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Partial Field Decomposition in Near-field Acoustical Holography by the Use of Singular Value Decomposition and Partial Coherence Procedures

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Partial Field Decomposition in Nearfield Acoustical Holography by the Use of Singular Value Decomposition and Partial Coherence Procedures

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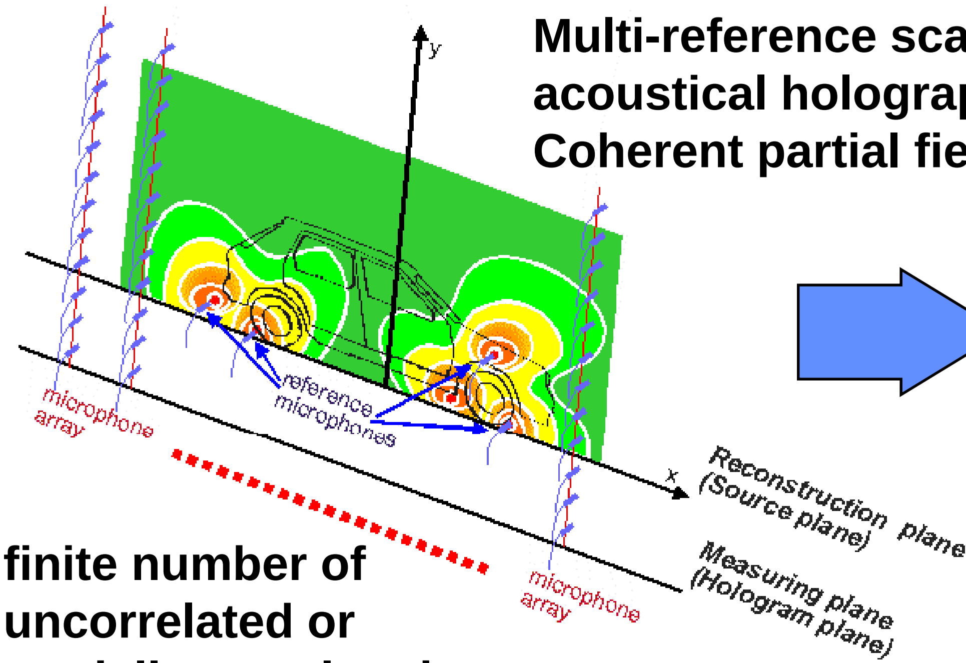
(Herrick Labs., Purdue Univ.)

INTRODUCTION

- **What's a partial field ?**

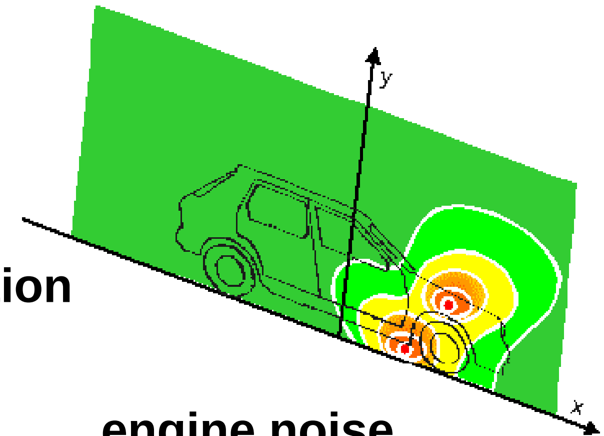


Multi-reference scanning acoustical holography: Coherent partial field separation

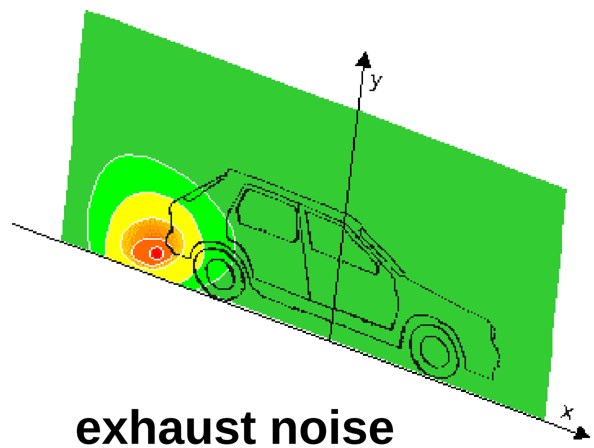
finite number of uncorrelated or partially correlated sources

total sound field

separation of sound field related to each noise source



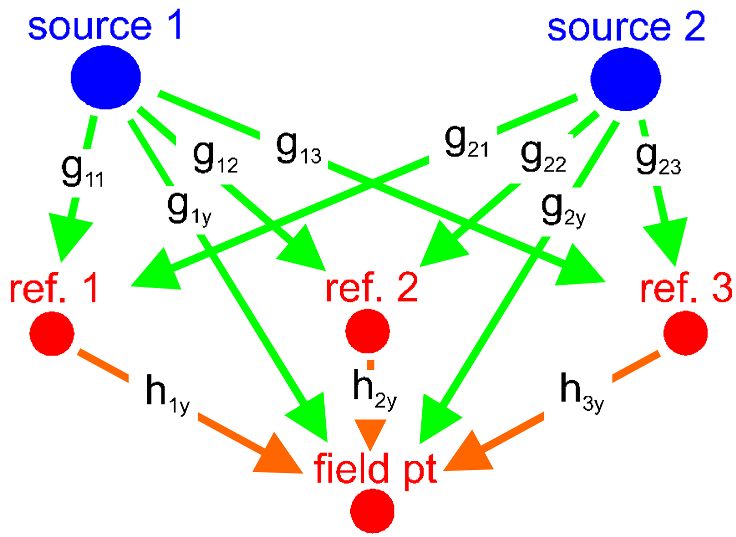
engine noise



exhaust noise

OBJECTIVES

- **How can we decompose partial fields ?**



$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{n} = \mathbf{q} + \mathbf{n}$$

$\mathbf{r}$: reference vector
$\mathbf{s}$: source vector
$\mathbf{n}$: noise vector
$\mathbf{G}$: geometrical transfer function matrix

$$\mathbf{S}_{\mathbf{rr}} = \mathbf{G}\mathbf{S}_{\mathbf{ss}}\mathbf{G}^{\mathbf{H}} + \sigma_n^2\mathbf{I} = \mathbf{S}_{\mathbf{qq}} + \sigma_n^2\mathbf{I}$$

  - Unable to directly measure source signals
  - Use of references close to sources; strong relation between each source and reference signals
    - **Identifying the number of incoherent sources**
    - **Choosing the best set of references: as many as sources**
    - **Decomposing the field into the best set of partial field**

