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Evaluation of the operating internal resistance, inductance, and capacitance of intact damped sine wave defibrillators

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Evaluation of the operating internal resistance, inductance, and capacitance of intact damped sine wave defibrillators

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Abstract

A method is developed for determining actual values of circuit elements in a damped sine wave (Lown waveform) defibrillator, solely from measurements of the output, using two or more power resistors and a storage oscilloscope. If a defibrillator containing capacitance, C, inductance, L, and internal resistance, R_i, is discharged into increasing 5- to 100-ohm resistive loads, R, it is shown for underdamped output waveforms that $\hat{a} = R_i/2L + R/2L$ and $\hat{c} = CR_i + CR$, where $\hat{a} = \pi/[t_2 \tan(\pi t_1/t_2)]$, $\hat{c} = 2\hat{a}/[\hat{a}^2 + (\pi/t_2)^2]$, t_1 = time from onset to peak, and t_2 = time from onset to first zero crossing of the output waveform on the oscilloscope trace. Linear plots of $\hat{a}$ vs. R are constructed for seven defibrillators, and values of R_i and L computed as intercept/slope and 1/(2 slope) respectively. C is given by the slope of a linear plot of $\hat{c}$ vs. R. Delivered energy is accurately predicted as stored energy × $R/(R_i + R)$.

Key words: defibrillator; delivered energy; internal capacitance; internal inductance; internal resistance; stored energy

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Introduction

Damped sine wave defibrillators contain a capacitor (10-100 microfarads) and an inductor (10-200 millihenrys), which are switched in series with paddle electrodes applied to the subject. During a cardiac defibrillation attempt, the capacitor, C, is charged to voltage V_0 (100-5000 volts), and a switching device allows current to flow through the inductor, L, the internal resistance of the defibrillator, R_i, and the subject resistance R. The current flow lasts only a few milliseconds and may be represented as either an overdamped or underdamped sinusoid as shown in Fig. 1.

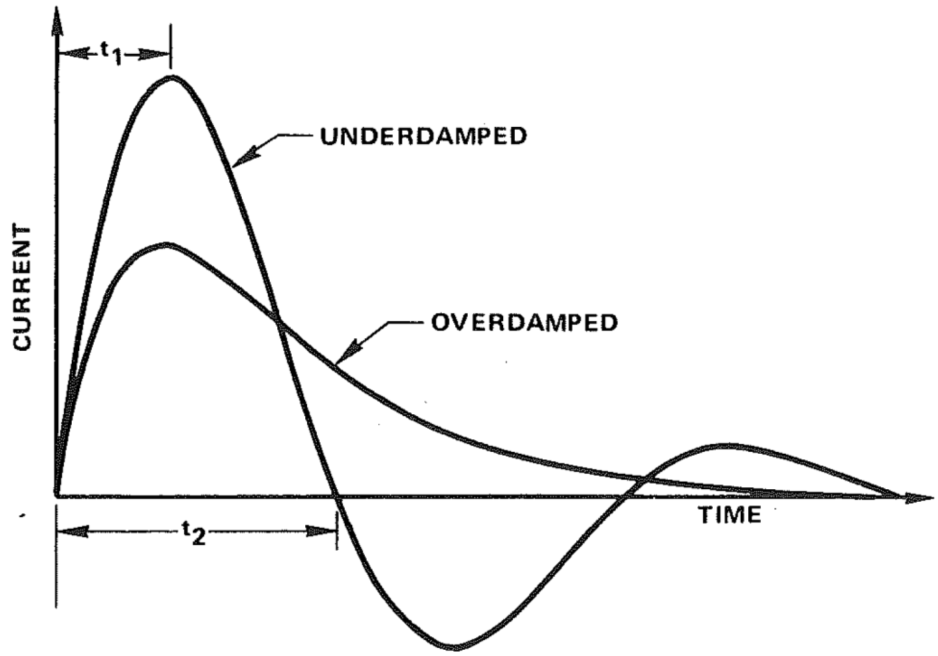

Fig. 1. Damped sinusoidal current waveforms.

The measurement of the operating R, L, and C of damped sine wave defibrillators would yield useful information that is not routinely provided by manufacturers, and that generally cannot be obtained by simple inspection of internal components. Knowledge of the internal resistance of the defibrillator would allow determination of delivered energy, given the dial setting in watt-seconds (joules) and the subject resistance.

