Components on Blade Excited Rigid Body Vibrations of Rotary Vane Compressors

V. Yee
W. Soedel

Follow this and additional works at: https://docs.lib.purdue.edu/icec

https://docs.lib.purdue.edu/icec/339

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/Herrick/Events/orderlit.html
COMMENTS ON BLADE EXCITED RIGID BODY VIBRATIONS OF ROTARY VANE COMPRESSORS

V. Yee, Graduate Research Assistant, and W. Soedel, Professor of Mechanical Engineering, Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University

ABSTRACT

Without loss of generality, but illustrated on a single degree of freedom rigid body vibration model, the excitation action of the rotating blades of rotary vane compressors or motors is investigated for the case of a cylindrical bore. Significant findings are that four blade compressors and multiples of four are excitation free, while odd numbers of blades produce the highest harmonic content in the excitation function.

INTRODUCTION

From a vibration and noise control viewpoint, it is important to study the excitation mechanism of a rotary vane compressor or motor. While similar studies may have been carried out in the past, results of such studies in the open literature have never appeared, or, at least, could not be found by the authors. This is the justification for this technical note.

This paper describes an application of the classical problem of rotating unbalance to rotary sliding-vane compressors. Figure 1 shows a simplified model of a rotary compressor with a single vane.

Examining the figure, one can intuitively see the analogy between this problem and the classical rotating unbalance problem. The primary difference lies in the fact that in the situation pictured the length of the eccentricity arm varies with time. In the following paragraphs (1) the equation of motion of the model shown in Figure 1 will be derived from basic principles, (2) in a similar manner the equation of motion for the diametrically opposite two vane case will be derived from basic principles, (3) subsequently, it will be shown that the equation of motion for the double vane situation can be easily obtained from the single vane case and can be extended in general to an arbitrary number of vanes at various angles with respect to the initial blade, and finally (4) an interesting result for the four vane case and multiples of four will be presented.

EQUATION OF MOTION SINGLE-VANE CASE

Shown in Figure 2 are the primary geometric points of interest for the single vane case.

Our first task is to form the position vector of the center of mass (G1) of the vane with respect to the fixed coordinate system OXYZ.

\[ \mathbf{r}_{G1/0} = \mathbf{r}_{CR/0} + \mathbf{r}_{G1/CR} \]  (1)

The position vector, \( \mathbf{r}_{G1/CR} \), is formed as follows:

\[ \mathbf{r}_{G1/CR} = d \cos \omega t \mathbf{i} + d \sin \omega t \mathbf{j} \]  (2)

where \( d = L-a \).
An expression for \( L \) which is valid for all time must be obtained. It is readily recognized that \((CB, CR, l)\) form a triangle. Therefore, applying the law of cosines to this triangle we have

\[
R^2 = (R-r)^2 + L^2 - 2L(R-r)\cos(90-\omega t)
\]

Making the trigonometric substitution

\[
\cos(90-\omega t) = \sin \omega t
\]

and rearranging, we obtain

\[
L^2 - 2(R-r)\sin \omega t]L + [r(r-2R)] = 0 \tag{3}
\]

We solve for \( L \) by applying the quadratic equation and obtain

\[
L = (R-r)\sin \omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \tag{4}
\]

where the positive value of the radical was chosen since \( L \) must be positive. Therefore,

\[
d = L - a = (R-r)\sin \omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} - a \tag{5}
\]

Substituting (5) into (2) and making a trigonometric substitution, we have

\[
\vec{r}_{GL/CR} = \frac{1}{2} (R-r)\sin 2\omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t - a \cos \omega t \vec{i} + \{(R-r)\sin^2 \omega t + \\
\sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \sin \omega t - \\
asin \omega t \vec{j}\}
\]

Let

\[
A(t) = \frac{1}{2} (R-r)\sin 2\omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t - a \cos \omega t
\]

Therefore,

\[
\vec{r}_{GL/CR} = A(t)\vec{i} + \{(R-r)\sin^2 \omega t + \\
\sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \sin \omega t - \\
asin \omega t \vec{j}\}
\]

The position vector, \( \vec{r}_{CR/O} \), is

\[
\vec{r}_{CR/O} = [\Delta st + (Y+K)]
\]

where

\[ \Delta st \] - location of center of mass of housing and rotor combination at static equilibrium

\[ Y \] - displacement of center mass of housing and rotor combination from static equilibrium

\[ K \] - a constant distance from housing - rotor mass center to CR

Substituting (6) and (7) in (1) and combining similar unit vector terms, we have

\[
\vec{r}_{GL/0} = A(t)[Y+K] + [(R-r)\sin \omega t + \\
\sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \sin \omega t - \\
asin \omega t]
\]

and

\[
\vec{r}_{CR/O} = \Delta st + K = \text{constant.}
\]