SELECTION OF REFERENCES

- **Number of incoherent sources**
  - Singular value decomposition

$$\mathbf{S_{rr}} = \mathbf{V}(\mathbf{\Lambda} + \sigma_n^2 \mathbf{I})\mathbf{V}^{\mathbf{H}}$$

N incoherent sources =
N singular values greater than

Pre-measurement of background noise level
Reference candidates, Great *SNR*

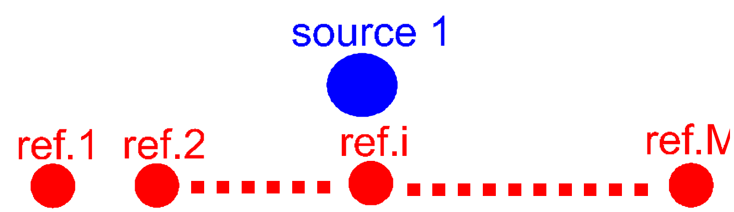
- **Selection of good references from the candidates**
  - Try to make the reference cross spectral matrix close to ***diagonal***
  - Use **MUSIC** algorithm with a trial vector $\mathbf{e_i} = \{0,0,\cdots,0,1,0,\cdots,0\}^{\mathrm{T}}$
  - Select based on MUSIC power

$$P_i = (\mathbf{e_i^H R_{noise} e_i})^{-1}$$

noise subspace

$$\mathbf{R_{noise}} = \sum_{n=N+1}^{M} \mathbf{v_n v_n^H}$$

$\mathbf{v_n}$: n-th singular vector



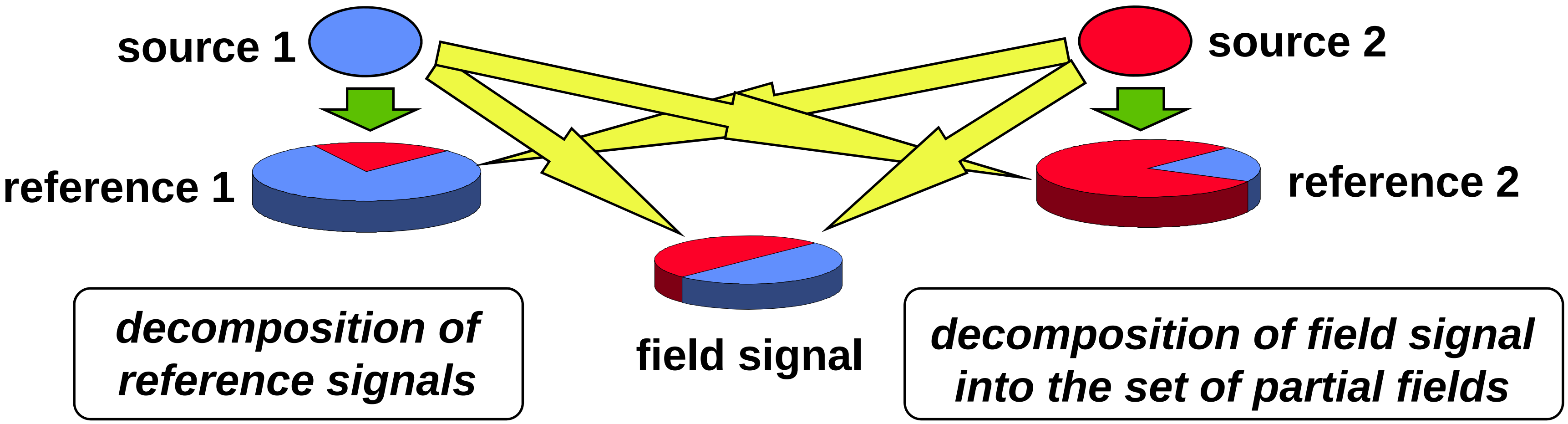
$\mathbf{r} = \{ r_1 , \quad r_2 , \ldots\ldots , r_i , \ldots\ldots\ldots , r_M \}^{\mathrm{T}}$

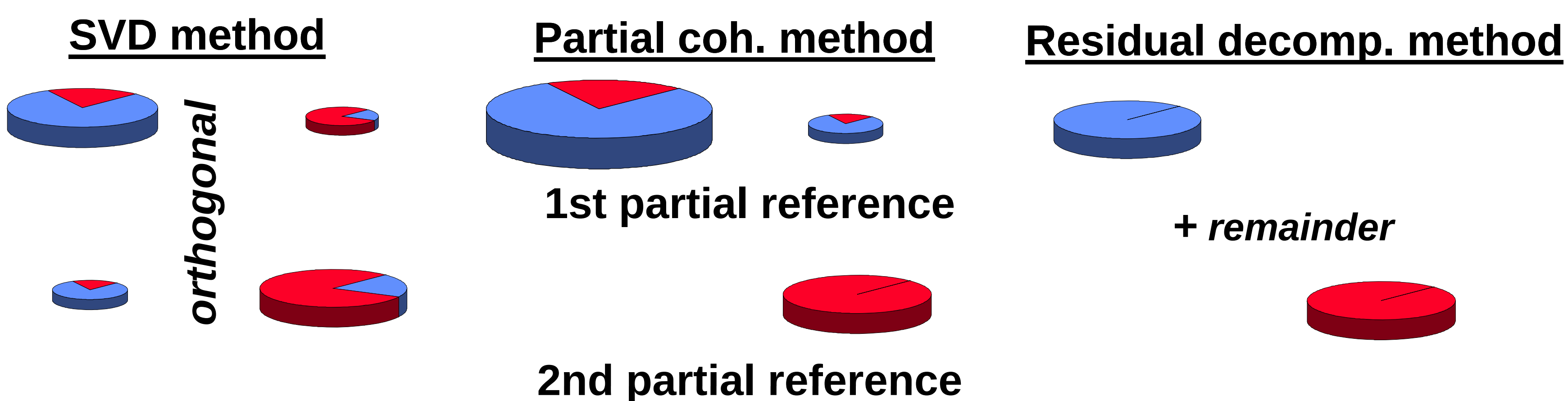
$\cong$ ***with normalization***

$\mathbf{e_i} = \{ 0 , \quad 0 , \ldots\ldots , 1 , \ldots\ldots\ldots , 0 \}^{\mathrm{T}}$

