

# Measuring cost efficiency of public and academic libraries in Poland - a methodological perspective and empirical experience (Keynote paper)

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**Measuring cost efficiency of public and academic libraries in Poland –  
a methodological perspective and empirical experience<sup>3</sup>**

**1. Introduction**

In the for-profit sector, market competition and self-interested profit maximisation are the forces society relies upon to push decision-makers to operate efficiently (in every sense). Neither of these forces is present to the same extent in the government or not-for-profit sector, so it is unlikely that libraries perfectly minimise cost. The aim of this study is the presentation of the Bayesian stochastic frontier approach, developed within econometrics, as well as its application to measuring cost efficiency of public and academic libraries in Poland. We apply a short-run cost function to panel data from 20 academic libraries (years 1997, 1998, 1999) and to cross-sectional data from 240 public libraries (year 2000). Within the stochastic frontier model framework, we decompose the error term into a symmetric around zero random variable for statistical noise, and a positive random component measuring economic inefficiency. Using Bayesian techniques, the average cost efficiency of public libraries is estimated to be around 92%, the average cost efficiency of academic libraries is estimated to be around 88%.

This article presents (in a shortened version) main assumptions and results of the research which the authors have undertaken since 1998 [12]. Starting with academic libraries, we have adopted a similar model as the one proposed by D. F. Vitaliano (1997) and we have estimated it using data from 13 academic libraries in Poland. An extended model, estimated on the sample of 18 academic libraries, was published in 1999 [13]. In other articles, which appeared in 2000 [15], and 2003 [17], we extended the model, so that we could consider (for example) acquisition profiles, the automation level that increases service quality, etc. Due to the lack of data, we decided to change the subject of our research from academic to public libraries and, with the great help from the Polish Librarian Association, we surveyed 240 public libraries. An article that came out in 2003 [16] shows many interesting facts about technology and (high) cost efficiency of Polish public libraries. All our previous papers were published in Polish and dealt with particular cases. Now we summarise those results and present them to international audience.

The structure of the paper is as follows. Section 2 summarises microeconomic foundations of the cost efficiency analysis. In section 3 we briefly present the stochastic frontier cost model from the econometrics literature. In section 4 we discuss the choice of variables for the library cost function. The statistical details are shortly presented in section 5 (with many references to the literature). Section 6 shows empirical results for academic and public libraries in Poland.

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## 2. Microeconomic foundations

From the microeconomic perspective, a library can be treated as a production unit that uses certain production factors to produce specific services. It is of great importance for managers to be able to objectively assess whether, at given factor prices, the observed cost is relevant to the production level and structure, or whether this cost is higher than necessary (hence some resources are wasted). When confronted with scarce financial resources, one would also like to know how a proportional increase in all production factors affects the scale of production (i.e., the returns-to-scale effect).

The assessment of economic performance of non-profit institutions can be carried out in two ways:

1. by comparing the present level of production with the maximum level which could be achieved at the same factor inputs (analysis of technical efficiency based on the frontier production function),
2. by comparing the observed cost with the minimum cost at which the same production level could be achieved (analysis of cost efficiency based on the frontier cost function).

As D. F. Vitaliano (1997) rightly noticed, the second approach seems to be more appropriate in the case of libraries as their products are exogenous, i.e., determined by the users' demand for library services (the library cannot decide on the number of loans or reading room visits). Thus, in our research we are using the frontier cost function that, in certain statistical sense, explains the data best. The deviation of the actually incurred costs from the frontier function is called cost inefficiency, which is the joint effect of technical and allocative inefficiency. From management perspective, cost inefficiency assessments are very important as they determine the starting point for proportional decrease of all costs at a fixed production level (technical inefficiency correction) and for changes in the structure of production inputs (allocative inefficiency correction).

Figure 1 presents the graphical decomposition of cost inefficiency into technical and allocative components under the following assumptions: there are two production factors, one product, the input prices are fixed; see J. Marzec, J. Osiewalski (1996-1997).

