1990

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AN ANALYSIS OF PIPE FAILURE CRITERIA 
UNDER ARBITRARY OPERATING DISPLACEMENTS 

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ABSTRACT 

This paper discusses two problems in the vast subject of piping system reliability. The first problem deals with increased pipe flexibility due to bends and their usual elliptical cross sections. The second problem discussed is how to measure the maximum bending strain with the minimum number of strain gages. 

While the increased flexibility of bent piping has been understood for many years, the combination of this effect with imperfect bend geometry does not seem to be widely recognized as critical by the practicing HVAC engineer. Many pipe failures which occur in bends, running along what is normally considered to be the pipe neutral axis, are arbitrarily ascribed to material defects, when in fact they are due to excessive pipe motion. This paper illustrates a method for determining the effect of the combination of increased flexibility and imperfect bend geometry on piping reliability. 

A second problem in assessing the reliability of piping under unknown displacement conditions, such as may occur inside a hermetic compressor, is in determining the maximum bending strain from a finite number of strain gage locations. This paper shows how this can be accomplished at a known critical location (a fixed point for example) with only two simultaneous gage readings. 

Using these ideas will make analysis of pipe failures more correct and informative and minimize the number of strain gage installations necessary to adequately determine piping reliability. 

LIST OF SYMBOLS 

a, ellipse major axis radius 
b, ellipse minor axis radius 
I, moment of inertia of a section, 

\[ I = \frac{\pi d^4}{4}, \text{Circular section} \] 

\[ I = \frac{\pi}{4} (r_o^4 - r_i^4), \text{Thin ring, tube, pipe} \] 

\[ I = \frac{\pi}{4} (a_b b_o^4 - a_i b_i^4), \text{Ellipse} \] 

Subscript i, inside 
Subscript o, outside 
R, pipe (tube) bend radius 
r, pipe (tube) radius 
t, pipe wall thickness 
a, The angle of the principal bending moment plane with respect to strain gage \#1
\( \lambda \), pipe characteristic, \( \frac{\tau R}{F} \)

+1, example of gage #1, reading tensile strain
-2, example of gage #2, reading compressive strain

Notation used in Figures 3 & 4

I. INTRODUCTION

Piping associated with refrigerant compressors can be troublesome unless carefully designed and fabricated for each specific situation. The following paper discusses two aspects of the vast general subject of piping system structural integrity using failure analysis as the basis for certain assumptions in each case.

The first topic deals with split pipe bends, a fracture problem that is often erroneously attributed to faulty material, such as metallurgical inclusions or welding, depending on the tube fabrication process. While these sources of failure must be considered, it is more often the case in the author's experience that this kind of pipe fracture is due to pipe motion causing opening and closing of the tube cross section as explained by von Kármán (1) on the basis of minimum energy principles for the deformation of curved hollow sections. Von Kármán showed how bent tubes or elbows with \( \lambda \) ratios less than about 1.0 are prone to failure along what would normally be considered the tube neutral axis, especially when the bend cross section is not carefully formed. The following discussion will clarify a common type of tubing failure often misunderstood by HVAC practitioners.

The second topic deals with making efficient and timely strain measurements on tubes where the bending moment is constrained from preferentially opening and closing the cross section. It will be shown that the maximum bending strain and the principal bending direction can be obtained with reasonable accuracy using only two strain gages on a transverse plane. It follows that three gages on the plane can determine the direct stress as well as the bending stress.

II. BENT TUBE FLEXIBILITY

Measurements of the stiffness of bent tubes always show them to be more compliant than elementary beam theory predicts. Von Kármán published the solution to this problem in 1911, and numerous authors have since referenced this work (2) (3). For tubing and pipe, the flexibility when compared to straight lengths is as shown in Figure 1. The flexibility factor, \( 12\lambda^2+10/12\lambda^2+1 \) is plotted with respect to the bend geometry characteristic, \( \lambda = \frac{TR}{FA} \). This particular approximate relationship is due to von Kármán; other similar relationships (4) have been derived since and give similar results. These flexibility factors are all based on "minimum energy" or "least work" principles, which explain the tendency for the tube walls in the plane of the bending moment to collapse or approach each other under the bending displacement. Opening and closing of the section produces cyclic strain which is maximum at the ends of the major axis of the elliptical deformed shape.