PARTIAL FIELD DECOMPOSITION METHODS

- **Decomposition of reference signals**



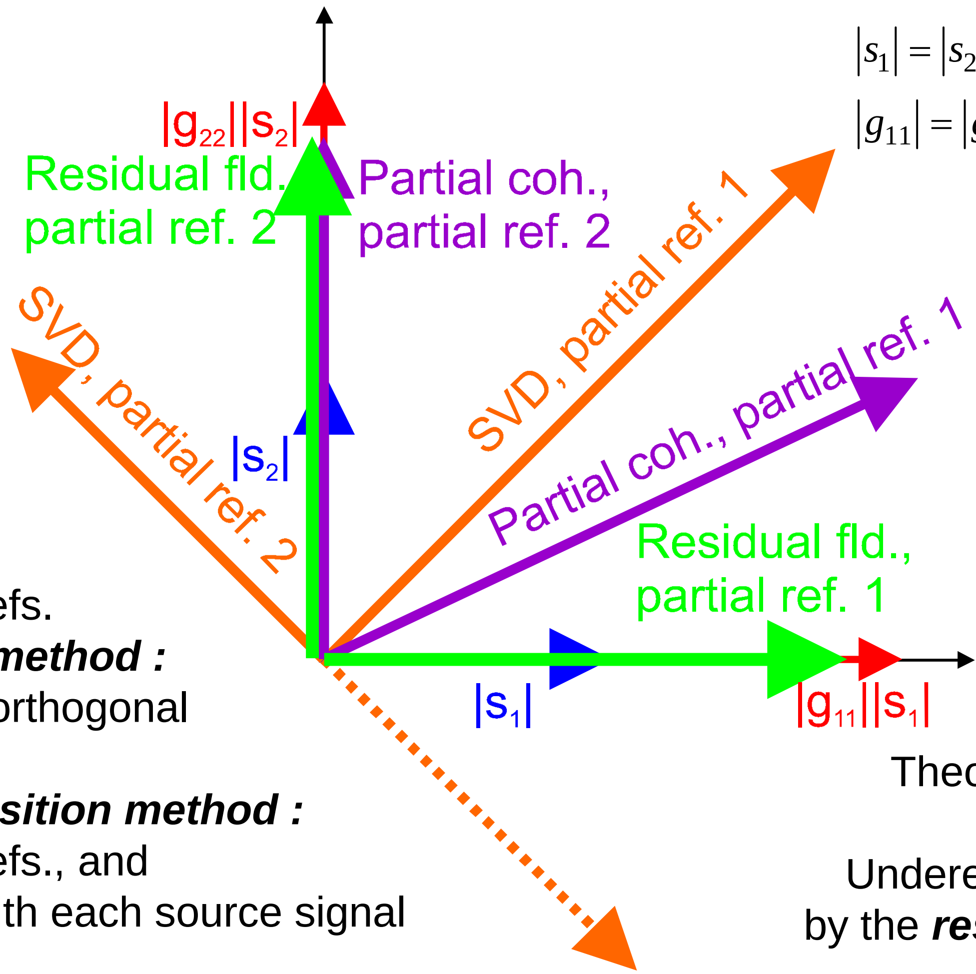


PARTIAL FIELD DECOMPOSITION

- **Comparison of partial reference signals**

$|s_1| = |s_2|$

$|g_{11}| = |g_{22}| = 2, \quad |g_{12}| = |g_{21}| = \frac{1}{2}$



SVD method :
Orthogonal partial refs.
Partial coherence method :
Incoherent, but not orthogonal partial refs.
Residual decomposition method :
Orthogonal partial refs., and closely correlated with each source signal

Theoretical partial refs.
$\cong$
Underestimated partial refs. by the ***residual decomposition***

DECOMPOSITION ALGORITHMS

- **Field signal**

$$S_{yy} = \mathbf{g_y S_{ss} g_y^H} + \sigma_n^2 = \mathbf{h_y S_{qq} h_y^H} + \sigma_n^2$$
$$= \mathbf{a a^H} + \sigma_n^2$$

$\mathbf{g_y}$: transfer function between source & field signal
$\mathbf{h_y}$: transfer function between reference & field signal
$\mathbf{a}$: set of partial fields decomposed from field signal

- **Partial fields**
  - Theoretical decomposition $\mathbf{a} = \mathbf{g_y S_{ss}^{1/2}}$
  - SVD method $\mathbf{a} = \mathbf{h_y V \Lambda^{1/2}} = \mathbf{S_{ry}^H V \Lambda^{-1/2}}$
  - Partial coherence method $\mathbf{a} = \mathbf{h_y L D^{1/2}} = \mathbf{S_{ry}^H U^{-1} D^{-1/2}}$

    (LU decomposition method, Cholesky factorization)
  - Residual decomposition method $\mathbf{a_{cond}} = \mathbf{h_y S_{cond}^{1/2}} = \mathbf{S_{ry}^H S_{qq}^{-1H} S_{cond}^{1/2}}$

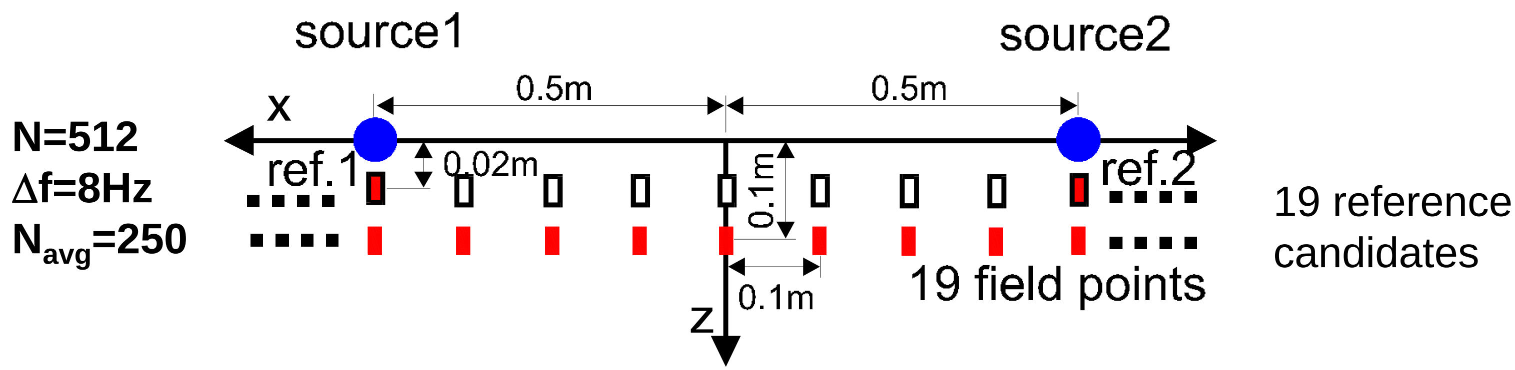
    (Conditioned autospectra of reference signals and the remainder)

$$\mathbf{S_{cond}} = \begin{bmatrix} S_{11\bullet 23\cdots N} & & & 0 \\ & S_{22\bullet 13\cdots N} & & \\ & & \ddots & \\ 0 & & & S_{NN\bullet 123\cdots N-1} \end{bmatrix}$$