If the peak voltage across the subject and the peak current through the subject are measured experimentally, the energy and charge actually delivered may be calculated when the defibrillator constants R_i, L, and C are known. Reporting of accurate R_i, L, and C values would provide a complete and concise description of defibrillators used in experimental studies, and would allow exact reconstruction of defibrillating stimulus waveforms, thus aiding the comparison of results from diverse laboratories. This paper describes a method whereby the operating values of internal resistance, inductance, and capacitance of a damped sine wave defibrillator may be determined entirely from measurements of its output, using only a storage oscilloscope and two or more power resistors. The defibrillator need not be dismantled for testing or removed from its normal site of use.

Theory

Application of Kirchhoff's voltage law to the output circuit of a damped sine wave defibrillator gives

$$L(di/dt) + (R_i + R)i + 1/C \int idt = 0,$$

from which one obtains the second-order differential equation for the damped sine wave defibrillator,

$$L(d^2i/dt^2) + (R_i + R)(di/dt) + (1/C)i = 0,$$

where

i = the instantaneous current and t = time.

Of the three real solutions to this equation, only one need be considered in this context, namely,

$$i = I \sin(bt)e^{-at},$$

where

$$a = (R_i + R)/2L,$$
$$b = \sqrt{1/LC - [(R_i + R)/2L]^2},$$
$$i = [(a^2 + b^2)/b]CV_o,$$

and

$$[(R_i + R)/2L]^2 < 1/LC$$

for the initial condition i = 0 at t = 0.

The condition $[(R_i + R)/2L]^2 < 1/LC$ specifies underdamped sinusoidal waveforms most often obtained in clinical practice (see Fig. 1). If $[(R_i + R)/2L]^2 > 1/LC$, the waveform is overdamped, and if $[(R_i + R)/2L]^2 = 1/LC$, the waveform is said to be critically damped (6). In our experience, underdamped waveforms can always be achieved in defibrillator testing by using sufficiently small values of R.

The constant $\hat{b}$ may be estimated experimentally as

$$\hat{b} = \pi / t_2,$$

t_2 being measured on an oscilloscope from the onset of the output pulse to the first zero crossing (Fig. 1). The constant $\hat{a}$ may be estimated from measurements of both the time to the first zero crossing t_2, and the time to the current peak t_1, using the expression

$$\hat{a} = \hat{b}/\tan(\hat{b}t_1) = (\pi/t_2)/\tan[\pi(t_1/t_2)].$$

This expression is easily obtained from the condition (di/dt) = 0 at t = 0.

Thus numerical values of the parameters a and b may be estimated from the measured time intervals t_1 and t_2. For clarity, such estimates of a and b calculated from empirical data are denoted $\hat{a}$ and $\hat{b}$. The values of a obtained in a given experiment are dependent upon the magnitude of the resistive load as indicated by the defining relationship,

$$a = (R_i + R)/2L = R_i/2L + 1/2L \cdot R.$$

If the defibrillator is discharged into differing known resistive loads (beginning with small values of resistance to ensure that the condition $[(R_i + R)/2L]^2 < 1/LC$ is satisfied), then a plot of the calculated values $\hat{a}$ as a function of R will be a straight line with slope 1/(2L) and intercept R/(2L). From this plot the theoretical values R_i and L may be calculated as L = 1/(2 slope) and R_i = intercept/slope.

A similar technique may be used to calculate the defibrillator capacitance, C. It is easy to show by rearrangement of the defining equations,

$$a = (R_i + R)/2L,$$

and

$$b = \sqrt{1/LC - a^2},$$

that the hybrid parameter, c, may be defined as

$$c = 2a/(a^2+b^2)$$

Hence, if the measured values $\hat{c}$ are calculated from the measured time intervals t_1 and t_2, and plotted as a function of R, the resultant straight line will have slope C and intercept CR_i. The defibrillator capacitance will be given by the slope and a second estimate of the internal resistance will be given by the ratio, intercept/ slope.

Experimental Method

Seven commercial defibrillators were tested. The defibrillators were discharged into a variable resistor box containing 100 Ω, wire-wound resistors having an inductance of approximately 0.1 mH per 100 Ω resistance. Resistance values R of 5, 10, 15, 25, 35, 50, 75, and 100 Ω, ± 1 percent, were available. Voltage across the resistive load was measured using a single-beam storage oscilloscope (model D-11, Tektronix, Inc., Beaverton, OR). The time base of the oscilloscope was calibrated using a crystal controlled sweep generator (model 183, Wavetek, Inc., Beech Grove, IN).

