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A REVIEW OF SIMPLE MATHEMATICAL MODELS
OF VALVES IN RECIPROCATING COMPRESSORS

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INTRODUCTION

Since 1950 a number of relatively simple mathematical models have been developed which describe a compressor, its valves and the working fluid. In general the analysis yields two non-linear differential equations which relate the many parameters involved: a "flow" equation which relates the pressure difference across the valve to mass flow rate and valve opening and a "dynamic" equation which describes the valve movement. Simultaneous solution of the equations by iterative methods yields pressure difference across the valve and valve displacement as functions of crankangle and subsequently allows the construction of a conventional pressure-volume diagram.

To describe the complex physical situation in mathematical form requires that a number of assumptions be made. It is usually assumed that infinite volume receivers exist at inlet and discharge and hence that pressures \( P_i \) and \( P_d \), Fig. 1, remain constant; this simplification permits the study of a compressor and its valves in isolation from the associated receivers and pipework system.

The value of any such model depends on its ability to describe actual events with sufficient accuracy. Hence, assessment of the validity of the model requires that accurate experimental data be available from a compressor for the purpose of comparison. This paper reviews briefly a number of investigations directed to this end.

SEQUENCE OF EVENTS IN THE CYCLE

A mathematical model of a compressor and its valves must account for the series of events in a cycle. This series of events may be designated (a) re-expansion, (b) suction, (c) compression, (d) discharge. A starting point must be chosen and one such is point 0, Fig. 1. Initially it is assumed that at point 0 the cylinder pressure is equal to the (constant) discharge pressure, \( P_d \), and that the piston is at inner dead centre.

Event (a): The clearance gas expands from point 0 to point 1 where the suction valve begins to open. If there is valve spring preloading or if significant oil stiction exists at the valve seat, valve opening is delayed and the clearance gas will continue to expand in the cylinder to a pressure less than the (constant) suction pressure, \( P_i \).

Event (b): At commencement of the suction process, near point 1, as the suction valve is opening, both the flow equation and the dynamic equation are solved simultaneously. When the valve reaches the stop, and remains there, the dynamic equation is not applicable and the flow equation alone is used to compute the pressure difference across the valve. Later in the stroke, due to the reduction in piston velocity, the pressure difference across the valve reduces and the drag force on the valve thereby reduces. When this force becomes less then the valve spring force the valve will begin to close and during valve closure both the flow and dynamic equations are applicable. However, when the valve reaches the stop it may bounce, or, if the pressure difference across the valve is insufficient to hold it open against the force exerted by the valve spring(s), the valve may flutter. Such flutter may continue throughout the suction process.

Zero coefficient of restitution due to, say, the damping effect of oil on the valve stop, would mean zero valve bounce, yet flutter could still occur. If valve bounce or flutter occurs, both equations are applicable.

The valve may flutter during the closing phase and may bounce after impact on its seat. At piston reversal at outer dead centre the suction valve may or may not be closed and the cylinder pressure may be greater or less than the suction line pressure. Thus under certain circumstances "blow-by" can occur before the valve is finally seated.

Event (c): The gas is compressed from point 2 to point 3 where the discharge valve begins to open. If there is valve spring preloading or if significant oil stiction exists at the valve seat, valve opening is delayed and the gas in the cylinder will continue to be compressed to a pressure greater than the (constant) discharge pressure, \( P_d \).
Event (d): After commencement of the discharge process near point \(3\), the appropriate flow and dynamic equations for the discharge valve are solved during opening, bounce, flutter and closure: if the valve remains open at rest against the stop, only the flow equation is applicable. At piston reversal at inner dead centre, the discharge valve may or may not be closed and the cylinder pressure may or may not be equal to the discharge pressure so the computation is continued till the valve is finally closed. This ends the sequence of events but the finish of the cycle may not be at the initial point \(0\).

After re-expansion from the finish of the cycle to point \(1\), Fig. 1, comparison may be made with point \(1\). If the two points, \(1\) and \(1\), do not coincide then the original assumptions about conditions at point \(0\) were in error. If the discrepancy is considered appreciable the computations for the cycle may be repeated using a revised starting point \(0\). Hence solution of the equations yields the displacement of each valve and the pressure in the cylinder during the cycle.