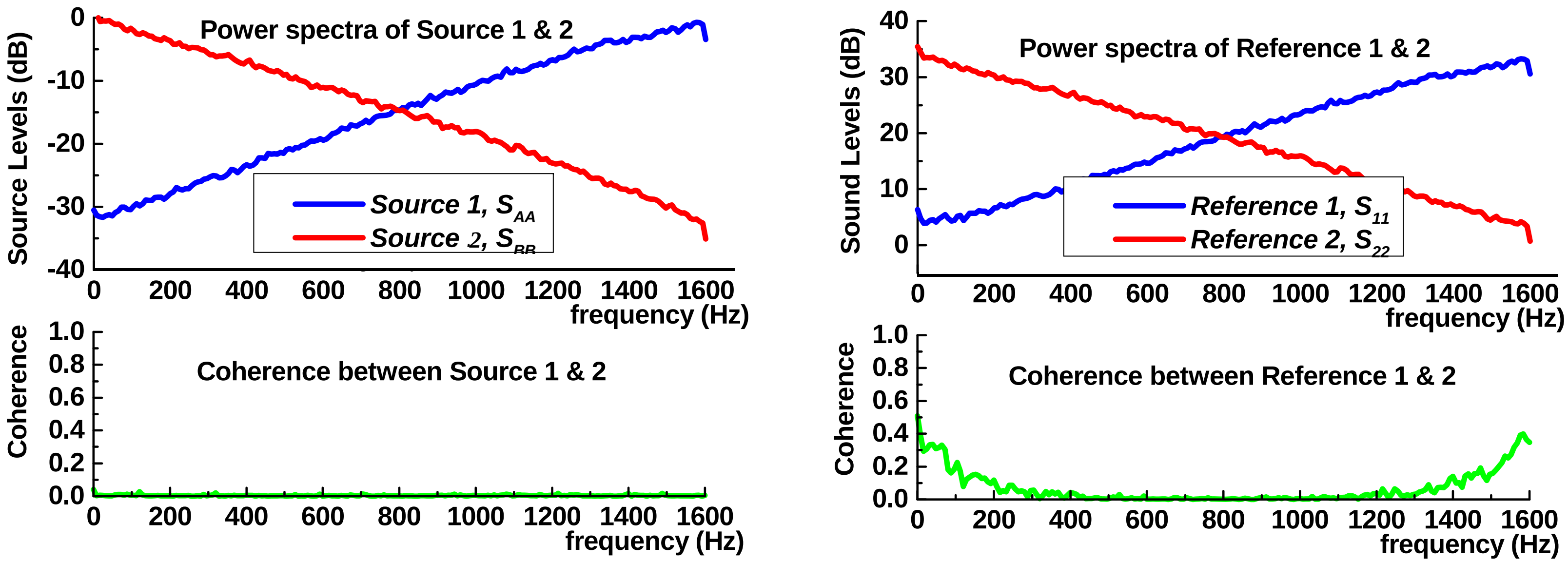
$$\mathbf{S_{qq}} = \mathbf{S_{cond}} + \mathbf{S_{remainder}}$$
$$= \mathbf{S_{cond}^{1/2} S_{cond}^{1/2}} + \mathbf{S_{remainder}}$$

$\mathbf{S_{remainder}}$: not positive definite,
therefore cannot be decomposed

NUMERICAL SIMULATION

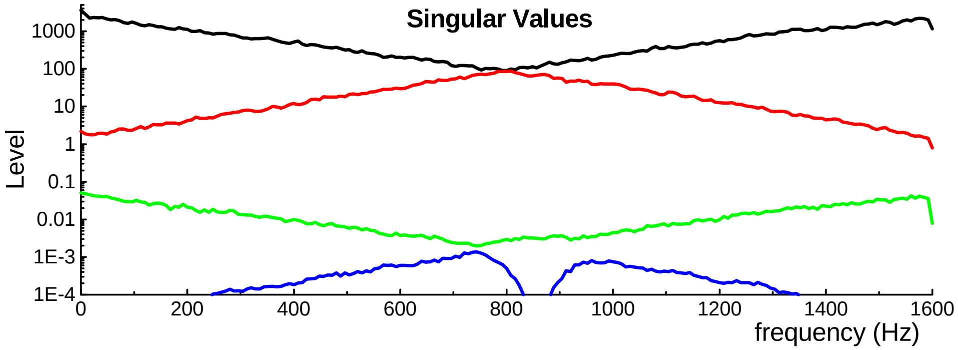


Characteristics of simulated signals



SELECTION OF REFERENCES

Determination of the number of references



2 dominant singular values =
2 incoherent sources =
2 references in selection

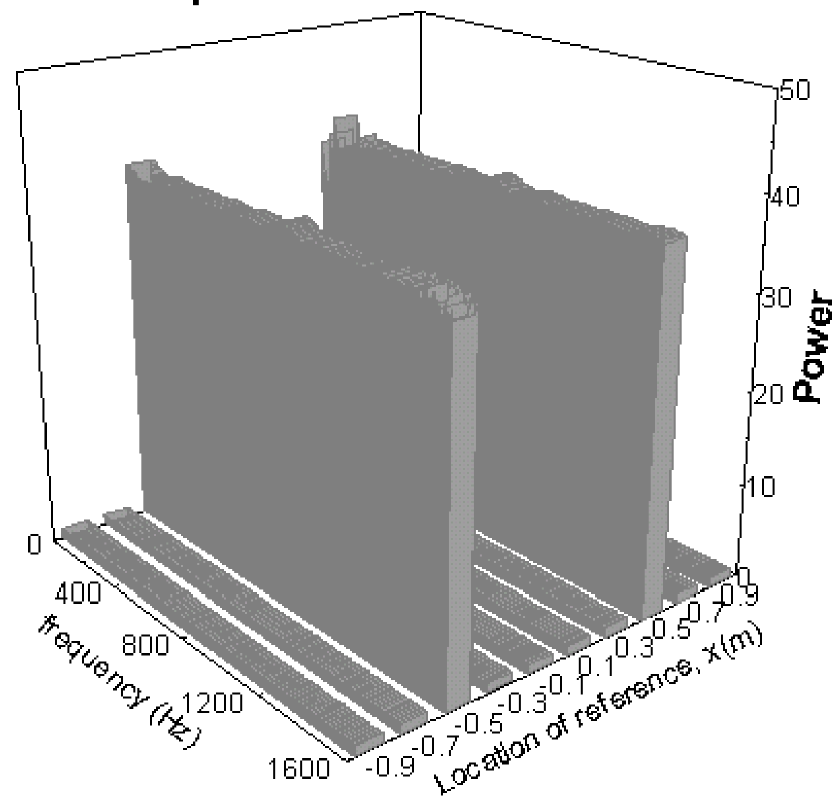
$$\mathbf{S_{rr}} = \mathbf{V}(\boldsymbol{\Lambda} + \sigma_n^2 \mathbf{I})\mathbf{V}^{\mathbf{H}}$$