The defibrillators were charged to a dial setting of 100 W-sec (joules) and discharged into successively higher resistive loads, beginning with 5 Ω. The values of t_1 and $t_2$were read from the oscilloscope trace, together with the peak voltage, e_1, delivered to the load. The values of $\hat{a}$, $\hat{c}$, and the slopes and intercepts of the linear regression functions best relating $\hat{a}$ and $\hat{c}$ to the known values of R were computed with a small programmable calculator. Delivered energy was calculated as described by Flynn et al. (4).

To corroborate the values obtained using the method proposed in this paper, the cabinets of the defibrillators were opened and whenever possible the rated values of capacitors and inductors were noted. When unobtainable by inspection, the rated values of the internal components were requested from the manufacturers. Details of the test procedure and sample calculations are presented in Fig. 2.

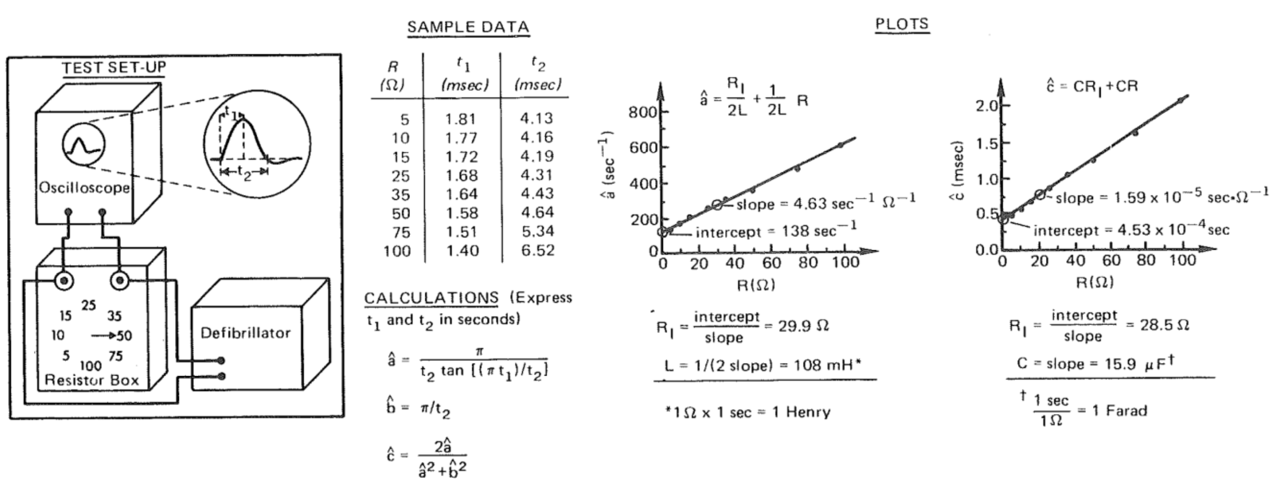

Fig. 2. Test procedure and sample calculations for determining defibrillator capacitance, inductance, and internal resistance.

Results

As expected from theory, the plots of $\hat{a}$ vs. R and $\hat{c}$ vs. R are linear within the limits of experimental error. Values obtained from the slopes and intercepts of such functions for seven defibrillators are shown in Table 1. The table also indicates the rated values of the capacitive and inductive circuit elements. In only one of seven defibrillators could both rated values be ascertained by direct inspection of the internal components.

Table 1. Summary of measured values of internal resistance, R_i, inductance, L, and capacitance, C, of Lown waveform defibrillators compared to rated values.

Defibrillator	*R_i from $\hat{a}$/from $\hat{c}$ (Ω)*	*L (mH)*	*C (μF)*	*Rated L (mH)*	*Rated C (μF)*
American Optical* dc Defibrillator					
Early Model	18.4/18.0	99.0	17.9	100	16
Model 670, #1	28.9/28.5	108.0	15.9	100	16
Model 670, #2	31.9/30.4	111.9	15.8	100	16
Model 10760R	33.1/31.5	107.1	16.0	100	16
Model 262012	15.3/17.0	99.4	16.7	100	16
Burdick† Model DC/150	32.3/32.2	91.1	15.4	100	16
Physio-Control‡ Model 092-00	12.0/11.5	43.8	36	42	35

*Framingham MA.
†Milton WI.
‡Redmond WA.