Further loss of section stiffness in tubing bends, not covered by the von Kármán effect, is caused by the elliptical distortion of all practical bend cross sections. This distortion also has its basis in the foregoing theory. Flexibility derivations are based on initially round sections. Since these do not occur in practice, it is of some interest to see how elliptical cross sections can affect tubing stiffness.
Figure 2. shows an example relationship for initially elliptical sections. This effect can be combined with the von Kármán flexibility by superposition. The combined effect can then be used to evaluate stresses throughout a piping system. It should be noted that finite element curved pipe elements may have the von Kármán or other flexibility options available for increased accuracy in numerical computation of pipe system displacement solutions.

An example of the combined effect of the flexibility factors is as follows: A tube with \( t = 0.035 \), \( r = 0.250 \) and \( R = 1.50 \) has \( \lambda = 0.84 \). From Figure 1, the flexibility of this pipe will be 1.95 times the flexibility of the same length of straight pipe. Now if the section is elliptic such that (a-b) as shown in Figure 2 is 0.100, the section is only 84% as stiff as an ideally circular one. The flexibility of this section of pipe will then be \( 1.95/0.84 \) or 2.3 times as flexible as a straight circular section. Calculation of the maximum strain at the ellipse major axis ends can be accomplished by following the procedure outlined in (2) for any fixed condition. Bending displacement of the pipe will cause an increase in ellipticity when the section is closing so the complete problem is geometrically non-linear.

### III. EFFICIENT BENDING STRAIN MEASUREMENT

**Assumptions** (2 gages)

a. Bending stress only, direct stress is not considered.

b. Bending is symmetrical about a plane coincident with the neutral axis.

c. Planes remain plane.

d. Gages may not be 180° apart.

e. Cross sections have rotational symmetry.

Where straight pipe is used or where pipe constraint points are far enough away from curved sections, maximum bending strains may be measured with strain gages oriented longitudinally and affixed at various angles around the pipe at the section of interest. During an investigation of a hermetic compressor discharge pipe failure problem, the general question of how to gage the pipe arose. It was found that transverse fracture occurred at a fixed braze joint and the microscopic analysis indicated that the cracks initiated in a preferential direction. The failures were of such high strain - low cycle fatigue character, however, that the exact location could not be confidently ascertained. In addition, it was by no means certain that the maximum or principal bending strain was always in the same direction.

Three strain gages were located at 120° around the pipe with the #1 gage placed as near the critical maximum strain location as possible. Flooded start tests showed that the pipe whip was so severe that individual strain reversals could approach 1% in amplitude. This meant that strain gage reliability was a serious problem. It was found, however, that only two gages were needed to determine the maximum bending strain and the principal bending directions. Figures 3 - 5 show the information required to quickly obtain these values with acceptable accuracy for this type of investigation.

This procedure is applicable to any section with rotational symmetry such as tubes and bars, and may be adapted to other nearly rotationally symmetric sections if desired. The example shown and the charts are for gages located at 120°, but other angles may also be used. A simple computer program could be generated to perform the analysis should this be desired. In the present case, the additional time required to perform this step was not taken since once the charts were generated, the data for any given measurement could be found by inspection.
IV. STRAIN ANALYSIS EXAMPLE

Figure 6. shows an example of data taken at start-up with a liquid charged compressor. An extremely large strain reversal occurs shortly after switch contact due to the pipe whipping through its available range of motion. This extreme pipe motion was found to be due to the liquid refrigerant momentum change as the liquid in the pipe and discharge chamber was struck and moved around the pipe bends by a stream of liquid ejected from the partially filled cylinder of a three phase motor-driven compressor. Further discussion of other aspects of this problem will be found in (5.).

The strains measured on this pipe were so large that low cycle fatigue analysis procedures were required. An example of the strain data analysis follows. Half cycles or reversals of strain were examined since each pipe motion of interest produced gross yielding of the mild steel material. Unrestrained motion could produce pipe fractures in a few hundred flooded starts. This means that the pipe operation was in the extremely low cycle, high strain region of the material fatigue performance range.

Referring to Figure 6., the first strain reversal occurred at 0.040 sec. after switch closure. The strain almost immediately went to a maximum value which was limited by mechanical interference within the compressor shell. Great care was taken to insure that the instrumentation ranges and frequency responses were adequate for this investigation once the nature of the problem was determined. The data trace shown in Figure 6. was obtained from a magnetic tape recorder with the playback speed reduced to allow a chart recorder adequate time to properly show the data excursions.

Example: referring to Figure 6.

1st reversal, time = 0.040 sec.

Gage #1 = +8700 \mu e

Gage #2 = -7500 \mu e

(From Figure 3b.) \( \frac{Gage \#1}{Gage \#2} = 1.16, \theta \approx 158^\circ \)

(From Figure 4.) Gage \#1/Gage \#2 = -0.79, \theta \approx 318^\circ

(From Figure 5.) Gage \#1 = 93\% of maximum strain on the section

The maximum bending strain = 0.94\% or 0.0094 \text{ in/in}.