MUSIC power : $P_i = (\mathbf{e_i^H} \mathbf{R}_{\mathbf{noise}} \mathbf{e_i})^{-1}$

with trial vector $\mathbf{e_i} = \{0,0,\cdots,0,1,0,\cdots,0\}^{\mathrm{T}}$

Selection of as many references as incoherent sources

MUSIC power



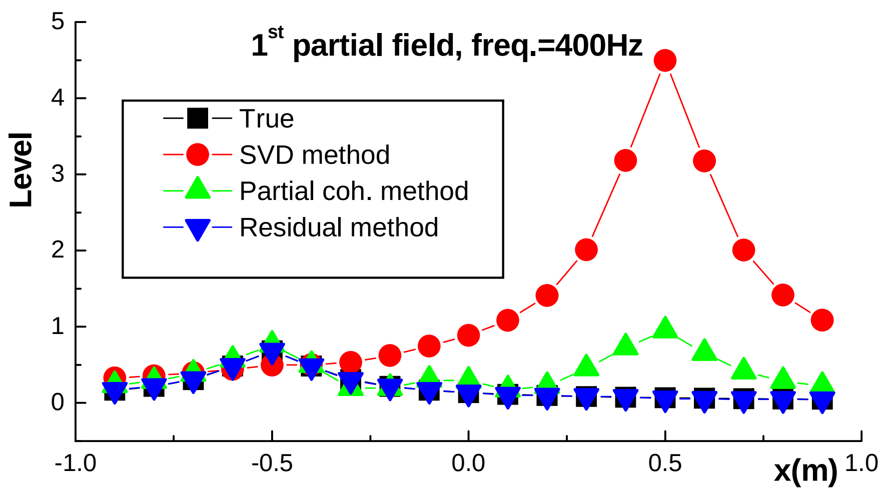
DECOMPOSED PARTIAL FIELDS

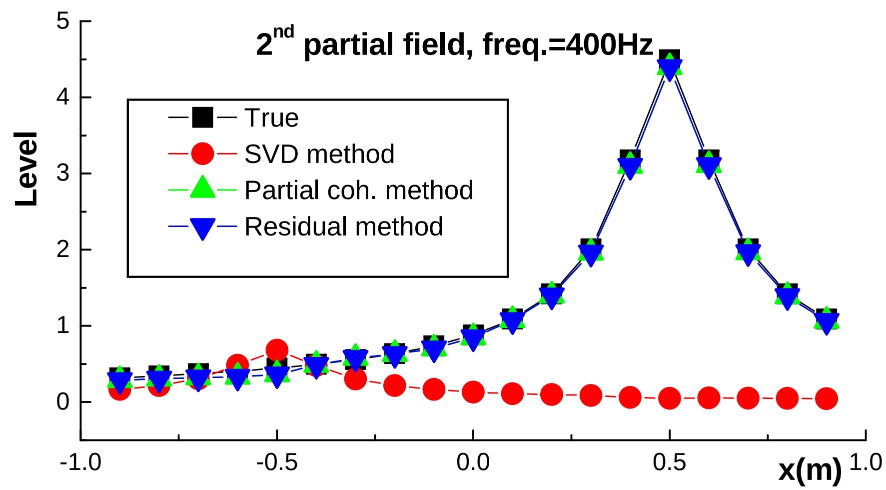
- **Comparison of decomposition methods**
  - Low frequency (400Hz)
    - **source 1 level < source 2 level**

    SVD method : changed order of partial fields
    Partial coh. method : remark of 2nd source
    Residual decomp. method : good estimation
  - Middle frequency (800Hz)
    - **source 1 level = source 2 level**

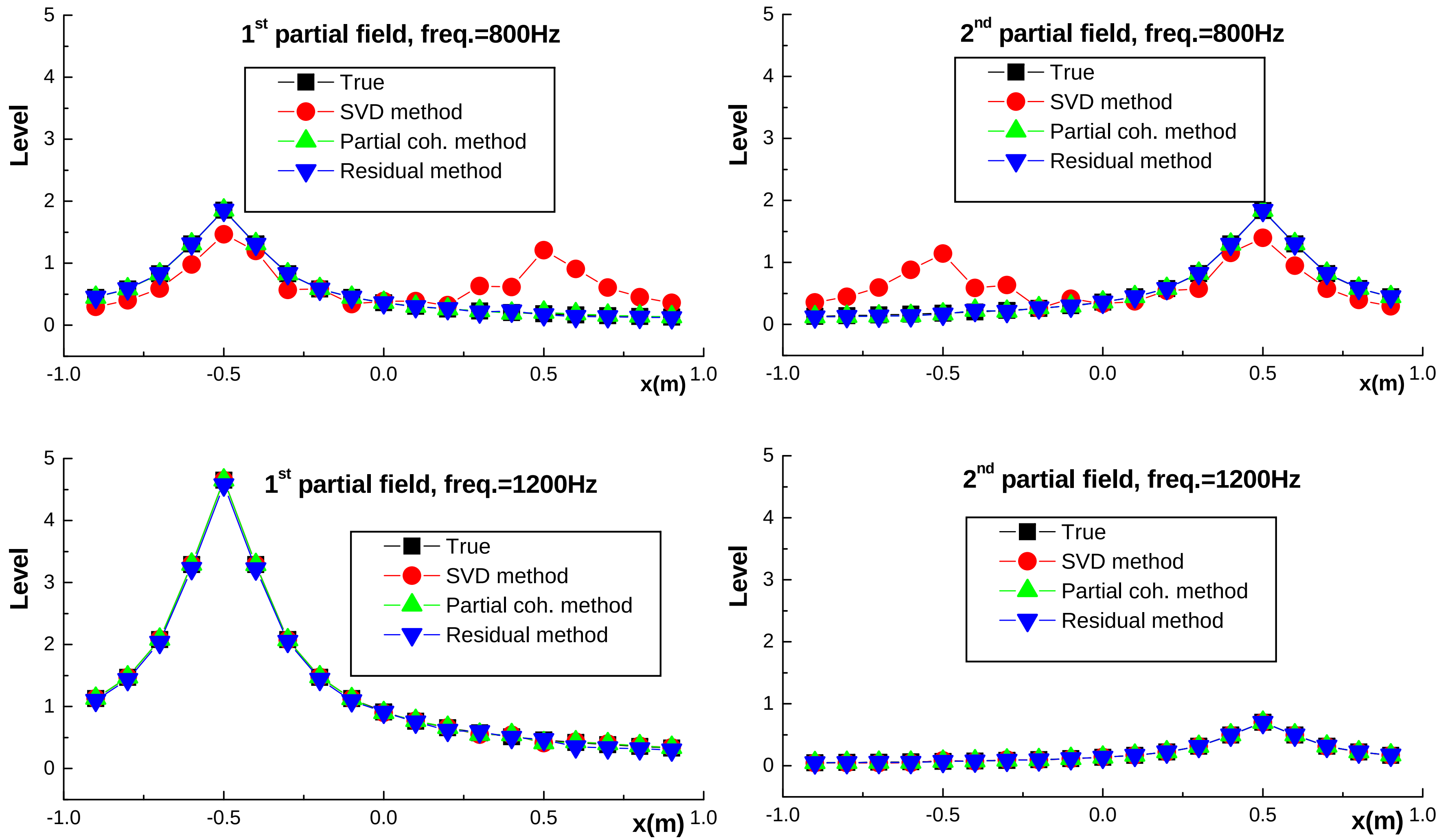
    SVD method : orthogonal, but mixed partial fields
    Partial coh. method : good estimation
    Residual decomp. method : good estimation
  - High frequency (1200Hz)
    - **source 1 level > source 2 level**

