1994

The Study of Geometry and Pressure in the Trochoidal-Type Machine Without Apex Seal

S. Li  
Louisiana Tech University

J. B. Shung  
Louisiana Tech University

Follow this and additional works at: https://docs.lib.purdue.edu/icec

International Compressor Engineering Conference. Paper 995.  
https://docs.lib.purdue.edu/icec/995

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.  
Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/Herrick/Events/orderlit.html
THE STUDY OF GEOMETRY AND PRESSURE IN THE TROCHOIDAL-TYPE MACHINE WITHOUT APEX SEAL

Shuangqing Li  
Graduate student  
Department of Mechanical and Industrial Engineering  
Louisiana Tech University  
Ruston, Louisiana 71272--0046

John B. Shung  
Assistant Professor  
Department of Mechanical and Industrial Engineering  
Louisiana Tech University  
Ruston, Louisiana 71272--0046

ABSTRACT

A peritrochoid and its type 2(bii)-Inner conjugate envelope are good candidates for chamber and rotor of a trochoidal-type machine that requires no apex seal. This paper presents their geometry attributes and pressure processes inside the pockets. First, the exact expressions of contact point coordinates and pocket volumes at any shaft angle are derived. And then, the pressure processes inside pockets are simulated with simplified valves. Finally, the effect of clearance between rotor and chamber on the pressure processes are studied. The results indicate that a trochoidal-type machine without apex seal is worthy of further research and development.

NOMENCLATURE

- \( \phi_0 \): characteristic angle of the rotor
- \( A_R \): cross-sectional area of the rotor
- \( \alpha \): parameter of the trochoid
- \( \beta \): angle measured from \( X_e \) axis to directed line \( O_e O_t \)
- \( \gamma \): shaft angle
- \( e \): the distance between \( O_t \) and \( O_e \)
- \( k_t \): angular velocity ratio for trochoid
- \( k_e \): angular velocity ratio for envelope
- \( r_c \): trochoid radius
- \( R_i \): regions for profile of envelope, \( i = 1,2,3 \)
- \( T_i \): temperature in the \( i \)th pocket
- \( \mu, \nu \): parameters of envelope
- \( A_v \): the effective flow area of the discharge valve
- \( A_{o1} k_v \): constants for the valve model
- \( P_{in} \): pressure of the inlet fluid
- \( n \): exponent describing polytropic process
- \( N \): shaft speed
- \( p_i \): pressure in the \( i \)th pocket
- \( \varepsilon \): clearance between rotor and chamber
- \( R \): thermal constant in ideal gas law
- \( P_{out} \): discharge pressure
- \( T_R \): thickness of the rotor
- \( g_c \): gravitational acceleration

INTRODUCTION

Trochoidal-type machines belong to the category of planetary rotation machines. They have some advantages such as simplicity and high reliability as compared to other types of planetary rotation machines. The primary problem of a trochoidal-type machine is the excessive friction on apex seals which leads to low efficiency and low reliability. Hoffmann (1985) designed, manufactured and tested a prototype of trochoidal-type compressor. He discovered that apex seals contributed approximately 25% of all friction drive due to seal, and suggested eliminating apex seals by keeping tight running clearance to remove this drawback forever.

There are two major components in a trochoidal-type machine, i.e. a rotor and a chamber. If one of them is a trochoid then the remaining one is its conjugate envelope. Systematic study has been done on the parametric equations of the conjugate envelope where the number of envelope lobes is either one more, or one less, than the number of trochoid lobes. Closed form equations for all nine types (Colbourne, 1974; Shung and Pennock, 1994) of conjugate envelope have been derived. It has been discovered that a peritrochoid with type 2(bii)-Inner conjugate envelope does not have cups and is a good candidate for trochoidal-type machine that requires no apex seal. First, it is easier to be manufactured with high accuracy than other types of envelope. Second, there will be less contact stress in case contact happens. This paper presented the geometry attributes and the simulation results of the pressure process in the pockets.

THE GEOMETRY ATTRIBUTES
Trochoid And Envelope

For the purpose of illustration, we consider a trochoid-type compressor shown in Figure 1. This compressor has five inlet and five discharge valves and is similar to a compressor manufactured by the Trochoidal Power Corporation [Hoffmann, 1985]. The centers of the moving and fixed pitch circles are denoted by O_e and O_r respectively. Two Cartesian reference frames are used: (i) an envelope frame denoted by \((X_e-O_e-Y_e)\) attached to the rotor and (ii) a trochoid frame denoted by \((X_r-O_r-Y_r)\) attached to the chamber. The angle measured from the \(X_t\)-axis to the directed line \(O_eO_r\) is denoted by \(\gamma\) and is referred to as the shaft angle. The angle measured from the \(X_e\)-axis to the \(X_r\)-axis is denoted by \(\beta\). The geometry of chamber is described by a trochoid, and its coordinates in the trochoid frame are:

\[
x_t = e \cos \alpha + r_c \cos \frac{\alpha}{k_e} \\
y_t = e \sin \alpha + r_c \sin \frac{\alpha}{k_e}
\]  