Taking a dynamic free-body diagram of the entire system of housing, rotor, and blade in Figure 3,
\[-kY-c\ddot{Y} = (M-m)\ddot{Y} + m \frac{d^2}{dt^2} \left[ Y + k_1 + (R-r)\sin^2 \omega t \right] + \sigma(R-r)\sin \omega t \]

\[
\sqrt{[(R-r)\sin \omega t]^2 - r(r-2R) \sin \omega t - \sigma \sin \omega t}
\]

Rearranging and performing the differentiation for the first two terms, we obtain

\[
MY + CY + kY = m \frac{d^2}{dt^2} \left[ \sigma \sin \omega t - (R-r)\sin^2 \omega t \right]
\]

Equation (10) is the governing differential equation of motion for the system. If we carry out the indicated differentiation, we obtain

\[
MY + CY + kY = m \frac{d^2}{dt^2} \left[ \sigma \sin \omega t - (R-r)\sin^2 \omega t \right]
\]

The position vectors of the centers of mass of the two vanes with respect to the center of the rotor are

\[
\begin{align*}
\mathbf{r}_{G1/CR} &= (R-r)\sin \omega t \cos \omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t + a \cos \omega t \mathbf{i} \\
\mathbf{r}_{G2/CR} &= -(R-r)\sin \omega t \cos \omega t - \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t + a \cos \omega t \mathbf{i}
\end{align*}
\]

At this point we attempt to find the position vector of the center of mass of the combined system of two vanes. Recalling

Following the same procedure as before we obtain for \(d_1\) and \(d_2\)

\[
\begin{align*}
d_1 &= (R-r)\sin \omega t \cos \omega t + \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t + a \cos \omega t \\
d_2 &= -(R-r)\sin \omega t \cos \omega t - \sqrt{[(R-r)\sin \omega t]^2 - r(r-2R)} \cos \omega t + a \cos \omega t
\end{align*}
\]
from basic principles the definition of the
center of mass of a system of particles
\[\sum_{i=1}^{N} m_i \frac{\mathbf{r}_i}{m_i} = \frac{\mathbf{r}_G}{m} \]
(15)
For this particular case with \(m_1 = m_2 = m\), we
have
\[2m \frac{\mathbf{r}_G}{CR} = \frac{\mathbf{r}_{G1}}{CR_1} + \frac{\mathbf{r}_{G2}}{CR_2} \]
(16)
Substituting (14) in (16) results in
\[\frac{\mathbf{r}_G}{CR} = \frac{\mathbf{r}_{G1}/CR + \mathbf{r}_{G2}/CR}{2} \]
(17)
Therefore, the position vector of the center
of mass of the system of two vanes with re­
spect to the fixed reference \(OXYZ\) is
\[\frac{\mathbf{r}_G}{O} = \frac{\mathbf{r}_{CR}/O + \mathbf{r}_G/CR}{2} \]
(18)
Applying Newton's 2nd Law to the system of
housing rotor and vanes shown in Figure 5

![Dynamic Free-Body Diagram, Double-Vane Case](image)

we have for the \(Y\) direction
\[M\ddot{y} + C\dot{y} + kY = -2m \frac{d^2}{dt^2} [(R-r)\sin^2 \omega t] \]
(19)
Performing the indicated differentiation we
have for the governing differential equation of
motion
\[M\ddot{y} + C\dot{y} + kY = -4m\omega^2 (R-r)\cos 2\omega t \]
(20)
The well known, but still interesting result
is that in the two vane case only the
second harmonic of the rotation frequency
excites the compressor.