**REVIEW OF SIMPLE MATHEMATICAL MODELS**

**Costagliola** (1) at M.I.T. in 1950 produced the first worthwhile mathematical model of a reciprocating compressor and its valves. The analysis of valve dynamics was the primary concern. Corresponding experimental work was conducted with a \(6\frac{1}{2}\) in bore x \(4\frac{1}{2}\) in stroke single cylinder air compressor, fitted with flexing reed "feather" type valves, in the speed range 900-1800 rev/min. The solution of the non-linear differential equations by graphical methods was too tedious for the model to be of interest as an industrial design tool. Many theoretical pressure and valve displacement diagrams were calculated, and although these were not shown superimposed on experimental diagrams, it was claimed that the model was "essentially correct".

Widespread use of digital computers has allowed later investigators to solve the equations rapidly and has allowed the basic model to be refined and extended. Virtually all the models now in existence are based to some degree on the pioneer analysis by Costagliola.

**Wambganss and Cohen** (2) of Purdue University in 1967 developed a similar model and made comparison with experimental records for a \(\frac{1}{2}\) hp, 3600 rev/min hemi-ethically sealed compressor fitted with reed type valves and pumping air or R12. Fig. 2 shows the extent of the correlation obtained between the analytical and experimental records. This correlation may be judged to be good when allowance is made for the difficulties inherent in instrumenting such a small compressor and its valves. (From a similar exercise MacLaren and Kerr (10) of Strathclyde University, concluded that an analytical model could provide qualitative results much more rapidly and cheaply than an extensive experimental programme. It should not be inferred from this statement that such models will replace all development testing). In the Investigations at Purdue University (2), attention was paid to the details of the dynamics of reed valves and several degrees of freedom were allowed. (In most other studies it has been assumed that the valve had only a single degree of freedom).

Amongst the conclusions drawn were (a) a single degree of freedom approximation was not sufficient to represent the valve reed dynamics in a high speed compressor (b) a condition of valve "stiction" existed which could have a large effect at low values of compressor pressure ratio and (c) a damping term in the dynamic equation for the valves was considered to be important.

Borisoglebski and Kuzmin (3) in Russia in 1965 combined the simultaneous flow and dynamic equations into a single non-linear differential equation as Costagliola (1) did in his doctoral thesis. The various geometric and operating dimensions were arranged into a small number of lumped dimensionless parameters which were evaluated from nomograms. The solution of the single general equation was by an iterative process using the Runge-Kutta procedure. Fig. 3 shows the extent of correlation achieved between analytical and experimental results for a multi-ring suction valve in a four stage air compressor.

Upfold (4) of the University of New South Wales in 1967 studied the behaviour of a ring-plate suction valve in one cylinder (5 in bore x 4 in stroke) of an eight cylinder double V type air compressor. Oscillograms of valve displacement were compared with those predicted by an analytical model similar to Costagliola's but extended to account for heat transfer and valve damping. Upfold measured the coefficient of restitution of the ring plate valves by using a high speed photography technique. Upfold used the model to examine the effect of changes in various dimensionless lumped parameters on compressor performance criteria, in particular their effect on valve impact velocity. From Fig. 4 (Test 719) it could be claimed that the correlation between analytical and experimental results was good. However, the valve displacement diagram when flutter was present, as in Test 723, indicated that this model would be seriously in error if used to predict the point of final closure of the suction valve, impact on the valve seat, or "blow-by" loss. In this case the experimental diagram was the more credible, suggesting errors in the complex computer program or errors in the values of empirical coefficients used.

Traversari and Lacitignola (5) in Italy in 1970 constructed a model based on those by Costagliola (1) and Maclaren and Kerr (6). Modifications were made to account for damping due to the pneumatic type of valve stop used with the multi-ring plate valves. As in most models, discharge, drag and damping coefficients were assumed to have constant values. Provision was also made for simulating delay in valve opening due to oil stiction effects. Fig. 5 (a) compares analytical and experimental records for a discharge valve mounted at the inner end of a double-acting cylinder. The pressure pulsations within the cylinder were quite small. Fig. 5 (b) shows this valve when it failed to reach its permitted lift and that severe valve flutter and corresponding large pressure fluctuations ensued. This unsteady situation creates a severe test of
of a model and the correlation between analytically predicted and experimental results appears to be good. Fig. 5(c) is the record of a suction valve for the inner end of the double-acting cylinder when late valve closure occurred. Fig. 5(d) shows the computation for a discharge valve when opening was assumed to be delayed by 15 crank-angle degrees. As a result the valve impact velocity at the stop increased from 1.42 m/s to 4.98 m/s. (Such a delay is unlikely to be due only to oil stiction: the rate of increase of pressure difference across a discharge valve is so large that oil stiction should not delay discharge valve opening by this amount.)