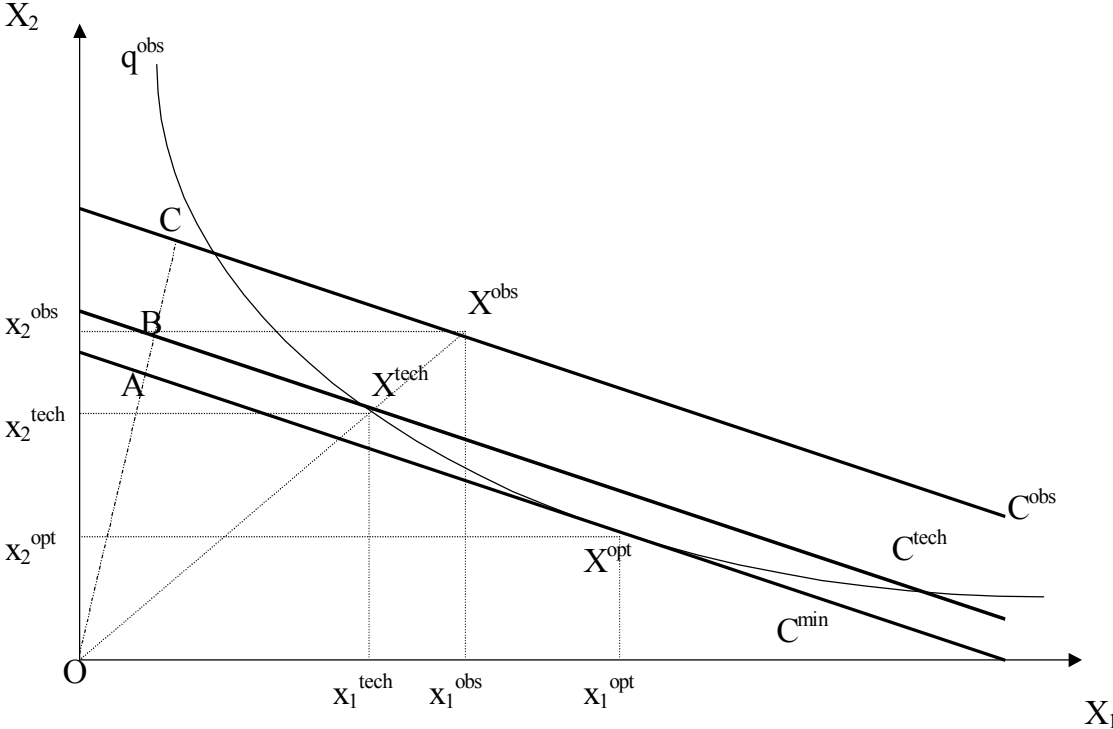
Suppose  $Q$  is the number of registered library users,  $X_1$  denotes books and periodical additions, and  $X_2$  denotes total library staff. By  $X^{\text{opt}} = (x_1^{\text{opt}}, x_2^{\text{opt}})$  we mark the optimal combination of these inputs, which (at fixed input prices) guarantees output level  $q^{\text{obs}}$  at the minimum cost  $C^{\text{min}}$ . By  $X^{\text{obs}} = (x_1^{\text{obs}}, x_2^{\text{obs}})$  we mark the observed inputs which produce the actual output  $q^{\text{obs}}$  incurring cost  $C^{\text{obs}}$ . Let us remind that the isoquant

$$\{(x_1, x_2) : f(x_1, x_2) = q^{\text{obs}}, f - \text{production function}\},$$

is the set of all input combinations which are technically necessary and sufficient for producing the output level  $q^{\text{obs}}$ . Technical inefficiency seen in Figure 1 is caused by the fact that  $X^{\text{obs}}$  is above the isoquant  $q^{\text{obs}}$ . This means the decision-making unit could proportionally reduce the inputs it employs and reach the point  $X^{\text{tech}}$ , producing the same output at reduced cost  $C^{\text{tech}}$ . The difference between  $C^{\text{obs}}$  and  $C^{\text{tech}}$  represents the cost of technical inefficiency. The allocation inefficiency seen in Figure 1 stems from the fact that the relation between input levels in  $X^{\text{obs}}$  and  $X^{\text{tech}}$  is far from cost minimisation; the structure of inputs is not related to their market prices (e.g., expensive factors are used too much). Taking into account input prices, the observed output  $q^{\text{obs}}$  could be obtained at lower cost  $C^{\text{min}} < C^{\text{tech}}$ , using optimal input combination  $X^{\text{opt}}$ . The ratio of two distances: OA/OB measures the degree of allocation

