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ANALYSIS OF COMPRESSOR BOLT FASTENERS
SUBJECTED TO DYNAMIC LOADING

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ABSTRACT
Predominant cause of fractures of bolts subjected to cyclic loading is attributable to a lack of attention to proper design considerations for elastic joints.

In this paper, fundamentals of bolted elastic joints subjected to cyclic loading are reviewed. A case history involving use of elastic joint techniques on the piston bolt of a small air compressor is given. In the example, analysis of dynamic loads acting on the joint, elastic joint properties and stress analysis of the bolt are presented.

In conclusion, factors of safety are developed that enable the designer to comparatively evaluate various bolt joint options that may be available. Application of this methodology may be further extended to the design of gasketed joints.

INTRODUCTION
In the past, numerous fasteners in clamped assemblies fractured when subjected to dynamic loads because insufficient attention had been given to proper joint design. Such previous design methods usually relied solely upon the determination of maximum direct tensile stress in the bolt. Direct stress was characteristically obtained by dividing the total maximum applied load by the minimum cross sectional area of the bolt. Generous factors (usually upwards of 10) were commonly ascribed for the ratio of bolt material tensile strength to this direct stress. Nonetheless, fatigue fractures of bolts having such misleadingly high numerical factors were not uncommon.

It has since become evident that the application of elastic joint analysis combined with dynamic stress analytical techniques have provided the compressor designer a far more reliable method. Interestingly, bolt factors of safety in the order of 1.4 when current analytical techniques are employed, afford the compressor designer far more assurance than did higher numbers associated with previous methods. On occasion, however, it remains difficult to convince some users accustomed to previous methods that such small factors of safety are adequate, particularly in critical applications.

Early investigators, (1,2) reported that when dynamic external loading is applied to pre-loaded bolted assemblies such loading is divided between the bolt and clamped members in a direct relationship to respective spring constants. These spring constants are derived from Hooke's law, as illustrated in elementary equations -1- and -2-:

\[ -1- \quad \Delta L = \frac{P L}{AE} \]

Since by definition
\[ K = \frac{P}{\Delta L} \]

\[ -2- \quad K = \frac{AE}{L} \]

Accordingly, spring constant for bolt is

\[ -3- \quad K_b = \frac{A_b E_b}{L_b} \]

and spring constant for clamped part member is

\[ -4- \quad K_p = \frac{A_p E_p}{L_p} \]

References are listed at end of paper.
Combined spring constant relationship for a bolted assembly preloaded to an initial load, $F_{in}$ is illustrated in Fig. 1.

\[ F_b = F_{in} + \frac{K_b}{(K_b + K_p)} \times P_{et} \]
\[ F_P = F_{in} - \frac{K_P}{(K_b + K_p)} \times P_{et} \]

In some dynamic load situations $P_{et}$ is other than a non-reversing type load, in many cases varying between positive and negative values. Maximum and minimum values of $F_b$ are calculated from equation -5-. Such boundary $F_b$ values are the basis for dynamic analysis of the bolt.

**STRESS ANALYSIS**

A modified Goodman diagram for 17-4 PH, condition H1075 steel is illustrated in Fig. 2. Typical ultimate tensile strength of this material is 145000 psi.

\[ \text{Stress concentration factors upwards of 4.5 are typical for cut thread having root radii. However, when manufacturing care is not taken higher stress concentration factors may exist. Factors of 2. to 2.5 are normal for well rounded roots of rolled threads.} \]
A partial cross-sectional view of piston and piston bolt assembly of a 100 cfm air compressor is shown in Fig. 3.

Function of the bolt, shown at the axial center, is to clamp the piston to the crosshead assembly through the guide tube. Present bolt material used is 17-4PH, condition H1075 steel. Specified pre-load torque on piston nut is 31 lbf-ft. Fatigue fractures had occurred through cut threads, and bolts with rolled threads had been substituted. A comparison of factors of safety between the two thread forms was required, along with recommendations for possible material substitution, if this was found to be necessary.

A number of bolts of both thread forms were examined under an optical comparator and findings were quite revealing, as illustrated in Fig. 4. Root radii of the 3/8-16 UNC2A cut threads were practically non-existent and were extremely sharp. Measured root diameter was 0.278". Rolled threads, on the other hand, were well rounded in the root area and measured root diameter was 0.288". As stated previously, a stress concentration factor of 4.5 was deemed appropriate for the cut threads. A stress concentration factor of 2.3 was used for the rolled threads.

