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THE IMPORTANCE OF WALL FRICTION IN THE COMpressible Flow of Gas Through A ComPressor VALve

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ABSTRACT
The life in service of a reciprocating compressor valve is determined largely by the number and violence of the impacts the valve makes with the stop and the seat. Attempts to determine the dynamic behaviour of compressor valves are hindered by the lack of detailed knowledge of how the gas force driving the valve varies as the valve moves. Earlier work by the authors on disc-shaped plates concentrically set over round ports [1] has shown the importance of the behaviour of the gas force at low values of valve lift. This fact combined with the lack of success [2] of models which do not take compressibility and wall friction into account led the authors to carry out the analysis presented here, which has the advantage that for radial flow in the gasket region one-dimensional duct flow can be used as a model.

THE EQUATIONS OF FLOW
The flow is assumed to be steady, one-dimensional, compressible adiabatic flow with area change, wall friction and an entrance loss as the gas moves from the port region into the gasket region. In the port region the cross-sectional area is relatively large and the velocity low. The gas is assumed to obey the perfect gas laws. A diametral cross-section of the valve is shown in Fig. 1 and a detail of the gasket region in Fig. 2.

Manipulating the equations derived from applying Newton's second law and the laws of thermodynamics and mass continuity to the infinitesimally small control volume shown in Fig. 3 leads to the following equations.

\[
\begin{align*}
\frac{\mathrm{d}p}{\rho} &= \frac{\gamma M^2}{1 - M^2} \cdot \frac{\mathrm{d}A}{A} - \frac{\gamma M^2}{2A} \left[ 1 + (\gamma - 1) \frac{M^2}{2} \right] \frac{C_f s}{2A} \frac{\mathrm{d}r}{r} \\
\frac{\mathrm{d}p}{\rho} &= \frac{M^2}{1 - M^2} \cdot \frac{\mathrm{d}A}{A} - \frac{\gamma M^2}{2A} \frac{C_f s}{2A} \frac{\mathrm{d}r}{r} \\
\frac{\mathrm{d}M}{\rho} &= \frac{[1 + (\gamma - 1) \frac{M^2}{2}]}{1 - M^2} \cdot \frac{\mathrm{d}A}{A} + \frac{\gamma M^2 [1 + (\gamma - 1) \frac{M^2}{2}]}{2A} \frac{C_f s}{2A} \frac{\mathrm{d}r}{r}
\end{align*}
\]
The coefficient of wall friction, $C_f$, is defined as:

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho V^2}$$

where $\tau_w$ = shear stress in fluid at wall
$\rho$ = fluid density
$V$ = fluid velocity
$P$ = pressure
$M$ = Mach number
$\gamma$ = ratio of specific heats $C_p/C_v$
$A$ = cross-sectional area of flow
$s$ = wetted perimeter

Outward radial flow in the gasket region of the valve is very like flow in a wide, low, gently diverging rectangular duct whose inlet and outlet cross-sectional areas are $2\pi rh$ and $2\pi r' h$ respectively (Fig. 2). The "duct" in this case has no sides, but the behaviour of the fluid in ducts where the width to height ratio is greater than about seven, should be very similar. A huge amount of experimental data for ducts of this kind is available and has been summarised by R.B. Dean [3]. Dean's summary gives the following relationships for the coefficient of wall friction $C_f$:

$$C_f = \begin{cases} \frac{12}{Re} & 0 < Re < 1300 \\ \frac{0.073}{Re^{0.25}} & 1300 < Re < 60000 \\ 0.002 & Re > 60000 \end{cases}$$

where $Re = \frac{\rho V h}{\mu}$ and $\rho$ = local fluid density
$h$ = valve lift
$V = M \times V_a$ = local fluid velocity
$\mu$ = average fluid viscosity

$$V_a = \sqrt{\gamma R T}$$ = the local acoustic velocity

and $T$ = local temperature (from $\frac{p}{\rho T} = \text{a constant}$)

The loss due to the sudden area change from the port region to the first stage of the gasket region may be calculated in the following manner:

$$P_1 - P_0 = \frac{1}{2} \rho \frac{V^2}{\gamma}$$

The loss coefficient $\xi$ may be obtained from reference [4].