efficiency, while the ratio  $OB/OC$  measures technical efficiency. Overall cost efficiency,  $OA/OC$ , is the product of the two component measures, i.e.,  $OA/OC = (OA/OB) * (OB/OC)$ .

Figure 1



In the empirical microeconomic literature there are two groups of methods for cost efficiency assessment: deterministic methods such as Data Envelopment Analysis (DEA), applied to libraries by T. Chen (1997) or D. F. Vitaliano (1998), and stochastic methods applied by D. F. Vitaliano (1997). Stochastic methods have been used in estimating, e.g., cost efficiency of hospitals – see Koop, Osiewalski and Steel (1997) or, very recently, technical efficiency of museums – see P. Bishop, S. Brand (2003). In our research we only use the stochastic frontier approach.

**3. The stochastic frontier cost model**

The basic stochastic model of the frontier cost function assumes that any deviation of the observed cost from the theoretical microeconomic cost function is caused by purely random disturbances and inefficiency. Such a model is usually formulated by the means of one equation which explains the logarithm of the cost observed at time  $t$  in the  $i$ -th production unit (library). This equation represents the log of actual cost as a sum of an microeconomic cost function (showing the minimum cost at given production level and factor prices) and two random components. One of these random variables, symmetric around zero, reflects the effect of purely random influences and the measurement error, while the other (nonnegative) captures potential inefficiency, which is the subject of our study. In econometrics, such models are called stochastic frontier models. They were proposed by D. Aigner, C. A. K. Lovell and P. Schmidt (1977), and independently by W. Meeusen and J. van den Broeck (1977). D. F. Vitaliano (1997) shows the use of such models in the research on cost efficiency

of libraries. He adopted the Cobb-Douglas microeconomic total cost function. In this study we also use Cobb-Douglas cost frontiers, but in modelling variable costs of libraries. Thus we treat data representing physical capital as measures of fixed factors of library production. Therefore, our cost model takes the following general form:

$$VC_{it} = f(Q_{it}, W_{it}, K_{it}; \beta) \exp(u_{it} + v_{it}) \quad (t=1, \dots, T_i; \quad i=1, \dots, n)$$

where

$VC_{it}$  - variable cost of library  $i$  at period  $t$ ,

$f(\cdot; \beta)$  - a short-run microeconomic cost function of the Cobb-Douglas form, parameterised by a vector  $\beta$  of free coefficients,

$Q_{it}$  - the vector of library  $i$  products (at time  $t$ ),

$W_{it}$  - the vector of prices of variable inputs,

$K_{it}$  - the fixed inputs levels,

$v_{it}$  - a normally distributed, zero mean random error,

$u_{it}$  - a nonnegative random term, representing cost inefficiency of library  $i$  at time  $t$ .

Statistical inference on cost efficiency consists of simultaneously estimating (with the use of real data) the cost function parameters ( $\beta$ ) and efficiency indicators  $r_{it} = \exp(-u_{it})$ . The latter are the ratios of minimal cost subject to purely random deviations,  $f(Q_{it}, W_{it}, K_{it}; \beta) \exp(v_{it})$ , and the observed variable cost  $VC_{it}$ .

#### 4. Choice of variables

The issue of measuring library output, as well as input factors and their prices, is widely discussed in the existing literature. L. DeBoer (1992) sheds light on this complicated issue by asking what a library would sell if it were a private firm. He answers: "It might charge for membership like a health club; it might charge admission at the door, like a movie theater; it might charge for each item checked out, like a video rental store". This suggests using as output measures the following observable variables:

- the number of library card holders (registered members),
- total library attendance and
- loan transactions, respectively.

