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Mechanical Loss Model of Rolling Piston Rotary Compressor with Special Importance Attached to Journal Bearing

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ABSTRACT

The gas forces are solved by means of a polytropic process with no over and undershooting loss taken into consideration. That for fixed vane rotary compressors very important nongeometrical determined piston movement is presumed to be subjected to a constant number of revolutions. The said piston is supposed to be only in sliding contact with the vane, as well as pure unloaded uniform oil friction on both ends, together with short bearing torque to the eccentric journal. Subsequently all three radial bearings - the two mains plus the roller-eccentric are calculated due to dynamic forces with the aid of Bookers Mobility Method and Ovirts Short Bearing Theory to find minimum oil film thickness and energy consumption. The computer model is verified by measurements, where the mechanical losses are determined as the difference between shaft power and indicated gas effect.

THE PRESSURE IN THE COMPRESSION CHAMBER

The one separation line between the suction and compression chamber is defined by the angular position of the rotor, which, assuming a constant speed of rotation of the crankshaft, can be written as

\[ \Phi = \omega t \times 0 < \Phi < 2\pi \]

If we now introduce the absolute eccentricity

\[ e = R - r \]

and then the relative eccentricity

\[ \epsilon = \frac{e}{r + p} \ll 1 \]

(3)

the position at the vane of the second dividing line between the two chambers mentioned can be expressed by the angle

\[ \alpha = \arcsin(\epsilon \sin \Phi) \]

(4)

and the distance between the centre of the crankshaft and the centre of curvature of the vane tip will be

\[ l = (r + p)(\cos \alpha + \cos \Phi) \]

(5)

where

\[ A_0 = 1 - \frac{1}{2} \epsilon^2 - \frac{5}{4} \epsilon^4 - \frac{19}{4} \epsilon^6 \]

\[ A_1 = \epsilon \]

\[ A_2 = \frac{1}{2} \epsilon^2 + \frac{5}{4} \epsilon^4 + \frac{19}{4} \epsilon^6 \]

\[ A_4 = -\frac{1}{2} \epsilon^2 - \frac{5}{4} \epsilon^4 - \frac{19}{4} \epsilon^6 \]

\[ A_6 = \epsilon^6 \]

The Fourier cosine series is appropriate when describing the movement of the vane. Using the vane displacement, the compression volume becomes
\[ V = \frac{3}{2}H((2t-a)(R^2-r^2)+(r^2-p^2)a + 1(\sin t+t)-t(R+p)+V_c \]

and the corresponding polytropic pressure is

\[ P_p = P_s \left( \frac{V(V_0)}{V} \right)^{\delta} \]

Thus if under and overshooting loss is disregarded the pressure in the compression chamber can be expressed by the selection formula

\[ P = \begin{cases} 
  P_s & \text{if } \theta < \theta_s \\
  P_p & \text{if } \theta_s < \theta < \theta_p \\
  P_d & \text{if } \theta > \theta_p 
\end{cases} \]

and the pressure in the compressor chambers is now explicitly described as a function of the rotating angle, theta, in that the suction chamber pressure is defined as being equal to the evaporation pressure.

GAS FORCES ACTING ON ROLLER AND CRANKSHAFT

Integration of the pressure distribution on the roller gives the force of this pressure in both x and y direction

\[ K_x = H(P_s - P_p)(\cos \alpha - \cos \beta) \]

\[ K_y = H(P_s - P_p)(\sin \alpha - \sin \beta) \]

It can be demonstrated that the torque of the gas force around the crankshaft is

\[ M_g = \frac{\delta}{2}h(P_s - P_p)(R^2 - r^2 - 1(1 - 2 \rho \cos \alpha)) \]

and thus the most substantial part of the shaft torque is found.

