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A Transfer Matrix Method for Estimating the Dispersion and Attenuation of Plane Waves in a Standing Wave Tube

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An improved transfer matrix approach for estimating the sound speed and attenuation constant of air in the tube

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Abstract

In this paper, an iterative method based on the transfer matrix approach was developed for evaluating sound speed and attenuation constant of air. If the air inside the impedance tube is treated as the tested material, such as porous materials, the transfer matrix approach can be easily used to identify the air's acoustic properties. Then an iterative method can be applied in the post-processing to estimate the complex air wave numbers accurately. The result from the experiments showed that the air temperature inside the tube was a little higher than the ambient temperature and that the corrected Temkin formula gave the best theoretical prediction of sound speed in the tube. In addition, the approach may be extended to the accurate measurement of viscosity and sound velocity of liquids and gases.

1. Introduction

Two-microphone standing wave tubes are widely used to measure the normal incidence absorption and impedance of acoustical materials,[1-4] and more recently a four-microphone implementation has been proposed to measure the transmission loss.[5-7] In these standing wave tube measurements, there are a number of places that errors can enter the calculation, including uncertainty in the measurements of relative positions of the microphones and samples, uncertainty in the magnitude and phase of the transfer function estimates, and uncertainty in either the knowledge or the estimates of the properties of the medium in the tube. The incident and reflected sound waves that propagate within the tube are subject to the attenuation due to the viscous and thermal losses. Both the two- and four-microphone techniques use the complex wave number to account for the tube attenuation. The standards suggest it is necessary take into account the effect of tube attenuation when the distance from the specimen to the nearest microphone exceeds three tube diameters. It is recommended that the complex wave number, be found experimentally by using the technique described in ASTM C384, or, alternatively, it can be expressed empirically.[1]

The complete solution to the sound attenuation was derived by Kirchhoff in a very complicated transcendental equation in 1868.[8] For more than a century, only analytical approximations of his complete solution were introduced and used to estimate the propagation constants. E. Rodarte gave a brief summary of this topic and implemented a simple program to solve Kirchhoff's transcendental equation for the attenuation constants.[9]

Recently, a direct attenuation constant measurement by using the two microphone method has been proposed.[11] The measured transfer function could be used to solve for attenuation constant numerically by means of Newton-Raphson iteration scheme. Alternatively, the four-microphone measurement may be a better method to evaluate the complex wave numbers, especially for the limp and rigid porous materials.[5] In this paper, the air column between the microphones in the empty tube was treated as the material to be tested in order to obtain the complex wave number experimentally. Then an iterative method was applied in this process to improve the accuracy of measurements.

Besides the accurate measurement of air acoustic property in the tube, it is intuitive to extend the application of the improved transfer matrix approach to the measurement of other materials, such as viscous liquid [12] and fluid mixtures.[13] It may provide an accurate measurement approach and quick iterative algorithm for this application.

2. Theory of the improved transfer matrix approach

Transfer matrix formulation

The four-microphone impedance measurement based on the transfer matrix is better suited for extraction of material properties, such as complex wave number and complex characteristic impedance, especially for the limp and rigid porous material.[5-7] The figure shows a plane wave tube with a loudspeaker to the left, a termination to the right and a sample of test material in the middle, separating the tube in two sections, each with two flush mounted microphones. An x-axis is also shown with the origin at the upstream face of the material sample. The sample depth d and the coordinates x1 to x4 of microphones 1 to 4 are also shown.
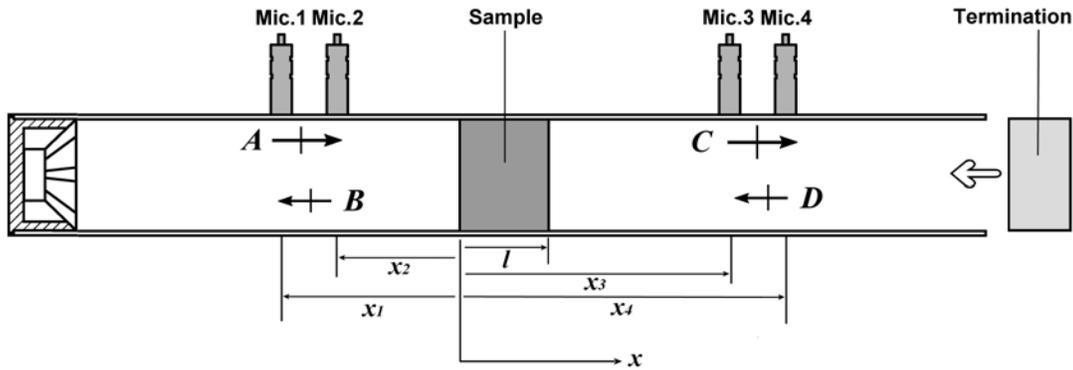