In the case of academic libraries in Poland, quality and quantity of available data made it necessary to omit the attendance variable. Instead, we have proposed the number of trained users, which stresses teaching as one more aspect (dimension) of the output of academic libraries in Poland. In this case the library might charge for courses like an educational organisation.

The following input measures were suggested in the literature: total library staff, new books purchased, total serial subscriptions currently active, area of library space. Taking the short-run perspective, we adopt

- total library staff (number of persons employed, full time equivalent),
- new books and periodical additions

as variable factors. The area of library space is a fixed factor (it could represent physical capital).

Variable cost function requires data on the following input prices:

- average salary,
- average prices of purchased books and periodicals.

Among the fixed factors (capital stock) we would like to see

- library space,

- total holdings (of all items for library users) at the beginning of the year surveyed,
- fixtures (e.g., tables and chairs).

Finally we have chosen seating capacities like T. Chen (1997).

In our questionnaire we also asked for the number of computer workstations for users (OPAC terminals, PCs for Internet and other on-line access and CD-ROM terminals). In current approach we treat this variable as reflecting quality. We assume that the unobserved true level of product  $g$  ( $Q_{g,ti}$ ) is a function of both the measured quantity ( $Y_{g,ti}$ ) and the number of computer workstations for users ( $Z_{ti}$ ), which increases the quality of library  $i$  services in the year  $t$ . Hence we specify:

$$Q_{g,ti} = Y_{g,ti} \exp(\alpha_g + \gamma_g Z_{ti}), \quad g=1, \dots, G, \quad (1)$$

where  $\gamma_g > 0$  and  $\alpha_g$  are unknown parameters. This approach is analogous to modelling so-called effective production factors, see G. Koop, J. Osiewalski and M. F. J. Steel (2000).

Finally our cost model takes the form

$$VC_{ti} = \alpha_0^* Y_{1,ti}^{\beta_1} \cdot \dots \cdot Y_{G,ti}^{\beta_G} K_{ti}^{\beta_{G+1}} W_{1,ti}^{\beta_{G+2}} \cdot \dots \cdot W_{H+1,ti}^{\beta_{G+H+1}} \exp(\beta_{G+H+2} Z_{ti}) \exp(u_{ti} + v_{ti}),$$

where  $G$  denotes the number of products,  $H$  is the number of observed prices of variable inputs (there can be an input, number  $H+1$ , with unobserved price, which can be treated as approximately constant over time and units), and

$$\beta_{G+H+2} = \beta_1 \gamma_1 + \dots + \beta_G \gamma_G, \quad \alpha_0^* = \alpha_0 \exp(\alpha_1 + \dots + \alpha_G).$$

It is easy to see that, in the Cobb-Douglas specification, we are not able to infer on individual coefficients  $\alpha_g$  and  $\gamma_g$ , appearing in (1). Only  $\beta_{G+H+2}$ , a linear function of parameters  $\gamma_g$  that measures joint cost effect of increasing service quality, is identifiable and can be estimated. Other  $\beta_j$  are cost elasticities with respect to products ( $j=1, \dots, G$ ), fixed input ( $j=G+1$ ) and prices ( $j=G+2, \dots, G+H+1$ ). They measure a relative cost change (in percentage terms) when a particular explanatory variable increases by 1% (keeping the remaining variables constant).

## 5. Stochastic assumptions and statistical inference

In our research we assume that all explanatory variables are exogenous (library does not decide on output levels and input prices) and treat them as if they were fixed (non-stochastic). The symmetric random components of variable cost,  $v_{ti}$ , are independent over time and units, and normally distributed with mean 0 and unknown variance  $\sigma^2$ . They are also independent of the one-sided inefficiency terms  $u_{ti}$ , which are independent (over time and units) Exponential random variables with unknown mean  $\lambda$ . These assumptions define the sampling component (i.e., the conditional distribution of observations and latent variables given parameters) of our Bayesian statistical model, which is the joint distribution of observations, latent variables (inefficiency terms) and parameters. The marginal distribution of the parameters, the so-called prior distribution, completes our Bayesian specification.