FORCES ACTING ON THE VANE

In addition to the dominant pressure load surrounding it the vane is subject to load from the vane spring force

\[ k = k_0 \frac{R^2 + r^2}{R^2 - R^2} \]

the friction forces in the thrust bearings

\[ f_i = \frac{1}{2}t_h(P_d - P) + u \frac{h_1}{\sqrt{t}} \frac{d}{dt}, \quad i = 1, 2 \]

and those in the two end clearances of the slot

\[ f_e = 2(\frac{1}{2}t_v h(P_d - P) + u \frac{b_1}{\sqrt{t_v}} \frac{d}{dt}) \]

since there is condensing pressure in the shell volume. Moreover a normal reaction takes place in the connecting line between the centre of curvature of the vane and centre of roller which is assumed to give rise to a sliding friction force of the order

\[ F = \mu \, N \text{sign}(r_w - \rho \cos(\theta + \alpha)) \]

The computer program also includes a feature for similar calculation of thrust bearing friction by means of a coefficient of Coulombs friction, since here too metallic contact is the most likely factor. However this feature will not be dealt with further in this paper.

FORCE BALANCE ON THE VANE

In order to set up the force balance of the vane it is expedient to know the distance from the separation line

\[ \Lambda = R - \rho \cos \alpha \]

to the cylinder circumference. The force balance in the direction of the x axis

\[ G_x - G_y + (P_s - P_p)Ah \cos \alpha - N \sin \alpha = 0 \]

and in the direction of the y axis

\[ -f_1 - f_2 + (\frac{1}{2}P_t + \rho \sin \alpha) + P_{d} - P_{a} \sin \alpha - P_{d} \]

\[ -f_e + N \sin \alpha + N \cos \alpha = m \frac{d^2 \theta}{dt^2} \]

plus the torque balance around the contact point

\[ G_x A - G_y (d + A) - P_s H \cos \alpha - N \sin \alpha = 0 \]

\[ + f_1 (\frac{1}{2}P_t + \rho \sin \alpha) - f_2 (\frac{1}{2}P_t - \rho \sin \alpha) + \rho \sin \alpha + f_e \rho \sin \alpha \]

\[ - P_{d} - P_{a} \sin \alpha = 0 \]

implicitly determine the two guide forces and the normal force between roller and vane. These reaction values can be determined when the speed of rotation of the roller and thus the friction force between vane and roller are given through (15).

THE INFLUENCE OF THE NORMAL FORCE ON CRANK MOVEMENT

The normal force influences the crankshaft in both x and y axis directions with the components

\[ K_{nx} = N \sin \alpha \quad \text{and} \quad K_{ny} = -N \cos \alpha \]

which together give the braking moment

\[ M_n = -N \sin \alpha \]

Note that the torque in the first half of the rotation accelerates the crankshaft whilst it decelerates the rotation in the latter half.
THE INFLUENCE OF VANE-ROLLER FRICTION FORCE ON CRANKSHAFT MOVEMENT

Like the normal force, the vane-roller friction force influences the crankshaft with the projection forces
\[ K_{rx} = -F \cos \alpha \quad \text{and} \quad K_{ry} = -F \sin \alpha \quad (21) \]

whilst the torque on the crankshaft will be
\[ M_r = F \cos (\theta + \alpha) \quad (22) \]

Since, as mentioned earlier, the friction force changes sign, cf. (15), this torque must, of necessity, be of a somewhat discontinuous nature.

THE INFLUENCE OF ROLLER HEAD FRICTION ON THE CRANK

The friction of the rolling piston against the cylinder head due to rotation of the eccentric will with the introduction of the constant
\[ \Theta = \frac{2 \pi \mu d_0}{r} (R_2 - R_1) \quad (23) \]
give rise to the force
\[ K_{rx} = -\Theta \cos \alpha \quad \text{and} \quad K_{ry} = \Theta \sin \alpha \quad (24) \]

and the torque
\[ M_r = \Theta \epsilon \quad (25) \]

This torque contribution is independent of the free rotation of the roller in relation to the compressor.

THE CENTRIFUGAL FORCE OF THE ROLLER

The only force contribution now lacking in the sum of forces acting on the crankshaft at the eccentric is the centrifugal force from the roller
\[ K = \mu \omega r^2 \quad (26) \]
or
\[ K_{cx} = \mu \omega \sin \phi \quad \text{and} \quad K_{cy} = \mu \omega \cos \phi \quad (27) \]

separated in the two axis direction.