The analytical computation of valve displacement (Fig. 5) ended at piston reversal. In the sequence of events in the cycle outlined in Fig. 1, the behaviour of one valve as it affects the commencement of opening of the other valve is accounted for. It would appear that the model of Traversari and Lactignola was not sufficiently complete to account for such valve interaction.

Traversari and Lactignola concluded that the differences between analytical and experimental results were due to (a) pressure pulsations in the suction and discharge piping, (b) delay in opening and closing on account of valve stiction (c) error in the selection of values of empirical coefficients and the assumption that they were independent of valve lift, (d) experimental errors.

Tauber and Blomsa (7) in Holland in 1971 examined the sequence of events through a complete cycle. The model included a simple simulation of inlet and discharge pipework. The suction and discharge valves studied were multi-finger reeds placed circumferentially round a cylinder, 160 mm bore x 110 mm stroke. Although comparison was made between analytical and experimental records when pumping R22, Fig. 6, shows the comparison when pumping air at two compressor speeds. The increased flutter of the suction valve at the lower speed is apparent. It could be concluded that this model described valve behaviour adequately for many practical design purposes.

McLaren and Kerr (8) of the University of Strathclyde in 1970 described a model developed from that of Costagliola. Comparison was made between theoretical and experimental results for a single stage, single-acting single cylinder air compressor, 6 in bore x 4½ in stroke, fitted with 2½ in o.d. spring loaded single ring-plate valves at both suction and discharge. The test series was conducted with a range of speed 345-800 rev/min and compressor pressure ratio 3.9-7.7. Fig. 7(a) shows that at the lowest speed there was considerable flutter of the suction valve. Again the presence of valve flutter provided a severe test of the analytical model to predict valve behaviour. The lowest speed was near to the compressor rated speed and the suction valve displacement diagram suggests that the valve design should be modified to reduce the effect on valve spring life of the excessive number of spring compressions per cycle. (The matter of suitable valve spring characteristics was discussed in a paper by Steindel (9) in Poland in 1964). The experimentally measured valve displacement diagram at A in Fig. 7(a) suggests that the valve was partially sticking on its guide, perhaps due to uneven departure from the stop. Non-uniform circumferential valve displacement may also account for the suction valve chatter near valve closure at 180°. It may be that the valve displacement transducer, sensing only one point of the valve, was recording a dying spinning penny action rather than a uniform valve bounce on the seat. Neither the partial sticking nor the uneven seating of the valve plate was simulated in the mathematical model.

During the experiments designed to assess the general validity of the model the suction valve operated without significant flow restriction at compressor inlet, the inlet filter and pipework having been removed. Hence the area of the suction "loops" in Fig. 7(a) and (b) corresponds with the suction loop in Fig. 1. (the lower shaded area), i.e. the suction plenum chamber pressure did not vary significantly from the atmospheric pressure P1. However, this simple situation could not be created experimentally for the discharge valve, i.e. an arrangement could not be made to maintain Pl constant. The cylinder pressure (X), Fig. 7(a) and (b), the plenum chamber pressure (Y), and the pressure difference across the discharge valve (Z = X - Y) were each measured separately. Accurate records were difficult to obtain and an element of uncertainty was involved in fixing the datum for the experimental traces. Hence it was not claimed that the experimental records of pressure difference (Z) were sufficiently accurate for meaningful comparison with the analytically predicted pressure difference ( Z'). It was apparent, however, that the plenum chamber pressure Y varied significantly and that the assumption in the analytical model that P1 remained constant was questionable. Nevertheless, the correlation between theoretical and experimental results for both valves was considered to be sufficient for many design purposes; for example, to estimate the loss of volumetric efficiency due to suction valve throttling, the power consumption due to the "plus-loading" by the valves, changes in valve impact velocities at seat and stop with alteration of compressor speed, pressure ratio, valve lift, valve spring stiffness and preloading.