The vector of cost function coefficients,  $\beta$ , has an improper uniform distribution truncated by economic regularity conditions imposed on a short-run variable cost function. These regularity conditions are very simple in the Cobb-Douglas case: the elasticities of variable cost with respect to products and prices are positive (cost raises when we increase production or if an input price raises), the elasticity with respect to fixed input is negative, and the sum of elasticities with respect to all variable input prices is exactly 1. Our interpretation of the number of PC units ( $Z_{ti}$ ) as a proxy for product quality leads to the non-negativity restriction on  $\beta_{G+H+2}$  (higher quality requires additional cost).

The prior distributions of the inverses of  $\sigma^2$  and  $\lambda$  are independent Gamma distributions, with mean 1 and variance  $2 \cdot 10^6$  in the case of  $(\sigma^2)^{-1}$ , and the same, equal to 4.48142, mean and standard deviation for  $\lambda^{-1}$ . The latter specification leads to the prior median efficiency 0.8, reflecting our initial belief that there are equal chances of each individual efficiency to be less or greater than 0.8 (there are equal prior chances that more or less than 80% of the actual variable cost is justified by the production level, fixed input and variable input prices).

The Bayesian approach to statistical inference is conceptually simple, as it amounts to deriving and summarising the so-called posterior distribution, i.e. the conditional distribution of quantities of interest (parameters, efficiency levels) given the observations. This approach is not based on asymptotic theory, so it does not require large samples. It can also easily incorporate researcher's initial information. Thus it can be very helpful in analysing even small data sets. The application of Bayesian inference in stochastic frontier models was proposed by J. van den Broeck, G. Koop, J. Osiewalski and M. F. J. Steel (1994), and developed further by G. Koop, J. Osiewalski and M. F. J. Steel (1997) and C. Fernández, J. Osiewalski and M. F. J. Steel (1997), who focus on the fundamental problem of the existence of the posterior distribution under improper priors. Our proper Gamma prior on  $(\sigma^2)^{-1}$  reflects very vague initial information, but it assures the existence of the posterior distribution. Practical applications of Bayesian inference require appropriate numerical tools to explore and summarise the posterior distribution. Here we use Gibbs sampling, described in detail by G. Koop, M. F. J. Steel and J. Osiewalski (1995) and J. Osiewalski and M. F. J. Steel (1998); see also J. Osiewalski and A. Osiewalska (2003b). In the empirical section we present the posterior means (point estimates) and standard deviations (measuring uncertainty) as useful summaries of the multivariate posterior distribution of the parameters and efficiency levels.

## 6. Empirical results

In both cases (academic and public libraries) we model the observed variable cost, which is the sum of annual expenditures on salaries and acquisitions. Thus, our dependent variable represents the most important part of library performance costs.

### 6.1. Academic libraries

Our short-run cost model for academic libraries is based on an unbalanced panel. Only nine libraries gave us full data for three consecutive years (1997-1999,  $T_i = 3$  for  $i=1, \dots, 9$ ), from three libraries we received full data for two years (1997-1998 or 1998-1999,  $T_i = 2$  for  $i=10, 11, 12$ ), and for the remaining eight libraries we had to rely on information from 1997 alone ( $T_i = 1$  for  $i=13, \dots, 20$ ). We do not impose any panel structure on our model, treating the data representing the same library in different years as if they related to different units. Thus, our model for 41 observations from 20 libraries looks as if we had individual data from 41 units. The equation we estimate takes the form

$$\ln VC_{ti} = \beta_0 + \beta_1 \ln y_{ti,1} + \beta_2 \ln y_{ti,2} + \beta_3 \ln y_{ti,3} + \beta_4 \ln k_{ti} + \beta_5 \ln w_{ti,1} + \beta_6 \ln w_{ti,2} + (1 - \beta_5 - \beta_6) \ln w_{ti,3} + \beta_7 Z_{ti} + v_{ti} + u_{ti},$$

where:

- $VC_{ti}$  is the sum of library  $i$  expenditures on salaries and acquisitions in year  $t$ ;
- $y_{ti,1}$  is the number of library  $i$  card holders (in year  $t$ );
- $y_{ti,2}$  is the number of library  $i$  loan transactions (in year  $t$ );
- $y_{ti,3}$  is the number of library  $i$  trained users (in year  $t$ );

- $k_{ti}$  is the seating capacity (of library  $i$  in year  $t$ );  
 $w_{ti,1}$  is the average librarian's salary (in library  $i$ , year  $t$ );  
 $w_{ti,2}$  is the average price of periodicals purchased in year  $t$  by library  $i$ ;  
 $w_{ti,3}$  is the average price of books purchased in year  $t$  by library  $i$ ;  
 $Z_{ti}$  is the number of computer workstations for users (in library  $i$ , year  $t$ ).

In this example we have imposed all economic regularity conditions:  $\beta_1, \beta_2, \beta_3, \beta_5, \beta_6 > 0$ ,  $\beta_5 + \beta_6 < 1$ ,  $\beta_4 < 0$ , and also  $\beta_7 > 0$ .

Table 1. Frontier academic library cost function.

Variable	Parameter	Posterior mean	Posterior standard deviation
Constant	$\beta_0$	-4.754	0.710
log library card holders	$\beta_1$	0.329	0.092
log loan transactions	$\beta_2$	0.167	0.076
log trained users	$\beta_3$	0.236	0.066
log seating capacity	$\beta_4$	-0.064	0.060
log average salary	$\beta_5$	0.329	0.098
log average price of purchased periodicals	$\beta_6$	0.261	0.089
log average price of purchased books	$1 - \beta_5 - \beta_6$	0.410	0.123
number of PC	$\beta_7$	0.0021	0.0012

Table 1 presents the posterior means,  $E(\cdot | data)$ , and standard deviations,  $D(\cdot | data)$ , of the parameters of the Cobb-Douglas cost frontier, which describes the technology of Polish academic libraries treated as production units. The results on elasticities of variable cost with respect to products show that the number of library card holders has the strongest influence on variable cost, which raises by 0.33% ( $\pm 0.09\%$ ) when the number of card holders increases by 1%. Interestingly, the 1% increase in the number of trained users leads to the change in variable cost by 0.24% ( $\pm 0.07\%$ ), which is more than the 0.17% ( $\pm 0.08\%$ ) effect of the 1% raise of loan transactions. The teaching function of Polish academic libraries seems important (and relatively costly). Changing the scale of all products by 1% requires the change in variable cost by only 0.731% ( $\pm 0.084\%$ ), which indicates short-run increasing returns to scale. This short-run scale effect measure assumes that we keep fixed inputs unchanged. However, the data do not show any significant influence (on variable cost) of our measure of fixed inputs (seating capacity). Thus we may informally infer about long-run increasing returns to scale. Increasing libraries' budgets would result in more than proportional increase in their services.

When interpreting the results for  $\beta_4$  we should remember that the negativity restriction was imposed on this parameter, and it turned out to be binding in the posterior distribution. Thus, the data may suggest some degree of overcapitalisation, at least in some libraries (the Cobb-Douglas specification represents the average case, as it assumes constant elasticities).

As regards the role of the number of workstations (according to our assumptions, they increase the service quality), one extra PC raises variable cost by about 0.21% ( $\pm 0.12\%$ ). This is not a precise estimate, so it should be interpreted with care.

The importance of different input prices is similar, although its estimates are not very precise (due to a very limited number of observations). A 1% increase in salaries leads to the raise in variable cost by 0.33% ( $\pm 0.10\%$ ), while a 1% change in the average price of



periodicals results in 0.26% ( $\pm 0.09\%$ ) change in VC. The average price of books has the largest impact on VC ( $0.41 \pm 0.12\%$ ).

Table 2. Cost efficiency estimates.