THE SUM OF FORCES ACTING ON THE CRANKSHAFT IN THE CYLINDER CHAMBERS

We can now plot the resultant of forces acting on the crankshaft in the centre plane of the cylinder chamber
\[ K = K_n + K_f - K_r + K_c \quad (28) \]

used here as the vector.

ROLLER-ECCENTRIC BEARING

The journal bearing between roller and eccentric is calculated dynamically by means of Dvorkin's infinitely Pi short bearing theory according to the mobility method outlined by J. F. Booker in [1] and [2]. In these papers it is demonstrated that the two independent variables - the relative eccentricity of the bearing and the attitude angle - can be expressed in two coupled ordinary differential equations
\[ \frac{d}{dt} \epsilon = N(t) \quad (29) \]

and
\[ \epsilon (t) = \frac{d}{dt} \phi - \omega = N(t) \quad (30) \]

The mobilities given here for the length-diameter ratio, equal to one, are fairly complicated, dimensionless algebraic functions in their two arguments which for reasons of space are not given at this point. When the force vector is known a priori, and thus the reciprocal time parameter is given
\[ \mu = \frac{4 \mu \epsilon^2}{\mu \epsilon} \quad (31) \]

the corresponding journal motion can be found. The name mobility was chosen certainly because it gives a good impression of the ratio between journal velocity and film load.

TORQUE IN ROLLER-ECCENTRIC BEARING

If the total crankshaft force from the cylinder chambers on the eccentric (28) is inserted in (31), Booker's equations (29) and (30) give the bearing eccentricity ratio of the roller-eccentric bearing. If this established value is again substituted in the steady state formula for bearing torque in accordance with the short bearing theory
\[ M_{\phi} = M_j = \frac{N(t)}{\epsilon} \quad (32) \]

we have at once both the torque on the roller from the crankshaft and vice-versa given as a function of the total crankshaft force.

ROLLER HEAD FRICTION TORQUE

Since the roller rotates around its own axis in relation to the cylinder head surfaces a torque of
\[ M_r = \frac{1}{r} (R_2 - R_1) \quad (33) \]

arises around the mentioned roller axis. This torque combined with (32) then
\[ R_1 = \frac{1}{3} \mu \epsilon + \epsilon \]
gives the designer a degree of freedom to reduce the rolling speed to a suitable level.
TORQUE EQUILIBRIUM OF THE ROLLER

In addition to the mentioned torques around the roller axis there is a third originating from the friction force at the vane

\[ \tau_F = Fr \]

(34)

Since this torque varies greatly in terms of value but was regarded as negligible in the computer program development phase compared with the other two almost constant torques, it was assessed that speed of rotation must be almost constant. Thus the average angular velocity is found from the torque equation

\[ \bar{\omega}_e = \bar{\tau}_r + \bar{\tau}_f \]

(35)

where the stroke denotes the average values of the time-integrated torques.

NUMERICAL SCHEME

Equations (1) to (14) and (23) to (27) can be solved explicitly as a function of time. Relations (15) to (22) on the other hand cannot be solved until a guess for angular frequency of the roller is given, after which the relative eccentricity of the roller-eccentric bearing can be found by applying (28) to (31). This knowledge renders (32) solvable whilst (33) and (34) were solvable with the knowledge gained from the above conjecture. Torque equation (35) is now over-determined. A new roller frequency is fixed and the iteration loop is complete.

SUBSTANTIAL SIZES OF THE THREE JOURNAL BEARINGS

The load on the roller-eccentric bearing is distributed on the two main bearings - 1 og 2 cf. plane statics

\[ K_{i,j} = -\frac{a_{i-j}}{a_{i+j}}K, \quad i \neq j \text{ and } i,j = 1,2 \]

(36)

since the crankshaft is, for the sake of ease, assumed to be completely balanced. Using the film loads just found the general complex of formulas (29) to (32) is calculated for all three journal bearings to determine the oil film thickness

\[ h = C(1-\epsilon) \]

(37)

and the energy consumption, which in the two outer main bearings constitutes

\[ D_i = \omega M_i, \quad i = 1,2 \]

(38)

whilst for the roller eccentric-bearing it is

\[ D_j = (\omega - \omega) M_j \]

(39)

Note that in formula (32) the roller frequency for the two main bearings is substituted as equal to zero, since no sleeve speed occurs. Hence the most important dissipation in the compressor is described.