Figure 1. The experimental setup representation

The sound pressure $P1$ to $P4$ at the microphone positions can be expressed in terms of the complex plane wave amplitudes A to D. So based on the four microphone measurements, the complex plane wave amplitudes A to D can be easily determined,

$$A = \frac{j(P_1 e^{jkx_2} - P_2 e^{jkx_1})}{2\sin k(x_1 - x_2)}, \qquad C = \frac{j(P_3 e^{jkx_4} - P_4 e^{jkx_3})}{2\sin k(x_3 - x_4)}$$

$$B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_2})}{2\sin k(x_1 - x_2)}, \qquad D = \frac{j(P_4 e^{-jkx_3} - P_3 e^{-jkx_4})}{2\sin k(x_3 - x_4)}$$

$$(1)$$

The partition of material under investigation can be described as a four-pole (two-port) passive, linear acoustic sub-system. Then, a transfer matrix can be used to relate the exterior, complex pressures P and the exterior, complex normal acoustic particle velocities V on the two faces of the sample as follows,

$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d} \tag{2}$$

The pressures and acoustic particle velocities can be expressed in term of A, B, C, D

$$P\big|_{x=0} = A + B, \qquad P\big|_{x=d} = Ce^{-jkd} + De^{jkd},$$

$$V\big|_{x=0} = \frac{A-B}{\rho_0 c}, \qquad V\big|_{x=d} = \frac{Ce^{-jkd} - De^{jkd}}{\rho_0 c} \tag{3}$$

Generally speaking, two different terminations are required to solve the 2 by 2 transfer matrix. When the plane wave reflection and transmission coefficients from the two surfaces of the sample are the same, it is possible to take advantage of the reciprocal nature of the layer to generate two additional equations instead of making a second set of measurements,

$$T_{11} = T_{22}$$
$$T_{11}T_{22} - T_{12}T_{21} = 1 \tag{4}$$

By combining Eqs. (2)~(4), the transfer matrix elements for a sample satisfying the above conditions can be expressed directly for one termination condition: i.e.,

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{P\big|_{x=0}V\big|_{x=d} + P\big|_{x=d}V\big|_{x=0}} \begin{bmatrix} P\big|_{x=d}V\big|_{x=d} + P\big|_{x=0}V\big|_{x=0} & P\big|^2_{x=0} - P\big|^2_{x=d} \\ V\big|^2_{x=0} - V\big|^2_{x=d} & P\big|_{x=d}V\big|_{x=d} + P\big|_{x=0}V\big|_{x=0} \end{bmatrix} \tag{5}$$

Once the transfer matrix elements are known, all of the other acoustical properties of a layer, e.g., the transmission loss and complex wave number, can be calculated, as will be demonstrated next.

Determination of wave numbers and characteristic impedance

A homogeneous, isotropic porous material that is (approximately) limp or rigid can be modeled as an equivalent fluid,[14]

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_p d & j\rho_p c_p \sin k_p d \\ j\sin k_p d / \rho_p c_p & \cos k_p d \end{bmatrix} \tag{6}$$

The complex wave number k_p and the complex characteristic impedance $\rho_p c_p$ can be calculated from the showed matrix equation,

$$k_p = \frac{1}{d}\cos^{-1} T_{11} \tag{7}$$

Quantities such as the sound speed c_p, and the attenuation constant can easily be determined when k_p are known.

Post-process iterative algorithm

If the tested material is assumed to be the air in the tube, a post-process iterative algorithm can be applied to obtain the optimized value for wave number. It indicates that we can substitute the obtained complex wave number of Eqs. (7) into the Eqs.(1) and (3) as to generate an iterative loop, i.e., (1)~(7)~(1)~(7)~⋯⋯, until the solution converges, as shown in Fig. 2. The algorithm is very flexible in the initial setting value of wave number about which will be talked later. It should be pointed out that the characteristic impedance is not able to be evaluated in this process. Note that the uncertainty of characteristic impedance doesn't have any effect on the evaluation of complex wave number because the calculation of wave number in this process is independent on the characteristic impedance. Clearly, this algorithm is straightforward and intuitive enough to give convincing experimental results.
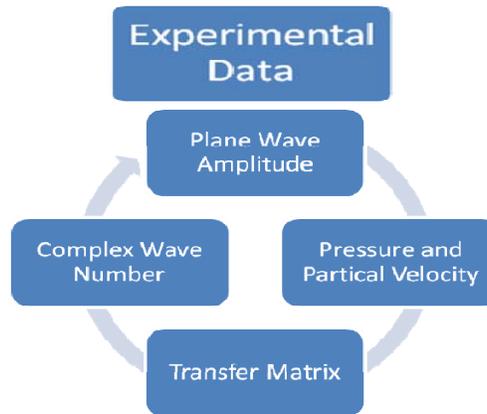