Library no. ( <i>i</i> )	Year	$\ln VC_{it}$	$E(r_{it} data)$	$D(r_{it} data)$	Ranks by time average of $E(r_{it} data)$
1	1997	7.313	0.765	0.182	17 (0.811)
	1998	7.288	0.844	0.138	
	1999	7.658	0.824	0.154	
2	1997	8.403	0.845	0.128	10-11 (0.891)
	1998	8.562	0.906	0.087	
	1999	8.739	0.922	0.074	
3	1997	7.900	0.914	0.079	6 (0.910)
	1998	8.068	0.912	0.081	
	1999	8.325	0.903	0.088	
4	1997	8.249	0.820	0.145	15 (0.844)
	1998	8.235	0.859	0.121	
	1999	8.419	0.852	0.125	
5	1997	7.526	0.912	0.081	4 (0.912)
	1998	7.703	0.898	0.094	
	1999	7.885	0.927	0.069	
6	1997	4.038	0.911	0.083	12 (0.890)
	1998	4.188	0.872	0.115	
	1999	4.499	0.886	0.103	
7	1997	7.070	0.886	0.100	13 (0.884)
	1998	7.125	0.894	0.095	
	1999	7.264	0.873	0.110	
8	1997	6.288	0.907	0.084	9 (0.894)
	1998	6.597	0.889	0.099	
	1999	6.744	0.886	0.101	
9	1997	8.662	0.873	0.110	10-11 (0.891)
	1998	8.763	0.909	0.083	
	1999	8.957	0.890	0.098	
10	1998	6.831	0.904	0.088	8 (0.901)
	1999	7.036	0.898	0.093	
11	1997	7.174	0.839	0.135	16 (0.821)
	1998	7.468	0.803	0.154	
12	1998	7.187	0.918	0.075	2 (0.919)
	1999	7.421	0.920	0.074	
13	1997	4.606	0.875	0.118	14
14	1997	8.392	0.805	0.155	19
15	1997	7.908	0.911	0.081	5
16	1997	6.031	0.913	0.080	3
17	1997	7.227	0.906	0.085	7
18	1997	7.342	0.798	0.159	20
19	1997	5.308	0.951	0.048	1
20	1997	7.502	0.806	0.153	18
Total average			0.879	0.105	

Table 2 shows the posterior means and standard deviations of the individual efficiency indices ( $r_{it}$ ) as well as their time averages. The latter serve us to rank the 20 surveyed libraries. The minimum of time averages of cost efficiency estimates is almost 0.8, the overall average is 0.88, and the maximum is 0.95. Thus, on average, 88% of the observed variable cost can be explained by the scale, structure and quality of library services, as well as by input prices. This is an evidence of good management in majority of 20 Polish academic libraries which have been included in our research.

## 6.2. Public libraries

In this subsection we model cross-sectional data from 240 public libraries; the data represent their performance in year 2000. Thus  $T_i$  is always 1 and we omit the time subscript  $t$ . The equation we estimate takes the form

$$\ln VC_i = \beta_0 + \beta_1 \ln y_{i,1} + \beta_2 \ln y_{i,2} + \beta_3 \ln y_{i,3} + \beta_4 \ln k_i + \beta_5 \ln w_{i,1} + \beta_6 \ln w_{i,2} + \beta_7 \ln w_{i,3} + \beta_8 Z_i + v_i + u_i$$

where:

$VC_i$	is the sum of library $i$ expenditures on salaries and acquisitions;
$y_{i,1}$	is the number of library $i$ card holders;
$y_{i,2}$	is the number of library $i$ loan transactions;
$y_{i,3}$	is the attendance at library $i$ ;
$k_i$	is the seating capacity of library $i$ ;
$w_{i,1}$	is the average librarian's salary in library $i$ ;
$w_{i,2}$	is the average price of periodicals purchased by library $i$ ;
$w_{i,3}$	is the average price of books purchased by library $i$ ;
$Z_i$	is the number of computer workstations for users in library $i$ .