(1a)

(1b)

The geometry of rotor is described by a type 2(bii)-Inner conjugate envelope and its coordinates in the envelope frame are:

\[
x_r = 2e \cos \nu \cos \mu + r_c \cos \frac{2\mu}{k_e} \quad \text{(2a)} \\
y_r = 2e \cos \nu \sin \mu + r_c \sin \frac{2\mu}{k_e} \quad \text{(2b)}
\]

Where \(\cos \nu = -1\) if \(\frac{ek_e}{r_e} \leq \cos(1 - \frac{2}{k_e})\mu\) \(\text{(2c)}\), \(\cos \nu = 1\) if \(\cos(1 - \frac{2}{k_e})\mu \leq -\frac{ek_e}{r_e}\) \(\text{(2d)}\), \(\cos \nu = -\frac{r_c}{ek_e} \cos(1 - \frac{2}{k_e})\mu\) if \(-\frac{ek_e}{r_e} < \cos(1 - \frac{2}{k_e})\mu < \frac{ek_e}{r_e}\) \(\text{(2e)}\).

Note that the angle measured from the \(X_e\)-axis to the directed line \(O_eO_r\) is denoted by \(\beta\) and the angle measured from the \(X_e\)-axis to \(X_r\)-axis is denoted by \(\frac{\beta}{k_e}\) in the previous paper [Shung and Pennock, 1994]. Following relations between angles are obtained from Figure 1.

\[
\delta = -\frac{\beta}{k_e} = \frac{\pi - \gamma}{k_e - 1} \\
\beta = \frac{k_e}{k_e - 1}(\gamma - \pi)
\]

(3)

(4)

Therefore the coordinates of the rotor in the trochoid frame are:

\[
x_{r_t} = e \cos \gamma + x_r \cos \frac{\beta}{k_e} + y_r \sin \frac{\beta}{k_e} \\
y_{r_t} = e \sin \gamma - x_r \sin \frac{\beta}{k_e} + y_r \cos \frac{\beta}{k_e}
\]

(5a)

(5b)

Furthermore there are linear relations among angles \(\alpha, \beta, \mu\) and \(\nu\), as shown below [Shung and Pennock, 1994].

\[
\alpha = (1 - \frac{1}{k_e})\mu + (1 + \frac{1}{k_e})\nu \\
\beta = \mu - \nu
\]

(6)

(7)

Contact Points

Finding out contact points is important for the analysis of both performance and reliability of the machine. First, the contact points determine the boundaries of the pockets, their locations directly affect the volumes of the pockets. The clearances between the rotor and chamber are critical to the performance of the machine. Their influence on the performance is the key area of this study. Second, a possible failure mode is the wear between rotor and chamber if the clearance is too small. The reliability of the machine is greatly affected by the clearance.

From the forming process of conjugate envelopes [Shung and Pennock, 1994], one can perceive that at any shaft angle, there are at least \(k_e\) number of points on the envelope contacting trochoid simultaneously and, when the shaft rotates 360 degree, the complete conjugate envelope is formed by these contact points. The profile of the envelope can be
classified into three regions according to the value of \( \cos \psi \). Thereafter, the three regions are denoted by R1, R2, R3. For a given \( \psi \), the \( \mu \)'s of the contact points are obtained in the following way. First, define an angle \( \phi_0 \)

\[
\phi_0 = \arccos \left( \frac{k_e}{r_c} \right) \quad (8)
\]

From equations (2c), (3) (7); (2d), (3), (7); and (2e), (3), (7), the following equations (9a), (9b), (9c) can be obtained respectively.

\[
R1: \quad f_1(\mu, \psi) = \cos[\mu - \frac{k_r}{k_s - 1}(\psi - \pi)] + 1 = 0 \quad (9a)
\]

where \( \frac{k_i}{k_s - 2}(2\pi i - \phi_0) \leq \mu \leq \frac{k_i}{k_s - 2}(2\pi i + \phi_0) \) for \( i = 0, 1, 2, \ldots \)

\[
R2: \quad f_2(\mu, \psi) = \cos[\mu - \frac{k_r}{k_s - 2}(\psi - \pi)] - 1 = 0 \quad (9b)
\]

where \( \frac{k_i}{k_s - 2}(2\pi i + \pi - \phi_0) \leq \mu \leq \frac{k_i}{k_s - 2}(2\pi i + \phi_0) \) for \( i = 0, 1, 2, \ldots \)

\[
R3: \quad f_3(\mu, \psi) = \cos[\mu - \frac{k_r}{k_s - 2}(\psi - \pi)] + \frac{r_e}{k_s}(1 - \frac{2}{k_s})\mu = 0 \quad (9c)
\]

where \( \frac{k_i}{k_s - 2}(\pi + \phi_0) \leq \mu \leq \frac{k_i}{k_s - 2}(\pi - \phi_0) \) for \( i = 0, 1, 2, \ldots \)