2nd reversal, time = 0.046 sec.

Gage #1 = -8,200 \mu e

Gage #2 = +10,400 \mu e

(From Figure 3b.) \( \frac{Gage \#1}{Gage \#2} = -1 \) This is in the 270 - 30\(^\circ\) sector

(From Figure 4.) Ratio, Gage \#1/Gage \#2 = -0.79, \theta \approx 318^\circ

(From Figure 5.) Gage \#1 = 74\% of maximum

The maximum bending strain = 1.1\% or 0.011 \text{ in/in}.
After about 0.19 seconds and a number of reversals not exceeding 5,000 με, another large event takes place.

3rd reversal, time = 0.19 sec.

Gage #1 = -9,400 με
Gage #2 = +4,800 με

(From Figure 3b.) \(-\frac{1}{\sqrt{2}}\) This is in the 270° + 30° sector

(From Figure 4.) Ratio, Gage #1/Gage #2 = -1.96, \(\alpha = 359°\)

(From Figure 5.) Gage #1 = 100% of maximum

The maximum bending strain \(\approx 0.94\%\) or \(0.0094 \text{ in} / \text{in}\)

Each of these strain reversals is on the order of 1%. Counting only the fatigue damage done by these would mean that the pipe life would be less than a thousand flooded start cycles. Adding the additional damage from the lesser strain reversals easily accounts for why the unrestrained pipe was failing after only a few hundred flooded starts.

As a point of interest, it was found that the pipe went into instantaneous full resonant motion as the compressor speed pulled up through the natural frequency of the pipe. This was a secondary source of fatigue damage, not large compared to the excursions examined above. This unusual feature of the pipe response was understood once it was realized that the available energy from the fluid momentum changes causing pipe whip were orders of magnitude greater than the usual sources of vibrational excitation. The forces on the pipe were found to be on the order of hundreds of pounds for each of the liquid pumping cycles rather than the much smaller "g" forces normally associated with dynamic excitation.

V. CONCLUSIONS

1. Control of bending motions in piping systems associated with refrigerant compressors is of paramount importance. Curved pipes with split type fractures running near the section neutral axis are most often due to von Kármán flexibility. This results in closing and opening the cross section with consequent maximum strain occurring at the ends of the ellipse major axis line.

2. Poor control of bent pipe cross sections resulting in large eccentricities or large aspect ratio ellipses makes matters worse. It is recommended that the value of 2 \((a-b)\) not exceed 0.10 on any pipe bend. This is an inspection measurement that is easy to make. Bending equipment of proper design and in good condition should easily keep this dimension in control.

3. Maximum strains due to arbitrary bending moments can be found on any circularly symmetrical cross section using only two gages. This assertion is subject to the assumptions noted at the beginning of section III. A third gage will provide direct strain information and is invaluable as back up when large strain cycles may cause gage failures.

4. Use of only two (or three) gages per pipe to measure the critical strain is extremely efficient and productive in a variety of situations. The pipe whip problem used here as an example required a large number of strain gaged pipes since a variety of aspects of the problem had to be studied. Confidence in obtaining the maximum strain values was of importance during the program in order to evaluate the piping fatigue performance.
FLEXIBILITY
(von Kármán)
\[
\frac{12 \lambda^2 + 10}{12 \lambda^2 + 1}
\]

\[\lambda = \frac{tR}{r^2}\]

**FIGURE 1.** The flexibility of pipe bends compared to simple beam theory.

RELATIVE STIFFNESS
\[
\% \quad \frac{I_{\text{ellipse}}}{I_{\text{circle}}}
\]

**FIGURE 2.** The relative stiffness of an elliptical cross section related to a perfectly circular section.
FIGURE 3. An example of strain gage locations with their response to bending motions on a tube. Gage #3 can be used for backup or to determine the direct strain in the tube wall.

C. Table of signs and comparative magnitudes.
FIGURE 4. Ratio of gage #1 to gage #2 readings. The bending moment angle, \( \alpha \), is referred to the gage #1 reading.

FIGURE 5. The proportion of the strain readings with respect to bending moment directions.
Figure 6. Partial flooded start test data. Reversals 1-3 are marked at their beginnings.

VI. REFERENCES


5. Quesada, J., "The Utilization of Contemporary Engineering Tools During the Development of Reliable Discharge Lines", International Compressor Engineering Conference Proceedings, Purdue, Univ., 1990