DISSIPATION BETWEEN ROLLER AND CYLINDER HEADS

It is evident that the roller movement precipitated by the crankshaft dissipates an instantaneous power corresponding to the expression

\[ D_T = \alpha M_T \]

(40)

whilst it requires a little more thought to see that the corresponding contribution from the absolute movement of the roller in relation to the cylinder heads results in the part

\[ D_r = \omega M_r \]

(41)

in the total friction loss.

DISSIPATION BETWEEN ROLLER AND VANE

The formulas for dissipation between roller and vane are completely analogous with the corresponding expressions for friction loss between roller and cylinder heads. Therefore the total loss between roller and vane can logically be divided up into two expressions

\[ D_r = \alpha M_f \]

(42)

and

\[ D_T = \omega M_f \]

(43)

for the same reason as earlier. Note the simple sum

\[ D_r + D_v + D_T + D_f = \alpha (M_f + M_T + M_r + M_v) \]

(44)

which, conversely, does not indicate where the losses are dissipated. Equation (44) is naturally only valid as an average reflection.

DISSIPATION IN VANE SLOT

In the vane slot dissipation is based on an assumption of pure hydrodynamic friction losses both against the two cylinder head shapes

\[ D_e = 2\mu \frac{bl_v}{t_v} \left( \frac{dI}{dt} \right)^2 \]

(45)

and in the thrust bearings

\[ D_t = 2\mu \frac{h_l}{t_t} \left( \frac{dI}{dt} \right)^2 \]

(46)

This contribution is doubtless larger on account of the periodic metallic contact.
ENERGY CONSUMPTION IN THE TWO AXIAL BEARINGS

When the crankshaft is lying horizontally in the compressor, an almost unloaded end bearing is located on each side of the eccentric to position the crankshaft longitudinally. With the dimensionless parameter

\[ \zeta = e/R_0 \]

the energy consumption can be expressed in the integral

\[ D_c = \frac{\rho a^4}{2t_c} \left( \frac{1}{R_0} \right)_a \int \left( \cos \theta + \sqrt{1 - \zeta^2 \sin^2 \theta} \right)^2 d\theta - 2\pi R_0 \]  

which is easily solvable numerically.

INDICATED GAS POWER

The only contribution lacking in order to be able to describe the total power requirement is the indicated gas power

\[ P_i = \rho (M + N) e^{-t} \]

(48)

Determination of this value is not based on the model mentioned in this paper but on the more accurate refrigerant performance program outlined in reference (3). With this all important energy requiring mechanisms in the compressor should be accounted for.

COMPUTER SIMULATION

The computer model is verified by measurements on the test compressor mentioned in (3). The following is a supplement to the data given there:

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<tbody>
<tr>
<td>( D_1 ) =</td>
<td>( D_2 ) =</td>
<td>( D_3 ) =</td>
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<tr>
<td>2.1 W</td>
<td>1.0 W</td>
<td>2.2 W</td>
<td>0.6 W</td>
<td>2.5 W</td>
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<td>( D_6 ) =</td>
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</tr>
<tr>
<td>3.2 W</td>
<td>0.1 W</td>
<td>11.7 W</td>
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It should be noted that thrust bearing friction is calculated using Coulomb's model.

The total shaft power of the compressor is measured as 81 watt whilst gas power is calculated as 67 watt, cf. (3), so that the bearing losses should constitute approx. 14 watt which is roughly 2 watt more than the values calculated using the mentioned computer models. This difference will increase with a rise in condensing pressure, and it is therefore assumed that the deviation is due to underestimated gas mass flowing into the compression cylinder chamber of the model of performance not having been described.

