# Global Sensitivity Analysis of a Multi-Cylinder Automotive Reciprocating Compressor

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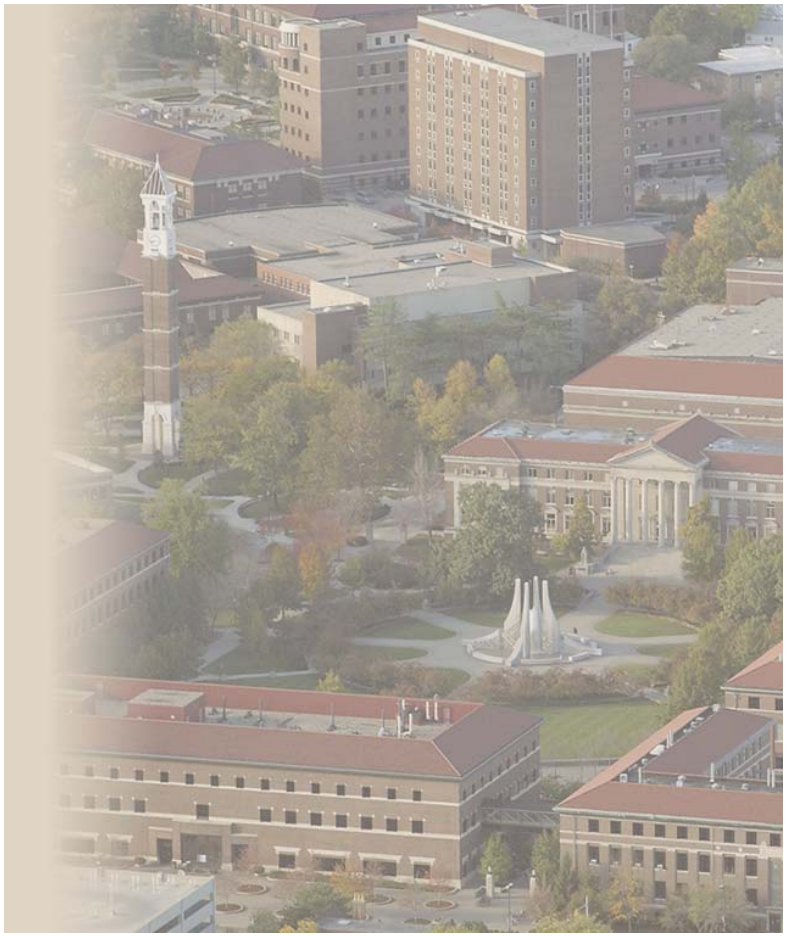

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# Statement of the Objectives

1. Sensitivity Analysis (SA) of gas pulsations of a compressor simulation model using a Global Sensitivity Analysis (GSA) method

2. Determine the sensitivity of gas pulsations in the suction manifold to three design parameters:
   i) radius; ii) width; iii) manifold depth

**3.** Calculate the first order effects and total effects of the manifold pressure response.

# Background

- An already developed and tested model for calculating gas pulsations is suction manifold of multi-cylinder automotive compressor is used. (Park 2004)
- The schematic of the basic simulation model is shown in Figure 2 and details could be found in (Park, 2004).
- Model pressure output response as a function of frequency is shown in Figure 3.