The 100 cfm air compressor in this example is of Y configuration with two 8.25" dia. single acting first stage cylinders and one 6.5" dia. single acting second stage cylinder. Piston stroke is 2.75" and crankshaft rotational speed is 830 rpm. All of the bolts that fractured had cut threads and were typically located at the second stage piston assembly. Applied joint force -vs- crank angle relationships were accordingly calculated for three cases of piston load: unloaded, stage I loaded and stage II loaded. These are illustrated in Figs. 5, 6 and 7, respectively. Since the piston weight for both stages is identical, Fig. 5 is common for both stages. Applied cylinder load P_{et} for piston inertia ranged from -161 lbf to 210 lbf. P_{et} for stage I loaded ranged from -1916 lbf to 184 lbf and P_{et} for stage II loaded ranged from -4192 lbf to -977 lbf.

In the example a series of washers and varying sectional geometry contributed to stiffness of overall clamped part in the relationship (7)

$$-8- \quad K_p = \frac{1}{K_n}$$

Numerical value of K_{p} was found to be 1.2301E6 lbf/in. Similarly, overall stiffness of bolt was 0.2477E6 lbf/in.
Bolt preload on clamped assembly, $F_{\text{in}}$, with dry threads was obtained from the generalized relationship

$$-9- \quad F_{\text{in}} = \frac{T}{0.018d}$$

With 3/8" bolt torqued to 31 lbf.ft, $F_{\text{in}}$ was 4593 lbf.

Simplifying equations -5- and -6- for present specific preload and spring constants result in

$$-10- \quad F_b = 4593 + 1.16761 P_{et}, \text{ and}$$

$$-11- \quad F_p = 4593 - 0.83238 P_{et}.$$  

Calculated forces and stresses acting on bolt are summarized on Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Load, lbf.</th>
<th>Nom. Stress $C$, PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{et}$</td>
<td>$F_b$</td>
</tr>
<tr>
<td>Unloaded, Max. $P_{et}$</td>
<td>210</td>
<td>4624</td>
</tr>
<tr>
<td>Unloaded, Min. $P_{et}$</td>
<td>-161</td>
<td>6566</td>
</tr>
<tr>
<td>Stg. I, Max. $P_{et}$</td>
<td>184</td>
<td>4624</td>
</tr>
<tr>
<td>Stg. I, Min. $P_{et}$</td>
<td>-1916</td>
<td>6272</td>
</tr>
<tr>
<td>Stg. II, Max. $P_{et}$</td>
<td>-977</td>
<td>4429</td>
</tr>
<tr>
<td>Stg. II, Min. $P_{et}$</td>
<td>-4192</td>
<td>3890</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load, lbf.</th>
<th>Nom. Stress $C$, PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut Thds.</td>
<td>$C_m$</td>
</tr>
<tr>
<td>-161</td>
<td>75370</td>
</tr>
<tr>
<td>-977</td>
<td>4160</td>
</tr>
<tr>
<td>-4192</td>
<td>3890</td>
</tr>
</tbody>
</table>

**TABLE 1**

Summary of Bolt Forces and Stresses ($F_{\text{in}} = 4593$ lbf.)

17-4 PH Condition H107S

Nominal stresses of cut threads are slightly higher due to their smaller minimum cross sectional area compared to the well rounded roots of rolled threads. Mean and cyclic loads $P_m$ and $P_r$ are graphically illustrated on Fig. 8 for unloaded and stage II loaded cases.
Equation -7- can be simplified for 17-4PH condition H1075 bolt material to:

$$-12- \quad F_{Sc} = \frac{58000}{4.5 E_F + .4 E_m} \text{ for cut threads}$$

and to

$$-13- \quad F_{Sr} = \frac{58000}{2.3 E_F + .4 E_m} \text{ for rolled threads.}$$

Values of bolt cyclic stress, bolt mean stress and factors of safety are summarized on Table 2 for 17-4 PH condition H1075 bolt material. As is evident, a significant increase in factor of safety was attained by use of rolled threads compared to cut threads.

Inasmuch as the application was considered to be highly critical, the customer opted for a factor of safety greater than 1.66 and questioned whether another bolt material was appropriate. Upon evaluation Multiphase Alloy MP35N* was considered as a possible alternate bolt material.