For flow in the gasket region:

$$A = 2\pi rh \quad \text{and} \quad s = 2 \cdot 2\pi r$$

thus $\frac{dA}{A} = \frac{dr}{r}$ and $\frac{s}{2A} = \frac{1}{h}$

So that equations (1) become:

$$\frac{dp}{dr} = \frac{P}{r} \left[ 1 - \frac{1}{2\pi rh} \right] \frac{C_f \gamma M^2}{h} \frac{1}{1 - M^2}$$

$$\frac{dp}{dr} = \frac{P}{r} \left[ 1 - \frac{C_f \gamma M^2}{h} \right] \frac{1}{1 - M^2}$$

$$\frac{dm}{dr} = M \left[ 1 + \frac{(Y-1) M^2}{2} \right] \frac{C_f - \frac{1}{2}}{h} \frac{1}{r - 1 - M^2}$$

SOLVING THE FLOW EQUATIONS

Equations (3) relate the Mach number change ($\Delta M$), pressure change ($\Delta P$) and density change ($\Delta \rho$) from one station in the gasket region to the next. So for a trial value of Mach number at station 1 and known port pressure and density, the pressure at exit (station 11) can be computed. Clearly, the outlet values from one station are the inlet values to the next station.

A computer program was written to evaluate the pressure at station 11 for a series of incremented trial values of inlet Mach numbers $M_1$. The coefficient of friction at each station was determined by equations (2).

When the pressure at station 11 arrived at a value within 5% of atmospheric pressure, the process was stopped and the integral $p \, da$ evaluated. This value was added to port pressure x port area. The value atmospheric pressure x valve plate area was then subtracted to give the net force exerted by the gas on the valve plate.

A four-step Runge-Kutta method was used in the computation.

The process was carried out for three values of port pressure above atmosphere and a range of valve lifts. The resulting values of gas force are shown graphed in Fig. 4 with measured values added for comparison. The working fluid was air.

DISCUSSION AND CONCLUSIONS

Fig. 4 shows a plot of the calculated values of gas force compared with measured values [5]. As can be seen, the agreement is fair and the curves are of the same form everywhere, except in the interesting region where the values of valve lift are very low where the Reynolds numbers are very low indeed. The most likely explanation of this lack of agreement is that the expression for the coefficient of wall friction, $C_f = \frac{12}{Re}$, does not hold at these very low values of Reynolds numbers, indicating that the flow in this very narrow gap is very sensitive to wall friction.

When the wall friction factor is set to zero, the mathematical model gives nonsensical results everywhere in the range covered by Fig. 4; the results bear no relation at all to measured values, i.e. arithmetical operations are being carried out which are not connected with physical reality. This establishes the importance of modelling wall
friction adequately at low values of valve lift. At higher values of valve lift the effect of wall friction will be less important, but more difficult to model since the one-dimensional duct flow model becomes inadequate, as computations and measurements have shown [5].

The critical parameter is the aspect ratio: the radial flow path length in the gasket region to the valve lift, i.e. the "duct" length to height ratio. The maximum aspect ratio used to calculate the values shown in Fig. 4 was about 17. When the aspect ratio falls much below this value, the agreement between the measured values and the predicted values of gas force becomes considerably poorer.

<table>
<thead>
<tr>
<th>Plate Diameter</th>
<th>Port Diameter</th>
</tr>
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<tbody>
<tr>
<td>16.8 mm</td>
<td>12.7 mm</td>
</tr>
</tbody>
</table>

\[ \Delta p = 62.0 \text{ kN/m}^2 \]

\[ \Delta p = 41.3 \text{ kN/m}^2 \]

\[ \Delta p = 20.7 \text{ kN/m}^2 \]

FIG. 4 GAS FORCE v VALVE DISPLACEMENT

REFERENCES