Figure. 2 Iterative algorithm

3. Experimental procedure

The experimental procedure used in the presented work was based on the proposed four-microphone measurement.[7] The Brüel and Kjær Transmission Loss Kit Type 4206T was used and Pulse 11.0 was responsible for the data acquisition. In the one-load measurement case, the measurements were performed with the anechoic terminations in place, by which the phase-mismatch could be minimized. Note that the termination impedance can be arbitrary and its value need not be known in principle.

Figure 3. The Brüel and Kjær Transmission Loss Kit Type 4206T is suitable for both low frequency (50 Hz to 1600 Hz) and high frequency (500 Hz to 6400 Hz) measurements since it includes both 10 cm (shown here) and 2.9 cm inner diameter tube sections.

Both the large tube and the small tube measurement were presented in the work. In order to speed up the convergence, the largest air column depths are chosen (the distance l from the microphone 2 to microphone 3), which are 35, 40, 45cm typically for low frequency 4206T tube and 20cm for the high frequency tube.

4. Results and discussion

Results of large tube measurement

Figure 4 represents the converged sound speed and attenuation constant of different assumed air column depths from the same measurement with the microphone distance 45cm (from microphone 2 to microphone 3). Obviously, the different assumed air depths give the same converged solution. Note that the peaks in the results are introduced by the "half-wave problem", and their presence does not affect the present conclusions. Since the air is full of the tube, any air column thickness could be assumed in principle. Note that the convergence would fail if the assumed air column thickness is shorter than some values. It is suggested that the assumed air depth is over three quarters of microphone distance l.
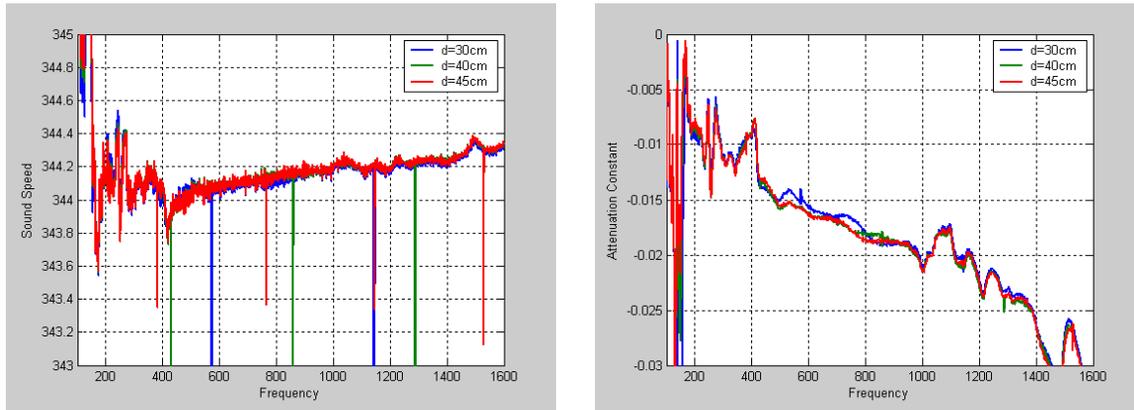

Figure 4. Sound speed and attenuation constant of different assumed air depths with the microphone distantce 45cm

Figure 5 shows the converged results for different microphone distances l (35cm, 40cm and 45 cm) and a comparison with the semi-empirical predictions.[1,10] In this case, all the assumed air depths is equal to the microphone distance. From Fig. 4 and Fig. 5, it shows that sound speed is independent on the assumed air depths and microphone distances.
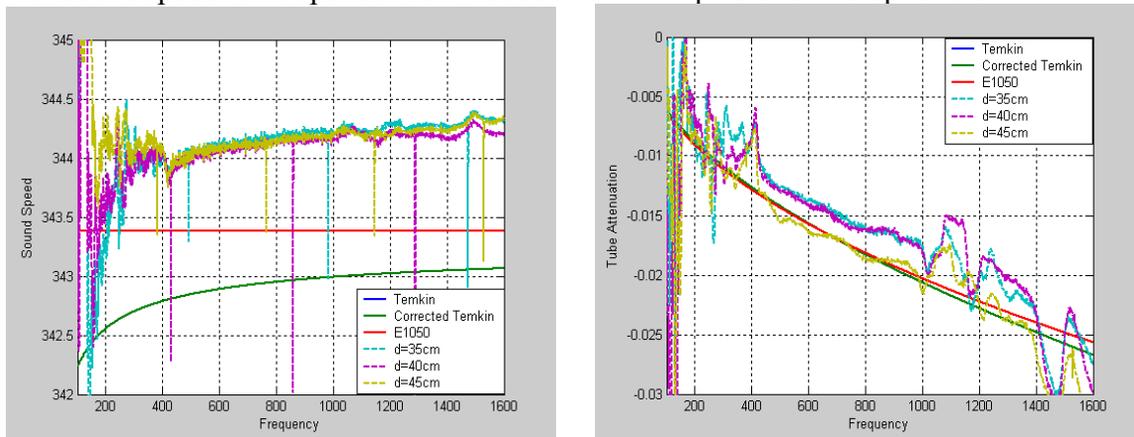