Applicable properties of MP35N material were (8)

$$\sigma_{ult} = 260,000 \text{ psi}$$
$$E' = .346$$
$$E' = 33.5 E6 \text{ psi @ } T_F = 175$$

On the basis of revised bolt modulus, $K_b$ was 0.2912E6 lbf/in., and

$$-10a- \quad F_b'' = 4593 + .19142 P_{et}, \text{ and}$$

$$-11a- \quad F_p'' = 4593 - .80858 P_{et}.$$
Bolt cyclic stresses, mean stresses and factors of safety for Multiphase alloy are summarized in Table 4. As is again evident by comparing Tables 2 and 4, a significant increase in factor of safety is obtainable with Multiphase alloy and rolled threads.

<table>
<thead>
<tr>
<th>Thread Form</th>
<th>Load Case</th>
<th>$S_m'$, psi</th>
<th>$S_m''$, psi</th>
<th>FS$'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut</td>
<td>Unloaded</td>
<td>590</td>
<td>75730</td>
<td>3.12</td>
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<tr>
<td>Cut</td>
<td>Stg. I</td>
<td>3310</td>
<td>72930</td>
<td>2.24</td>
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<tr>
<td>Cut</td>
<td>Stg. II</td>
<td>5070</td>
<td>67510</td>
<td>1.95</td>
</tr>
<tr>
<td>Rolled</td>
<td>Unloaded</td>
<td>550</td>
<td>70570</td>
<td>3.50</td>
</tr>
<tr>
<td>Rolled</td>
<td>Stg. I</td>
<td>3080</td>
<td>67960</td>
<td>2.94</td>
</tr>
<tr>
<td>Rolled</td>
<td>Stg. II</td>
<td>4730</td>
<td>62910</td>
<td>2.76</td>
</tr>
</tbody>
</table>

**TABLE 4**

**Summary of Cyclic and Mean Stresses and Factors of Safety**

Multiplier Phase Alloy MP3SN*, $F_m$ = 4593 lbf.

**CONCLUSION**

Use of elastic joint analysis, in conjunction with dynamic stress analysis, affords the compressor designer a reliable method in determining fastener factor of safety. A bolt material option was offered that provided significant increase in bolt factor of safety. Elastic joint analytical techniques may be further applied in cases of gasket design. (9)

**NOTATION**

- **A** Element area, in$^2$
- **$A_b$** Bolt cross sectional area, in$^2$
- **$A_p$** Effective cross sectional area of clamped part, in$^2$
- **$d$** Bolt nominal diameter, in
- **$E$** Modulus of elasticity, lbf/in
- **$E_b$** Modulus of elasticity of bolt, lbf/in
- **$E_p$** Modulus of elasticity of clamped part, lbf/in
- **$E_R$** Endurance ratio of material
- **$F_b$** Total load on bolt, lbf
- **$F_m$** Bolt pre-load on clamped assembly, lbf
- **$F_p$** Net residual load on clamped part, lbf
- **$F_S$** Factor of safety
- **$F_{S_C}$** Factor of safety for cut threads
- **$F_{S_R}$** Factor of safety for rolled threads
- **$K$** Element spring constant, lbf/in
- **$K_b$** Bolt spring constant, lbf/in
- **$K_r$** Spring constant of clamped part element, lbf/in
- **$K_p$** Spring constant of clamped part, lbf/in
- **$L$** Element length, in
- **$L_b$** Effective length of bolt, in
- **$L_p$** Effective length of clamped part, in
- **$P$** Load, lbf
- **$P_b$** Double amplitude cyclic load on bolt, lbf
- **$P_{et}$** Applied external dynamic load, lbf
- **$P_p$** Double amplitude cyclic load on clamped part, lbf
- **$P_m$** Mean load on bolt, lbf
- **$P_r$** Cyclic load on bolt, lbf
SCF  Stress concentration factor of bolt thread
T  Set-up torque, lbf-ft
Tf  Operating temperature on bolt, °F
ΔL  Change in element length, in
Em  Mean stress, lbf/in²
Er  Cyclic stress, lbf/in²
Ult  Ultimate tensile strength of bolt material, lbf/in²

Prime notation denote values using alternate material

REFERENCES


(4) "Precipitation Hardening Stainless Steels," Bulletin, Republic Steel Corp.

(5) RE Peterson, "Stress Concentration Factors," J. Wiley & Sons, N. Y.


