motor model of Marriott and Griner, which includes the effects of rotorbar currents.

Sidepull forces for the four motors were calculated by Equation (3) using the maximum airgap flux densities derived from conventional techniques similar to those used by Veinott [5] for motor design. Uncorrected motor saturation factors, i.e., total motor amp-turns divided by airgap amp-turns, were used to calculate Covo's 'effective rotor eccentricity' for all cases.

RESULTS

The results of rotor sidepull force calculations for four single phase induction motors are shown in Figures 3 through 6, where sidepull force has been plotted versus rotor eccentricity fraction. Smooth curves show the forces calculated from Equation (3), and discrete symbols the finite element solutions. It is clear that the no-load forces as calculated by both the Covo and finite element methods are in substantial agreement. 'No-load' in a single phase induction motor is an unrealizable state since rotorbar currents caused by the backward revolving field exist even if the motor is driven at synchronous speed with only the main winding energized. This is in contrast to three phase motors which have no countervolving field and hence a true no-load state. Be that as it may, the single phase no-load case still offers a convenient reference point for discussion of the full load and locked rotor results.

Calculated full load rotor sidepull forces for the 1/4 and one horsepower motors for 0.82 rotor eccentricity are shown in Figures 3 and 4. These forces are substantially less than calculated for no-load. One might think that, as load on an induction motor increases, increasing stator current would increase airgap flux density and consequently, sidepull forces. This is not the case, and three things suggest why this is not so. First, with increased load, rotorbar currents increase and tend to drive flux toward the rotor surface. This creates 'zigzag' flux linking the rotor and stator, and reduces the effective airgap flux density. Second, leakage flux in both stator and rotor increase as motor current increases, also tending to reduce airgap flux density. Third, increased saturation in the steel limits the obtainable airgap flux density. Forces calculated for the locked rotor cases are much nearer those for no-load as also shown in Figures 3 and 4. Evidently, the large currents in the stator and rotor drive the effective airgap flux density to values much beyond those of full load. Based on these admittedly limited number of cases, it appears that the Covo formula, which computes sidepull forces based on no rotor current, yields estimates of peak sidepull forces for single phase induction motors that are not exceeded during motor operation.

The time variation of rotor sidepull force of the one horsepower motor at full load conditions, with rotor displaced at 0° from the main winding axis and with an eccentricity of 0.82 is shown in Figure 7. A harmonic analysis of these forces is shown in Figure 8. Strong 2nd and 4th harmonics of line frequency are evident along with several fairly large frequency components at rotor and stator slot frequencies. The frequency spectrum at the higher harmonics does not correspond exactly to the number of stator (24) and rotor (30) slots in this motor due to rotor slip. The same analysis for this motor, with the rotor displaced at 90° to the main winding magnetic pole axis, indicated that the 2nd harmonic was the largest component of force, although a figure is not shown here for this case. Classical analysis for three phase machines states that motor sidepull force occurs at the 2nd harmonic of line frequency for the case of the rotor revolving about a displaced centerline [1]. Apparently, the frequency of sidepull force in single phase PSC machines depends both on the location of rotor centerline displacement and also upon the effects of auxiliary winding current on airgap flux density.

Although no experimental study was performed to verify the sidepull results of this paper, numerous papers on three phase machines have shown that the Covo method gives results which are usually within 20% of measured values. One paper by Ohishi, Sakebe, Tsumagari and Yamashita [6] presented experimental data for a 280 MVA, 24 pole generator
indicating agreement, at worst, within 13% of Covo's formula. Also, torque calculations on numerous motors using the Marriott and Griner FEA method have agreed quite closely with experiment. Equally accurate force calculations would be expected since essentially the same computer code is used in the virtual work method for both torque and force calculations.

Both of the preceding analyses are strictly two-dimensional in nature and cannot deal with the case of rotor cocking in the stator bore, as would be the situation with a cantilevered rotor. An analysis of this phenomenon could perhaps be made by taking an incremental 'stack of coins' approach and calculating an effective sidepull for each coin. Also, due to energy considerations in distorted finite elements, the motor model of Marriott and Griner cannot at present solve the case where the motor bearings are concentric to the shaft, but the rotor is eccentric with respect to the shaft. That is, when the rotor is 'whirling' in the stator bore. Only the case where the rotor rotates about a displaced shaft centerline has been considered in this paper.

CONCLUSIONS

Rotor sidepull forces for four single phase induction motors have been calculated by the finite element method and compared to those calculated by classical methods developed for three phase induction machines. The results show:

- No-load rotor sidepull forces for single phase induction motors calculated by the finite element method agree closely with those of the classical method developed by Covo if the uncorrected motor saturation factors are used to calculate effective rotor eccentricities.

- Based on the finite element results for two single phase permanent split capacitor motors, peak rotor sidepull forces for full load and locked rotor conditions are less than or equal to those calculated for the hypothetical no-load case.

- The finite element model of Marriott and Griner shows sidepull force harmonic components which differ from those calculated for three phase motors.

REFERENCES


2. Mathcad is a registered trademark of MathSoft, Inc.


The Figures for this paper are on on the following two pages.
FIGURE 1. Calculated no-load air-gap flux density for 1 hp, 230 volt, single phase PSC motor with rotor centered in the air-gap.

FIGURE 2. Calculated no-load air-gap flux density for 1 hp, 230 volt, single phase PSC motor with rotor displaced 90° elect. to magnetic pole axis, and eccentricity equal 0.81.

FIGURE 3. Calculated peak rotor sidepull force versus eccentricity for a 2-pole, 1/4 hp, 115 volt, 60 hz, PSC motor. Finite Element solutions compared with Covo formula.

FIGURE 4. Calculated peak rotor sidepull force versus eccentricity for a 2-pole, 1 hp, 230 volt, 60 hz, PSC motor. Finite Element solutions compared with Covo formula.
FIGURE 5. Calculated peak rotor sidepull force versus eccentricity for a 2-pole, 2.7 hp, 230 volt, 60 hz, PSC motor. Finite Element solutions compared with Covo formula.

FIGURE 6. Calculated peak rotor sidepull force versus eccentricity for a 2-pole, 4 hp, 230 volt, 60 hz, PSC motor. Finite Element solutions compared with Covo formula.

FIGURE 7. Calculated rotor sidepull force versus time for 2-pole, 1 hp, 230 volt, 60 hz PSC motor over one electrical cycle. Rotor displaced at 0° to main winding magnetic pole axis. Eccentricity fraction equal 0.81.

FIGURE 8. Calculated rotor sidepull force harmonic content for motor of Figure 7.