Therefore, the contact points on the rotor can be obtained from equations (2a) and (2b) where \( \mu \)'s satisfy equations (9a), (9b), and (9c). The contact points on the chamber can be obtained from equations (1a), (1b), where \( \alpha \)'s satisfy the following equation.

\[
\alpha = 2\mu - \frac{k_r}{k_s - 1}(\psi - \pi) \quad (10)
\]

For \( K_e = 6 \), \( e = 0.55 \), \( r_e = 6 \), a plot of \( \mu \) vs \( \psi \) is shown in Figure 2. At any \( \psi \), there are at least five contact points and as the shaft rotates, the number of contact points may change between five and six. The sixth contact point always occurs when the pocket volume is close to minimum. So, its influence on the pressure process can be neglected.

The pocket area The area of a pocket is a periodic function of shaft angle. As the shaft rotates 360 degree, the areas of all pockets experience one cycle. The only difference among them is a angle delay. The area of pocket 1 vs shaft angle is shown in Figure 3. The area of the \( i \)th pocket is:

\[
A_i = A_{ct} - A_{ri} \quad i = 1, 2, \ldots, k_s \quad (11)
\]

\[
A_{ct} = \frac{1}{2} \int_{a_1}^{a_2} (x_i \frac{dy_i}{dx} - y_i \frac{dx_i}{dx}) \, dx \quad (12)
\]

\[
= \frac{1}{2} \left( \left( e^2 + \frac{r_s^2}{k_s} \right) (\alpha_2 - \alpha_1) + \frac{k_s + 1}{k_s - 1} r_c \sin(1 - \frac{1}{k_s}) \alpha_1^2 \right)
\]

\[
A_{ri} = \frac{1}{2} \int_{a_1}^{a_2} (x_i \frac{dy_i}{d\mu} - y_i \frac{dx_i}{d\mu}) \, d\mu = I_1 + I_2 \quad (13a)
\]

\[
I_1 = -\frac{\beta}{2} \sin(\gamma + \frac{\beta}{k_s}) [x_i(\mu_2) - x_i(\mu_1)] + \frac{\beta}{2} \cos(\gamma + \frac{\beta}{k_s}) [y_i(\mu_2) - y_i(\mu_1)] \quad (13b)
\]
\[ I = \frac{1}{2} \int \left( x_n \frac{dy_n}{du} - y_n \frac{dx_n}{du} \right) d\mu \]

\[ = \frac{r_x^2 \Delta \mu + 2e^2(\Delta \mu - \mu_{21})}{k_1} \mu_{31} - \frac{r_x^2 e(2 + k_2)}{2-k_1} \sin^2 \frac{2}{k_1} - 1) \mu_{31} \]

\[ + \frac{r_x e(2 + k_2)}{2-k_1} \sin^2 \frac{2}{k_1} - 1) \mu_{31} - \frac{2(k_2 - 2)}{k_1} \mu_{31} \]

\[ \Delta \mu = \mu_2 - \mu_1 \]

\[ \Delta \mu_1 = \mu_{12} - \mu_{11} \]

\[ \Delta \mu_2 = \mu_{22} - \mu_{21} \]

\[ \Delta \mu_3 = \mu_{32} - \mu_{31} \]

\[ A_R = \frac{1}{2} \int \left( x_n \frac{dy_n}{du} - y_n \frac{dx_n}{du} \right) d\mu \]

\[ = k_1 - 1 \frac{r_x^2 e}{k_1} + 2(r_x^2 + 2e^2k_2) \frac{\phi_0}{k_1} - 2(k_2 + 2)e \sin \phi_0 + \frac{k_1 - 1}{k_1} r_x^2 \sin 2\phi_0 \]

**THE PRESSURE IN THE POCKETS**

**Assumptions** The change in state of the pocket fluid is the result from three distinct thermodynamic processes. The three processes are: (1) an expansion through inlet valve, (2) a compression or expansion inside the pocket, and (3) a compression through the discharge valve. In deriving the thermodynamic equations, the following assumptions were made:

1) The properties of the fluid instantaneously propagate throughout the pocket, i.e., they are uniform throughout the pocket at any instant of time.

2) The fluid changes from one state to another by a polytropic process, and the fluid follows the ideal gas law.