DERIVATION OF DOUBLE-VANE EQUATION OF
MOTION FROM SINGLE-VANE CASE

Let us now obtain the governing Equation
(19) for the double-vane case from Equation
(10), the governing equation for the single
vane case. To be more specific, we will
attempt to obtain Equation (19) by adding
to the right hand side of (10) a second
mass with angular position increased by \(\pi\).
We obtain
\[M\ddot{y} + C\dot{y} + kY = \frac{m}{d^2} [\sin \omega t - (R-r)\sin^2 \omega t]
- \sqrt{[(R-r)\sin \omega t]^2 - (R-2R) \sin \omega t} \]
(21a)
\[M\ddot{y} + C\dot{y} + kY = \frac{m}{d^2} [\sin(\omega t + \pi) - (R-r)\sin^2 (\omega t + \pi)]
- \sqrt{[(R-r)\sin(\omega t + \pi)]^2 - (R-2R) \sin \omega t} \]
(21b)
Making the trigonometric substitution
\[\sin(\omega t + \pi) = -\sin \omega t \]
(21b) reduces to
\[M\ddot{y} + C\dot{y} + kY = -2m \frac{d^2}{dt^2} [(R-r)\sin^2 \omega t] \]
(19)
This is the same equation as obtained by
application of basic principles. Through
the same process (20) can be obtained from
(11). If this process is continued for
higher number of vanes, one would find that
the process described is totally general
for extension to an arbitrary number of
vanes at various angles with respect to the
initial vane.

EQUATION OF MOTION FOUR-VANE CASE

A very interesting situation occurs when
we consider the four vane compressor shown
in Figure 6.

Applying the superposition technique de­
scribed previously we obtain
Figure 6. Simplified Model Four-Vane Rotary Sliding-Vane Compressor

\[
\ddot{Y} + CY + kY = m \frac{d^2}{dt^2} \left[ a \sin(wt) - (R-r) \sin^2(wt) \right] \\
- \sqrt{[(R-r) \sin(wt)]^2 - r(r-2R) \sin(wt)} \\
+ m \frac{d^2}{dt^2} \left[ a \sin(wt+\pi) - (R-r) \sin^2(wt+\pi) \right] \\
- \sqrt{[(R-r) \sin(wt+\pi)]^2 - r(r-2R) \sin(wt+\pi)} \\
+ m \frac{d^2}{dt^2} \left[ a \sin(wt+3\pi) - (R-r) \sin^2(wt+3\pi) \right] \\
- \sqrt{[(R-r) \sin(wt+3\pi)]^2 - r(r-R) \sin(wt+3\pi)} \right] \tag{22}
\]

Using the following trigonometric substitution:

\[
\sin(wt+\pi/2) = \cos wt \\
\sin(wt+\pi) = -\sin wt \\
\sin(wt+3\pi/2) = -\cos wt
\]

Equation (22) reduces to

\[
\ddot{Y} + CY + kY = m \frac{d^2}{dt^2} \left[ -2(R-r) \right] - 4m \frac{d^2}{dt^2} \left[ \frac{1}{2}(R-r) \right] \\
or
\ddot{Y} + CY + kY = 0 \tag{23}
\]

The preceding equation indicates that for the four-vane case no rotating unbalance due to the vanes exists. To understand why this situation occurs one must return to basic principles and form the position vector of the center of mass of the system of four vanes with respect to the center of the rotor. If this is done, the resulting position vector is found to be

\[
\frac{r_G}{CR} = \frac{1}{2}(R-r) \tag{24}
\]

This implies that for the four vane case the location of the center of mass of the system of four vanes is midway between the center of the rotor (CR) and the geometric center of the cylinder bore (CB) for all time. This is shown in Figure 7.

Figure 7. Dynamic Free-Body Diagram

DISCUSSION AND CONCLUSION

When discussing the excitation of a rotary vane compressor or motor by its blades, it should be remembered that this is often only a secondary effect, especially as the number of vanes increases. The primary effect may very well be an unbalance of the compressor and electromotor rotor, which of course always occurs at the frequency corresponding to the rotational speed. However, the primary unbalance of a rotary vane compressor can be completely eliminated, in contrast to the reciprocating compressor, where this is economically impossible since it would require a gearing system. In this case, all that is left is the blade unbalance, and one is justified to take a closer look at it.

A second point is that this paper confined itself to equally spaced blades of equal mass. In the real world manufacturing
tolerances interfere. Thus, one would suspect that a careful Fourier analysis of experimental data taken from, for instance, a four bladed compressor would still reveal small response spikes at frequencies where there should be none, however, this does not invalidate the general trend predicted by the theory.

In conclusion, the paper has presented the vibration excitation function for rotary vane compressors of any number of blades, but has discussed mainly the single, double and quadruple blade cases. The approach is valid for equally spaced blades, but, of course, also for unequal spacing, since the excitation function is generated by a superposition of single blade cases. While it might be of interest to repeat this analysis for the case where the compressor is allowed to move in three or even six degrees of freedom, the single degree of freedom case is sufficient to illustrate how the number and spacing of blades affects the vibration excitation.