The following is an extract of computer model plotting made as a function of angle of rotation:

1. The sum of \( M_g \) and \( M_n \) in Nm

2. The guide force \( G_a \) and \( G_b \) in N
3. Vane-roller friction force $F$ in N

4. The total dissipation $D_f + D_r$ between vane and roller in W

5. The trajectory for a peripheral point on the roller

6. The force $K$ on the roller-eccentric bearing in N

7. Predicted journal motion for roller-eccentric bearing

CONCLUSION

It will be seen that the model - despite the simplification with the constant speed of roller - describes the total friction loss satisfactory. Moreover, it gives an additional benefit in the form of the critical loads on the vane - the normal force and the two guide forces - plus the important minimum oil film thickness in the three radial bearings.
NOMENCLATURE

a_1 distance between main i and cylinder middle most
b vane thickness, equal to t
d distance between guide forces, equal to l_v
e roller eccentricity
f_i friction force in vane thrust bearing i
f_e friction force in vane cylinder-head bearing
h vane height
h_r roller height
h_f film thickness in journal bearing
k spring force
k_1 spring force in inner position
k_0 spring force in outer position
l distance between centre of cylinder and centre of curvature of the vane tip
l_v length of vane slot
m mass of vane
m_r mass of roller
n specific heat ratio of refrigerant gas
r roller radius
t thickness of vane, equal to b
t_c clearance of axial bearing
t_r clearance between roller and cylinder head
t_t clearance in vane thrust bearing
t_v clearance between roller and cylinder head

A_i Fourier coefficients, i=1-6
C clearance in journal bearing
D journal diameter
D_c dissipation in axial bearings
D_e dissipation in the vane end bearings
D_f dissipation of F due to crank movement
D_i dissipation in main nr. i
D_j dissipation in roller-eccentric bearing
D_k dissipation on roller ends due to crank movement
D_l dissipation in thrust bearings
D_p dissipation of F due to roller movement
D_q dissipation on roller ends due to roller movement
F roller-vane friction force
G_a vane guide force
G_b vane guide force
H cylinder height
K film load of roller-eccentric bearing
K_i film load of main nr. i
K_c centrifugal force of the roller
K_f vane-roller friction force on the crank
K_g gas force on the crank and on the roller
K_n roller normal force to the crank
K_T roller head friction force on the crank
L journal bearing length
M_f torque of F on the crank
M_g gas torque on the crank
M_j roller-eccentric torque on the crank
M_n torque on crank due to crank movement of roller
M_I journal mobility
M_f journal mobility
M_e roller-eccentric bearing torque on the roller
M_T torque of F on the roller
M_T torque on roller due to roller movement
N normal reaction between roller and vane
N time parameter
P pressure in the compression chamber
P_d discharge pressure, condensing pressure
P_p polytropic pressure
P_s suction pressure, evaporating pressure
P_i indicated gas power
R cylinder radius
R_1 inner radius of roller
R_2 outer radius of roller
R_1 inner radius of axial bearing
R_0 outer radius of axial bearing
V compression chamber volume
V_c clearance volume

a angle of vane-roller contact point
c relative eccentricity, equation (3)
c journal bearing eccentricity ratio
c relative eccentricity of axial bearing
o angle of vane-roller contact point, from y-axis
q position angle of suction port
h position length of vane-roller contact point
v film viscosity
w Coulombé friction coefficient
u radius of curvature of vane tip
t the time
p attitude angle of journal bearing
w angular velocity of roller
w average angular velocity of journal bearing and sleeve relative to K
w angular velocity of crankshaft
NOMENCLATURE FOR EXAMPLE ONLY

\( C_i \) journal clearance in main nr. i
\( C_j \) clearance in roller-eccentric bearing
\( D_i \) journal diameter in main nr. i
\( D_j \) diameter of roller-eccentric bearing
\( L_i \) bearing length of main nr. i
\( L_j \) bearing length of roller-eccentric

REFERENCES