    SVD method : good estimation
    Partial coh. method : good estimation
    Residual decomp. method : good estimation





DECOMPOSED PARTIAL FIELDS



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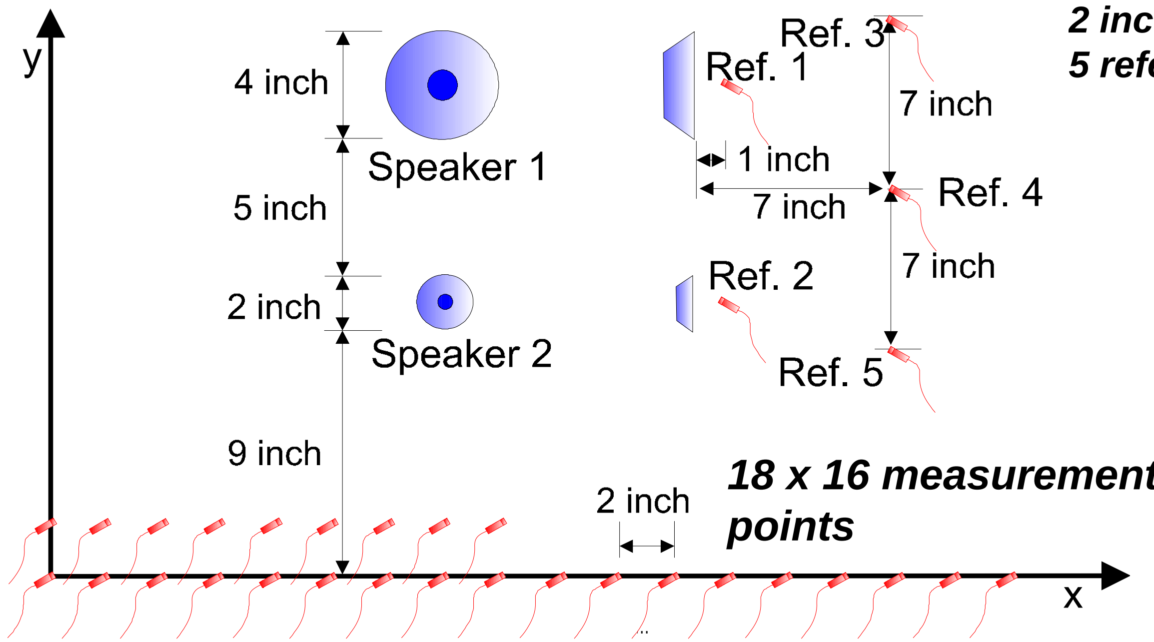
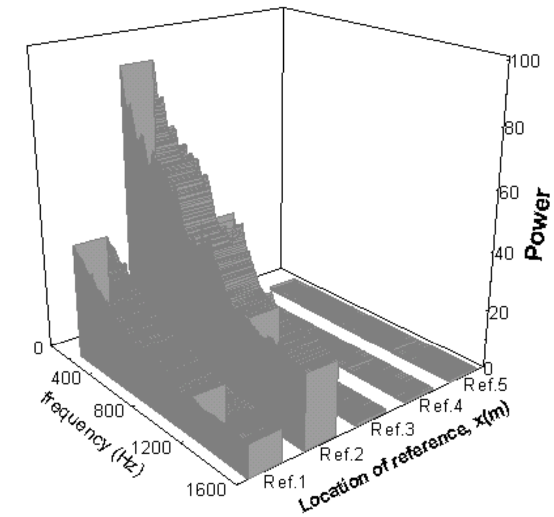
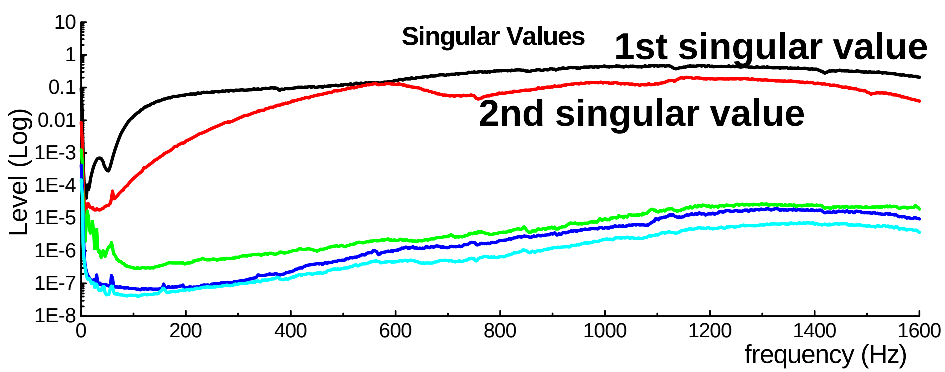
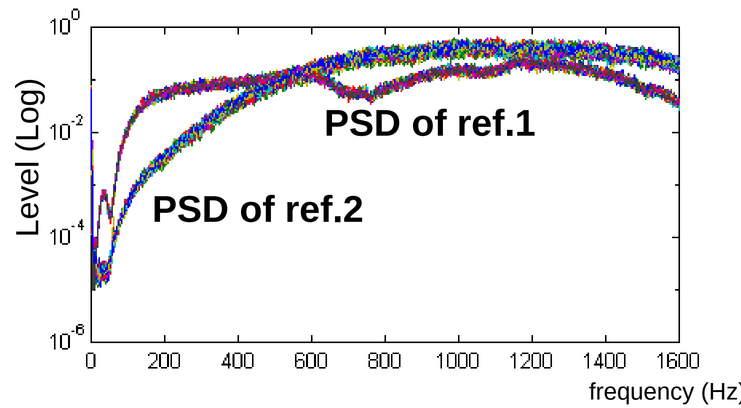
EXPERIMENTAL DESIGN & REFERENCES