3) In the suction process, the pressure in the pocket equals the inlet pressure.

4) The flow through the valve follows one-dimensional polytropic process.

**The model of discharge valve** For simplification, valve is treated as a simple orifice of a certain effective cross-sectional area which is described as follows:

\[ A_D = A_0(p_i - p_{out})^{k_v} \quad \text{when} \quad p_i > p_{out} \]

\[ = 0 \quad \text{when} \quad p < p_{out} \]

The constants \( A_0, k_v \) are determined by the following conditions:

1) Minimum residual gas. Thus, at the end of discharge process, the pressure in the pocket equals the discharge pressure. 2) The maximum pressure occurs at the shaft angle where the ratio \( L_1/L_2 \) (Figure 4) is specified from experiment such as one in Hoffmann's report. The mass flow equation through the discharge valve is [Soedel, 1972]

\[ \frac{dm}{dt} = -p_A A_D \sqrt{\frac{2n_6}{(n - 1)RT} \sqrt{\frac{r^2}{\mu^2 - r_{\mu 1}^2}}} \]

\[ (16) \]
where \( r = \frac{p_{ou}}{p_i} \) if \( \frac{p_{ou}}{p_i} > r_0 \)
\[ r = r_0 = \left( \frac{n+1}{n-1} \right)^\frac{n}{n-1} \] if \( \frac{p_{ou}}{p_i} \leq r_0 \)

The simulation of pressure process The whole pressure process consists of four pressure stages, i.e., the suction stage, the pure compression stage, the compression with discharge stage and the expansion stage as shown in Figure 4.

In the suction stage, the pressure in the pocket is assumed to be the same as the inlet pressure, i.e., \( p_i = p_{in} \). In the pure compression and expansion stages, the pressure is described by the polytropic process equation. In the compression with discharge stage, the pressure is determined by the following equations:

\[
\frac{dp_i}{dt} = -\frac{dA_i}{A_i} \frac{np_i}{nRT_i} + \frac{nRT_i dm_{i+1}}{A_i T_k \ dt} \quad (17)
\]

When the fluid is steam, the inlet pressure and discharge pressure are 14.5 and 87 psig respectively, shaft speed is 1500 rpm and clearance is zero, the shaft angle vs pressure is shown in Figure 4.

The effect of the clearance on the pocket pressure Because of the existence of the clearance between rotor and chamber, there will be mass flow between adjacent pockets through the orifices formed by the clearances. Let \( \varepsilon \) be the clearance, then \( A_\varepsilon = \varepsilon T_R \) is the effective flow area between two adjacent pockets. The mass always flows from pocket of higher pressure to the one of lower pressure. The mass flow rates through the clearances are:

\[
\frac{dm_{i+1}}{dt} = -A_{p_i} \sqrt{\frac{2ng_{c}}{(n-1)RT}} \sqrt{r^2 - \frac{r^{n+1}}{r^n}} \quad r = \frac{p_{i+1}}{p_i}, \quad \text{if} \quad \frac{p_{i+1}}{p_i} > r_0 \quad (18a)
\]

Assume \( p_i > p_{i+1} \)
\[ r = r_0 \quad \text{if} \quad \frac{p_{i+1}}{p_i} \leq r_0 \]

\[
\frac{dm_{i+1}}{dt} = A_{p_i} \sqrt{\frac{2ng_{c}}{(n-1)RT}} \sqrt{r^2 - \frac{r^{n+1}}{r^n}} \quad r = \frac{p_i}{p_{i-1}}, \quad \text{if} \quad \frac{p_i}{p_{i-1}} > r_0 \quad (18b)
\]

Assume \( p_i < p_{i-1} \)
\[ r = r_0 \quad \text{if} \quad \frac{p_i}{p_{i-1}} \leq r_0 \]

After considering the flow through the clearances, the pressure differential equation becomes:

\[
\frac{dp_i}{dt} = -\frac{np_i}{A_i} \frac{dA_i}{dt} + \frac{nRT_i}{A_i T_k} \left( \frac{dm_{i+1}}{dt} + \frac{dm_{i}}{dt} + \frac{dm_{i-1}}{dt} \right) \quad (19)
\]

The pocket pressures under different clearances are shown in Figure 4 and 5. The analysis reveals that when the clearance is greater than 0.01 inch, the discharge stage will experience a sudden change. Therefore, the clearance should be kept less than 0.01 inches under this particular operating condition. A trochoidal-type compressor without apex seal is quite feasible.

ACKNOWLEDGMENTS

Both authors greatly appreciate the support for this research from the Louisiana Education Quality Support Fund for Research and Development programs.

REFERENCES

4. Soedel, W., *Introduction to Compressor Simulation of Positive Displacement Type Compressor*. Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University (1972).