The internal resistance of defibrillators accounts for the fact that delivered energy is less than the stored energy (indicated by dial setting values in older units). Table 2 indicates the correspondence of the fraction of stored energy delivered by the defibrillators tested and the ratio, $R/(R_i + R)$, for the case R = 50 Ω.

Table 2. Correspondence of the fraction of stored energy actually delivered by Lown waveform defibrillators to the ratio $R/(R_i+R)$.

Defibrillator	$\frac{R}{R_I+R}$	$\frac{E_{delivered}}{E_{stored}}$*
American Optical		
dc Defibrillator Early Model	0.73	0.74
Model 670, #1	0.64	0.69
Model 670, #2	0.62	0.65
Model 10760R	0.61	0.65
Model 262012	0.77	0.80
Burdick Model DC/150	0.66	0.66
Physio-Control† Model 092-00	0.75	0.76

R = 50 Ω.
R_I = the average of the two values of internal resistance obtained.
*Corrected for meter error.
†This unit only was tested using load R at 35 Ω rather than 50 Ω.

Discussion

The technique reported in this paper provides a method for accurate measurement of the internal resistance, inductance, and capacitance of intact defibrillators under field conditions. Many defibrillators presently in use deliver significantly less than the indicated energy (1, 3) because of their internal resistances. Although the meters of newer units are calibrated in terms of energy delivered into a 50-Ω test load, the actual delivered energy is load-dependent. Delivered energy is given by the product of stored energy and the expression, $R/(R_i + R)$ and may vary from subject to subject. The ratio of delivered energy to stored energy most certainly varies from internal (open chest) to external (closed chest) applications. Accordingly, better estimates of delivered energy may be obtained with knowledge of operating internal resistance.

The technique described in this paper permits investigators to characterize damped sine wave defibrillators used in experimental studies. All too often, reports of defibrillation studies do not allow the reader to reconstruct the waveform of the stimulus applied to experimental subjects. Since there is no agreement at present as to the essential property of the electrical stimulus that causes defibrillation (2, 5), the lack of exact descriptions of the voltage and current waveforms employed in experimental studies seriously compromises the future utility and comparability of much hard-won experimental data. If the defibrillation circuit constants R_i, L, and C are reported together with the peak voltage, e_1, and peak current, i_1, applied to the subject, the exact current waveform, as well as delivered energy and delivered charge, may be calculated at any time by a computer program based on the relationship $a = (R_i + e_1/i_1)/(2L)$ and those in Table 3.

Table 3. Computational formulas for analysis of damped sine wave defibrillator output, given reported values R_i, L, and C, for the defibrillator; peak voltage e_1;and peak current i_1 for a particular trial.

	Underdamped Case if $a^2 < 1/LC$, $b = \sqrt{1/LC - a^2}$	Overdamped Case if $a^2 > 1/LC$, $b = \sqrt{a^2 - 1/LC}$
Instantaneous Current i(t), Time t, from Onset of Pulse	$i(t) = I \sin(bt) e^{-at}$ $I = i_1 \frac{\sqrt{a^2+b^2}}{b} \exp\left(\frac{a}{b} \tan^{-1} \frac{b}{a}\right)$	$i(t) = I(e^{-(a-b)t} - e^{-(a+b)t})$ $I = i_1 \left[\frac{a+b}{a-b} - 1\right]^{-1} \left(\frac{a+b}{a-b}\right)^{(a+b)/(2b)}$
Delivered Energy W	$W = \frac{e_1 i_1}{4a} \exp\left(\frac{2a}{b} \tan^{-1} \frac{b}{a}\right)$	$W = \frac{e_1 i_1}{4a} \exp\left(\frac{a}{b} \ln \frac{a+b}{a-b}\right)$
Delivered Net Charge Q	$Q = \frac{i_1 \exp\left(\frac{a}{b} \tan^{-1} \frac{b}{a}\right)}{\sqrt{a^2+b^2}}$	$Q = \frac{i_1 \exp\left(\frac{a}{2b} \ln \frac{a+b}{a-b}\right)}{\sqrt{a^2+b^2}}$

$a = (R_I + e_1/i_1)/2L.$

Finally, the technique reported in this paper may offer industry an additional tool for quality control and field testing of this important species of life-saving emergency equipment.

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