2 incoherent sources & 5 reference candidates

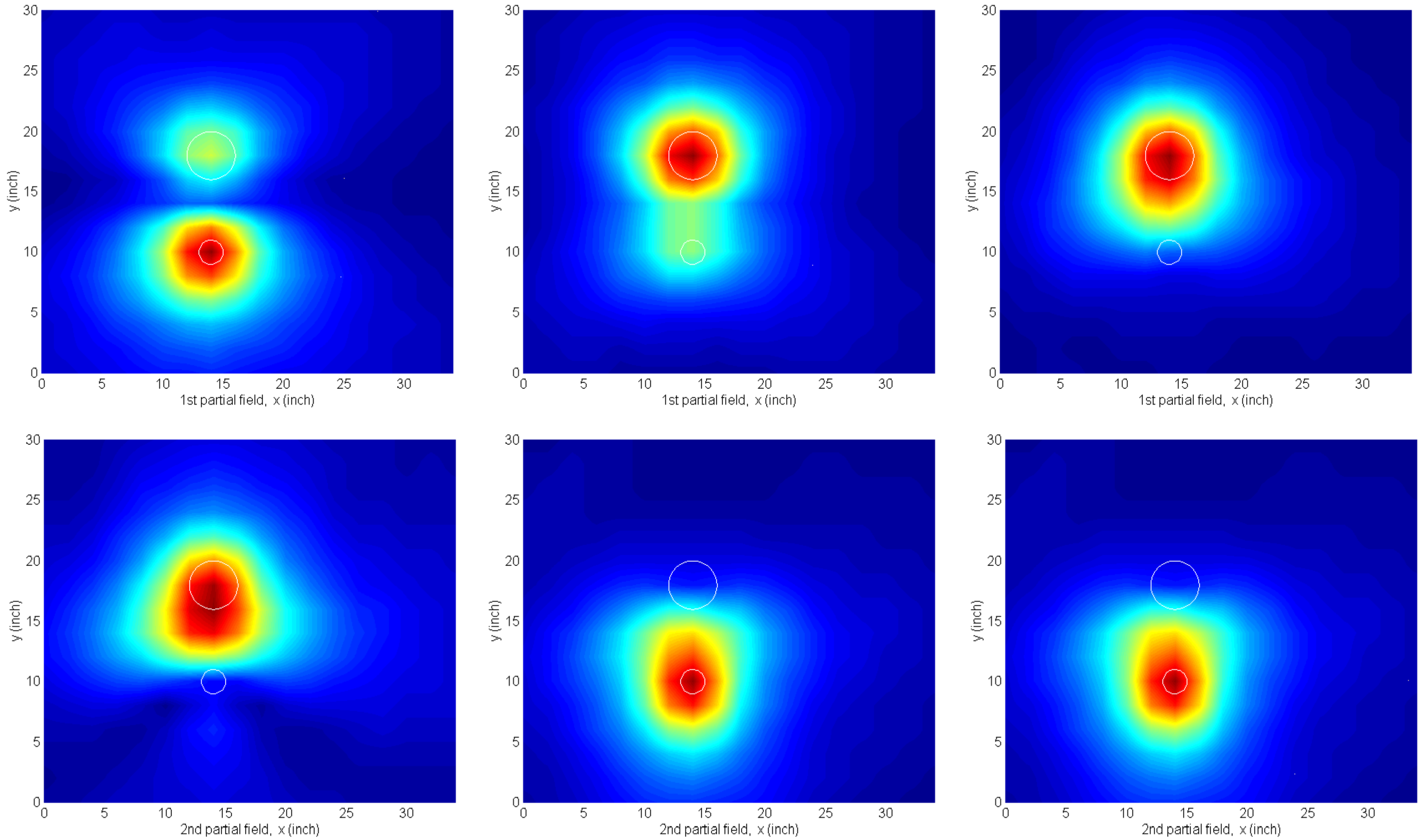






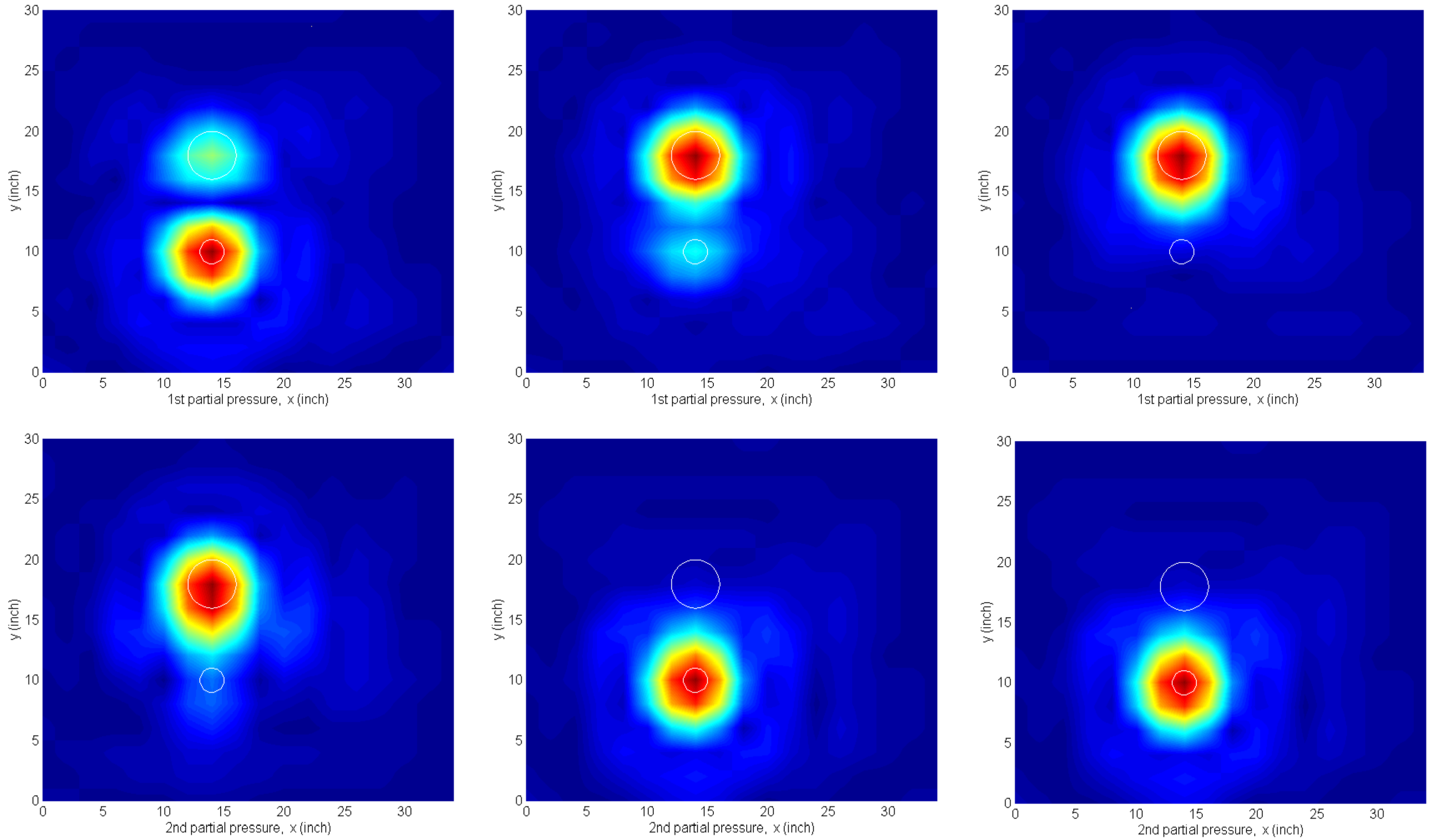
DECOMPOSED HOLOGRAMS (1200Hz)