Figure 5. Sound speed and attenuation constant of different microphone distances (large tube)

It can be seen that the sound speed in the tube was slightly higher than the initially predicted value, based on the measured air temperature exterior to the tube. In contrast, the measured tube attenuation was very close to the theory prediction. It may be

necessary to correct the temperature originally obtained from an exterior thermometer since the tube's interior temperature may be different. It was also found that the shape of Temkin's theoretical prediction was very similar to the experimental results. This means that we can establish a mean square error function based on temperature between the Temkin sound speed prediction and the measured value, and then minimize the error by adjusting the temperature to obtain the most accurate temperature and sound speed estimates in the tube. From the corrected Temkin curve in Fig. 4, the best-fit temperature was found to be $22.8\,^{\circ}C$ in this case; the measured value by using an exterior thermometer was $20.9\,^{\circ}C$. It is thus suggested that the adjusted Temkin sound speed and attenuation results be used in place of the directly measured estimates.

Results of small tube measurement

Small tube measurement was also implemented in the work. Figure 5 gives the results of small tube with the microphone distance 20cm. The similar conclusion could be drawn from the Fig. 6.
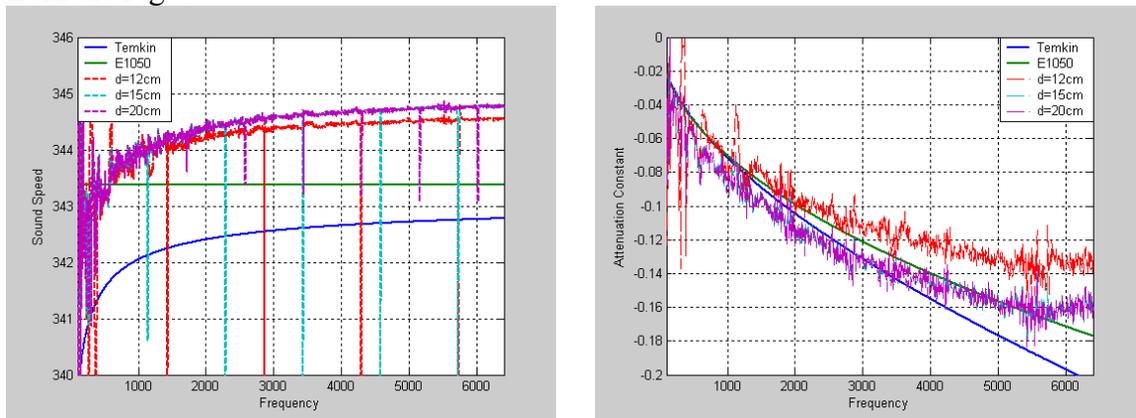

Figure 6. Sound speed and attenuation constant of different assumed thickness (small tube)

Remarks

It should be pointed out the iterative algorithm is so simple and fast that the CPU time is negligible in this process. All the cases could lead to convergent solution within 20 iterative loops whatever the initial sound speed is set. This approach is very flexible in the initial settting value, ranging from at least $1m/s$ to $10,000m/s$ .The main drawback of this iterative algorithm: half wavelength problem encountered in the post-process due to the periodic function has an effect on the convergent speed; in some cases it leads to non-convergent and unsteady solutions. The solution is to choose the air thickness as long as possible to speed up the convergence.

5. Conclusion

An experiment approach to estimate the attenuation constant sound speed of air in the circular tube was described in this study. Transfer matrix formulation has been implemented, because of its advantage of the extraction of material properties, such as complex wave number and complex characteristic impedance for limp or rigid materials. This improved transfer matrix algorithm is convenient and simple enough to apply in the impedance tube measurement and evaluate the medium property within the tube very

accurately. The approach has its potential in the accurate measurement of gases and liquid property. Also it is easy to be used to investigate the acoustical material performance of ducting lining.

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