In this case we have also considered another variable input, namely materials (e.g., computer accessories). Its price, which we do not observe, is assumed equal for all libraries, so its effect is captured by the intercept ( $\beta_0$ ). We have imposed most economic regularity conditions:  $\beta_1, \beta_2, \beta_3, \beta_5, \beta_6, \beta_7 > 0$ ,  $\beta_5 + \beta_6 + \beta_7 < 1$  and  $\beta_8 > 0$ . However, we have not restricted  $\beta_4$ .

Table 3. Frontier public library cost function..

Variable	Parameter	Posterior mean	Posterior standard deviation
Constant	$\beta_0$	-2.155	0.391
log library card holders	$\beta_1$	0.634	0.063
log loan transactions	$\beta_2$	0.153	0.055
log library visit	$\beta_3$	0.020	0.014
log seating capacity	$\beta_4$	0.126	0.031
log average salary	$\beta_5$	0.842	0.056
log average price of purchased periodicals	$\beta_6$	0.068	0.037
log average price of purchased books	$\beta_7$	0.034	0.026
number of PC units	$\beta_8$	0.014	0.007

Table 3 presents the posterior means and standard deviations of the parameters of the Cobb-Douglas cost frontier for Polish public libraries. The results on elasticities with respect to products show that the number of library card holders has the strongest influence on variable cost. The latter raises by 0.63% ( $\pm 0.06\%$ ) when the number of card holders increases by 1%. The 1% increase in the number of loan transactions leads to a change in variable cost by 0.15% ( $\pm 0.055\%$ ), which is much more than the almost negligible effect – equal to 0.02% ( $\pm 0.01\%$ ) – of the 1% raise of attendance. Changing the scale of all products by 1% requires a change in variable cost by only 0.806% ( $\pm 0.026\%$ ), which indicates increasing returns to scale in short-run (keeping fixed inputs unchanged).

As regards the role of the number of workstations (increasing the service quality, at least in our interpretation), one extra PC raises variable cost by about 1.4% ( $\pm 0.7\%$ ). This is much more than in the case of academic libraries – quite obviously, as public libraries are much smaller on average.

The importance of wages for variable cost is prevailing. The 1% increase in average librarian's salary leads to the raise in variable cost by 0.84% ( $\pm 0.06\%$ ), while the 1% change in the average price of periodicals results in a much smaller change in VC: 0.07% ( $\pm 0.04\%$ ). The average prices of books and materials have even smaller (and imprecisely estimated) impact on VC:  $0.034 \pm 0.026\%$  and  $0.056 \pm 0.047\%$ , respectively.

The positive estimate of the elasticity with respect to fixed input (seating capacity) suggests too large seating capacity (relative to the scale of library activities).

Table 4 shows the posterior means and standard deviations of individual efficiency indices ( $r_i$ ) for the most, medium and least efficient libraries. The minimum posterior mean of cost efficiency is 0.81, the overall average is 0.92, and the maximum is 0.965. Thus, on average, 92% of the observed variable cost can be explained by the scale, structure and quality of library services, as well as by input prices. This is an evidence of successful management (with very limited resources) in most of 240 Polish public libraries under study.

Table 4. Cost efficiency estimates.

library no. ( $i$ )	$VC_i$ (in zloty)	$E(r_i data)$	$D(r_i data)$
33	70708	0.965	0.034
147	32398	0.960	0.038
98	112037	0.955	0.043
214	23393	0.955	0.043
...	...	...	...
133	23740	0.925	0.069
82	304403	0.925	0.068
...	...	...	...
6	916026	0.919	0.074
...	...	...	...
22	301874	0.842	0.126
46	1387614	0.842	0.126
61	987074	0.837	0.129
121	80440	0.837	0.134
155	143811	0.836	0.129
139	151939	0.811	0.142
Total average		0.919	0.072

The posterior means of cost efficiency levels do not show significant correlation with either the observed variable cost or library characteristics. So there is no clear evidence of any systematic pattern in cost efficiency. Even the highest correlation coefficient (+0.24) (between efficiency estimates and shares of librarian salaries in variable cost) does not indicate a strong relationship. So lower efficiency can be attributed to worse management.

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