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An Approach to Finding Cyclic Covers in Undirected Graphs Without 2-Cycles

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Understanding ways to interconvert similar optimization problems plays an important role in developing algorithms to solve complex problems. Problems derived from areas as diverse as network flow optimization, protein folding, and DNA sequencing are combinatorial, meaning that solving them can grow in complexity exponentially with problem size. A well-known problem that can require exponential time to solve in the worst case is the symmetric traveling salesman problem. This problem relies on solutions to simpler problems, such as finding a cyclic cover or a set of edges that link nodes in a graph. The relaxed problem consists of undirected edges that can then be reexpressed, or transformed, into directed arc representations, and then solved using algorithms designed for directed arcs. However, applying directed arc algorithms to undirected problems tends to result in the formation of 2-cycles, or pairs of edges that loop between only 2 nodes. This project used an assignment problem formulation with an additional constraint to find a cover in an undirected graph, while avoiding 2-cycles. Using Excel, undirected problems were generated and then translated to a directed problem with symmetric edge weights. Then OpenSolver, a linear programming solver, was applied to answer the modified assignment problem to form edges. The proposed method found a cover in problems up to 400 nodes without 2-cycles in every case, with average solution time scaling as $O(n^4)$. While this simple approach is beneficial for avoiding 2-cycles, additional techniques will be required to obtain a solution to the symmetric traveling salesman problem, which in terms of our work requires forming a cover with a single Hamiltonian cycle across all nodes (no 3-cycles, 4-cycles, etc.).

Research advisor Joseph Pekny writes, "Many of the most important decision problems in science, information technology, and engineering are complex in the sense that the only means of solving them exactly scale poorly for instances of realistic sizes. Algorithm engineering techniques for solving these problems often involve transforming the original problem to a similar form to take advantage of existing methods that can form the basis of an effective approach. Exposure of students to methods for solving complex problems leads to ideas for new approaches and makes them aware of this exciting area for graduate school."

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