Other similar analytical models have been constructed by manufacturers but do not appear to have been described in published literature.

**EMPIRICAL COEFFICIENTS**

All the models referred to above are semi-analytical: several empirical coefficients have been included and generally these were evaluated by experiment. Coefficients to account for valve bounce (restitution) damping and stiction could, in principle, be estimated by fitting analytical results from the mathematical model to accurate experimental results from a compressor but this would be difficult to achieve in practice.
Published values of coefficient of restitution vary from 0.2 (measured by Upfold (4) for a metallic ring-plate valve) to 0.365 (quoted by Borisoglebski (3) from a Russian source). In the author's experience an accurate value of this coefficient need not be known when computing valve displacement. However, impact velocity and the coefficient of restitution are very relevant to valve plate surface stressing and valve spring surging and hence to valve life.

Values of damping coefficients were assumed and accounted for by Wambsganss and Cohen (2) who stated that damping was a significant effect in high speed compressors fitted with flexing light-weight reed valves. Traversari and La Pignola (5) also had to account for this effect due to the significant pneumatic damping incorporated in their ring valves.

MacLaren and Kerr (10) showed, Fig. 8, the computed delay in the opening of a reed suction valve due to an assumed value of oil stiction force. This diagram shows the resulting change of peak pressure drop across the valve and illustrates that at low evaporating pressures there is a possibility that a suction valve may open in the oil free condition but fail to do so when oil is present at the seat. (The same values of oil stiction force estimated from the suction valve were applied to the discharge valve and found to have negligible delaying effect on the commencement of opening of the discharge valve).

Important empirical coefficients are those relating to gas flow, e.g. the coefficient of discharge for a valve and its associated passages and the coefficient of pressure drag resulting from the flow over the moving valve element. Such coefficients have usually been determined for a particular geometry by steady flow tests within or without the compressor. Although most investigators have treated flow coefficients as constants, these coefficients are functions of several parameters, principally the instantaneous value of valve lift. Wambsganss and Cohen (2), Ucer (11) and Kerr (13) expressed these coefficients as functions of valve lift.

Fragmentary information relating to these coefficients is available in many publications. To survey the information would be a difficult task since authors seldom define fully the coefficients used. For the same reason, too much should not be made of the difference between numerical values quoted since each may be valid within its particular definition. A welcome but perhaps wishful outcome of a review would be to find that the flow coefficients, expressed in dimensionless form, such as used by Davis (12), did not vary much from one type of valve to another. If this were so, the mathematical model could be used with some confidence prior to any manufacture for a computer aided design study of a proposed compressor and its valves.

**CONCLUSIONS**

Mathematical models of a reciprocating compressor and its valves now exist in several countries. In general the validity of these models has been adequately established for a particular design by making comparison between analytical and experimental results. A cheap procedure is now available for the rapid study of some aspects of a design or the likely effects of proposed modifications to a design. This should permit more rational design and shorter experimental development programs.

All models have been semi-analytical, containing a number of empirical coefficients. Most investigators have devoted considerable effort to evaluating these coefficients by experiment. It is suggested that a comprehensive review of the considerable fragmentary data available is required and that the significance of each coefficient be more fully assessed before time and expense is allocated to further experimental study of them.

In the majority of models available to date, the effect of pressure variations due to the pulsating nature of the flow in the system adjoining the valves has been neglected. These pulsations can be of considerable amplitude and models should now be developed to include a simulation of the gas flow pattern in the pipe-work of the system. To implement this proposal would constitute a major extension of the study of compressors and valves and would require considerable computer capacity. Due account would have to be taken of the interaction between pulsation effects and valve behaviour. Many more boundary conditions would have to be satisfied.

**REFERENCES**


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**FIG. 1.**
Pressure - Volume and Valve Displacement Diagrams.

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**FIG. 2.** Theoretical-Experimental Correlation: Wambmans and Cohen.
FIG. 3.

Comparison of Predicted and Experimental Displacement Diagrams for Multi-Ring Suction Valve in 4-Stage Compressor EK10-1:

Borisoglebski and Kuzmin

FIG. 4. THEORETICAL-EXPERIMENTAL CORRELATION: UPFOLD.