SVD method | **Partial coherence method** | **Residual decomposition method**

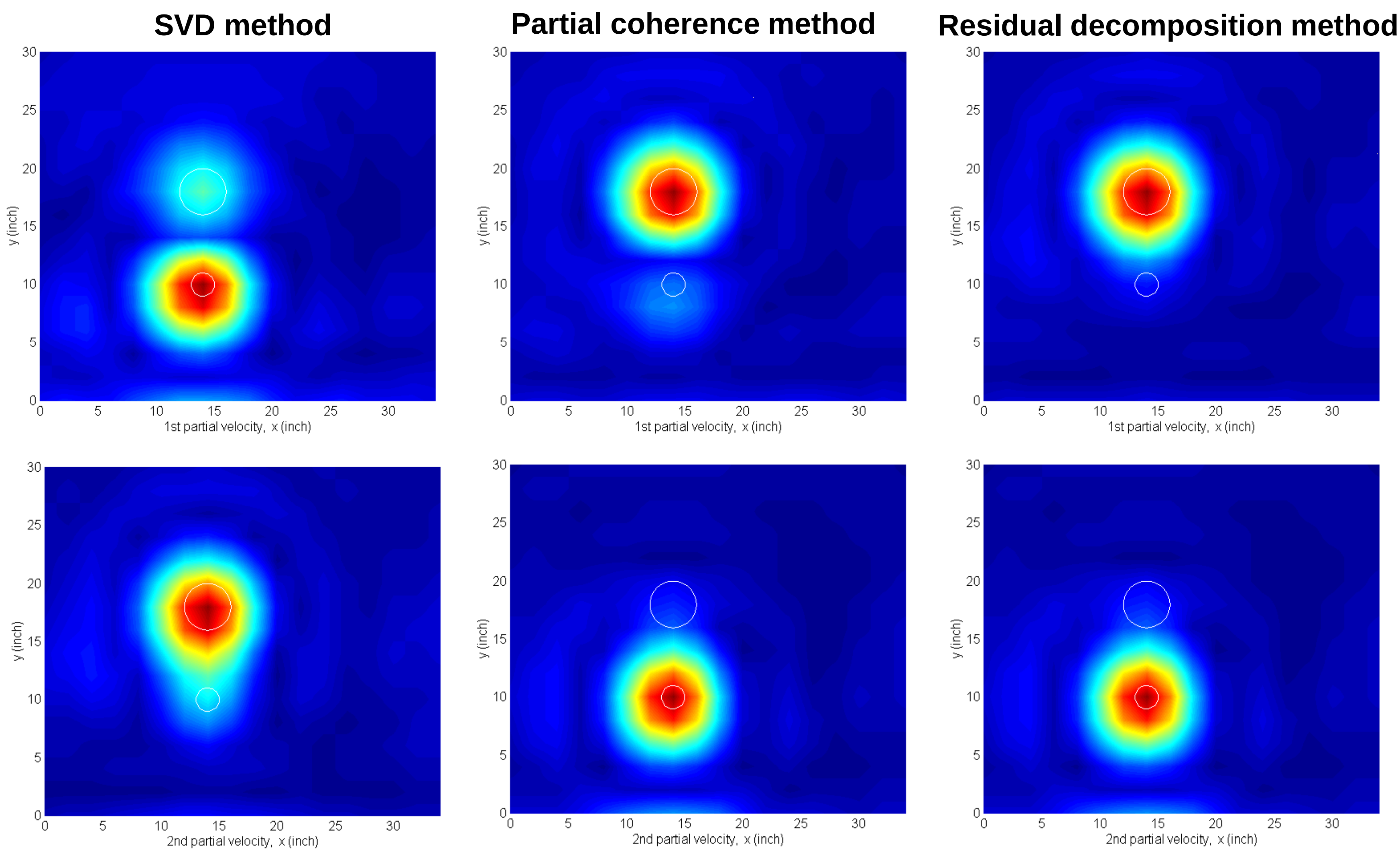


RECONSTRUCTED PRESSURES (1200Hz)

SVD method | **Partial coherence method** | **Residual decomposition method**



RECONSTRUCTED NORMAL VELOCITIES (1200Hz)



CONCLUSIONS

- **Selection of the set of references**
  - Use as many references as incoherent sources, and place them close to incoherent sources
    - **More reference candidates than incoherent sources**
    - **Number of incoherent sources; dominant SV's**
    - **Set of references; MUSIC power w/ trial vectors**
- **Separation of partial fields**
  - SVD method
    - **Orthogonal partial fields, but mixed information from all sources**
  - Partial coherence method
    - **Remarks due to other source signals**
  - Residual field method
    - **Perfect separation of partial field related to each source**
    - **Underestimated partial fields**