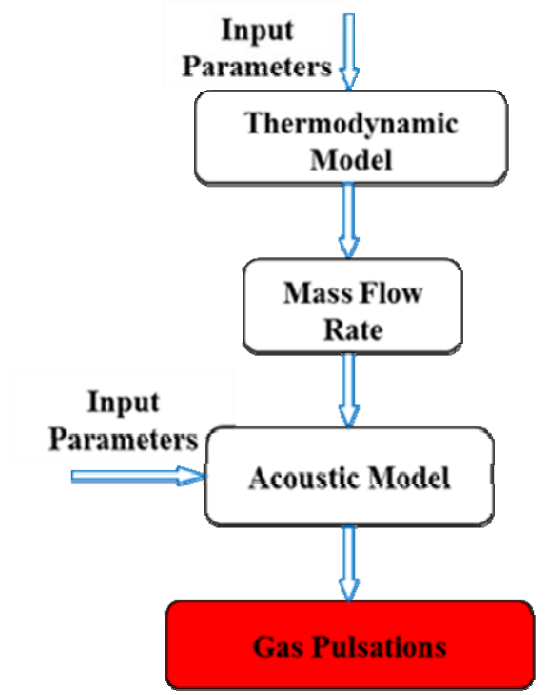



Figure 2


Figure 1

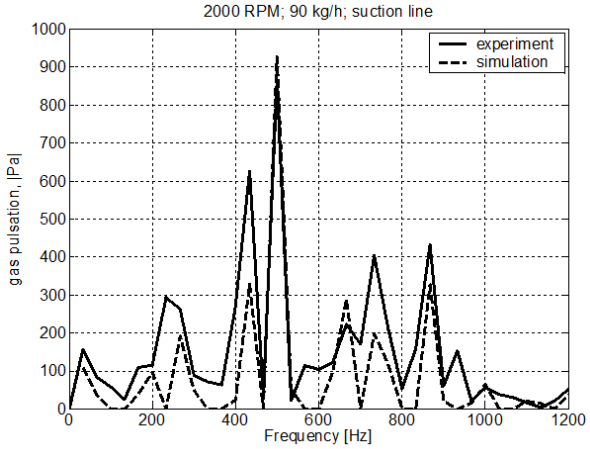



Figure 3

# *Sensitivity Analysis*

- SA is defined as, "*the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variations, and of how the given model depends upon the information fed into it.*"
- Two main approaches:

  (a) Local Sensitivity Analysis (LSA) , (b) Global Sensitivity Analysis
- LSA method also called one-at-time (OAT) is performed by varying one parameter at a time.
- GSA methods vary all the variables simultaneously and each varied through its entire range.
- Most GSA are variance-based methods and depend on the decomposition of variance.

# *VARIANCE BASED SENSITIVITY ANALYSIS*

- The variance based methods depend on the estimation of the following quantity:

$$\frac{Var_X\left[E(Y \mid X)\right]}{Var(Y)}$$

where *Y* denote the output variables,

- o *X* denotes an input variable,
- o *E(Y|X)* denotes the expectation of *Y* conditional on a fixed value of *X*, and the variance is taken over all possible values of *X*, and
- o *V(Y)* is total output variance of the output *Y*.

- The importance of the given input factor can be measured by a term defined as the sensitivity index,
  - which is the fractional contribution to the output variance due to uncertainties in the inputs. While there are many methods available for analyzing the decomposition of variance as a sensitivity measure

# *Sobol's Methods of Global Sensitivity Analysis*

- Sobol's method of GSA– a variance based methods of SA method is used in the study.
- Uses decomposition of the model output function $y = f(x)$ into summands of variance using combinations of input parameters in increasing dimensionality
- Main Idea: decomposition of the function $f(x)$ into summands of increasing dimensionality is given by:

$$f(x_1,\ldots,x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1\le i\le j\le k} f_{ij}(x_i,x_j) + \cdots + f_{1,2,\cdots,k}(x_1,\cdots x_k).$$

- $f_0 = E(Y)$
- $f_i(X_i) = E(Y|X_i) - f_0$
- $f_{ij}(X_i,X_j) = E(Y|X_i,X_j) - f_0 - f_i - f_j$

where $f_i$ is the effect of varying $X_i$ alone, and

$f_{ij}$ is the effect of varying $X_i, X_j$ simultaneously

- First Order Effects
  - o $f_0 = E(Y)$
  - o $f_i(X_i) = E(Y|X_i) - f_0$

  where
  - The first order effect, or the main effect, of a single input parameter is the effect of a single parameter on the model output variance.
- First Order Indices

$$S_i = \frac{V_i}{Var(Y)}$$

where, $V_i = Var_{X_i}\left(E_{X_{\sim i}}(Y|X_i)\right)$

- Second Order Effects

o $f_{ij}(X_i, X_j) = E(Y|X_i, X_j) - f_0 - f_i - f_j$

where, $f_{ij}$ is the effect of varying $X_i, X_j$ simultaneously, plus the effect of their individual variations.

- **Total Effects**
  - The total effects include the first order effects and all the interactions involving that parameter.
  - It measures the contribution to the output variance of $X_i$, including variance caused by its interactions of any order, with any input variable.
- **Example:**
- if there are only three input parameters, *A*, *B*, and *C*, then the total effects of a parameter *A*, for instance, on the output is,

$$TS(A)=S(A)+S(AB)+S(AC)+S(ABC)$$

where TS(i) is the total sensitivity index of parameter i,

- **Total-effect Index**

$$S_i^{tot} = \frac{D_i^{tot}}{D}$$

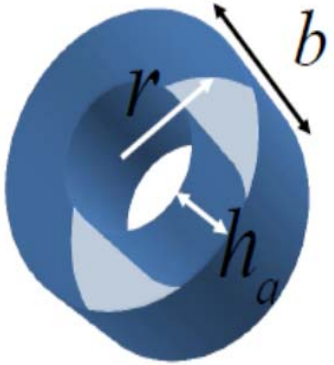
## *Implementation of the Sobol's Method to Compressor Simulation Model*

The method of Sobol was implemented by following a series of steps as outlined below:

- The parameters and their ranges of variation were defined as shown in Table 1.
- A probability distribution to sample the variables was defined. In this case, a uniform distribution to generate the input vector.
- Simulations were run for 295 iterations.

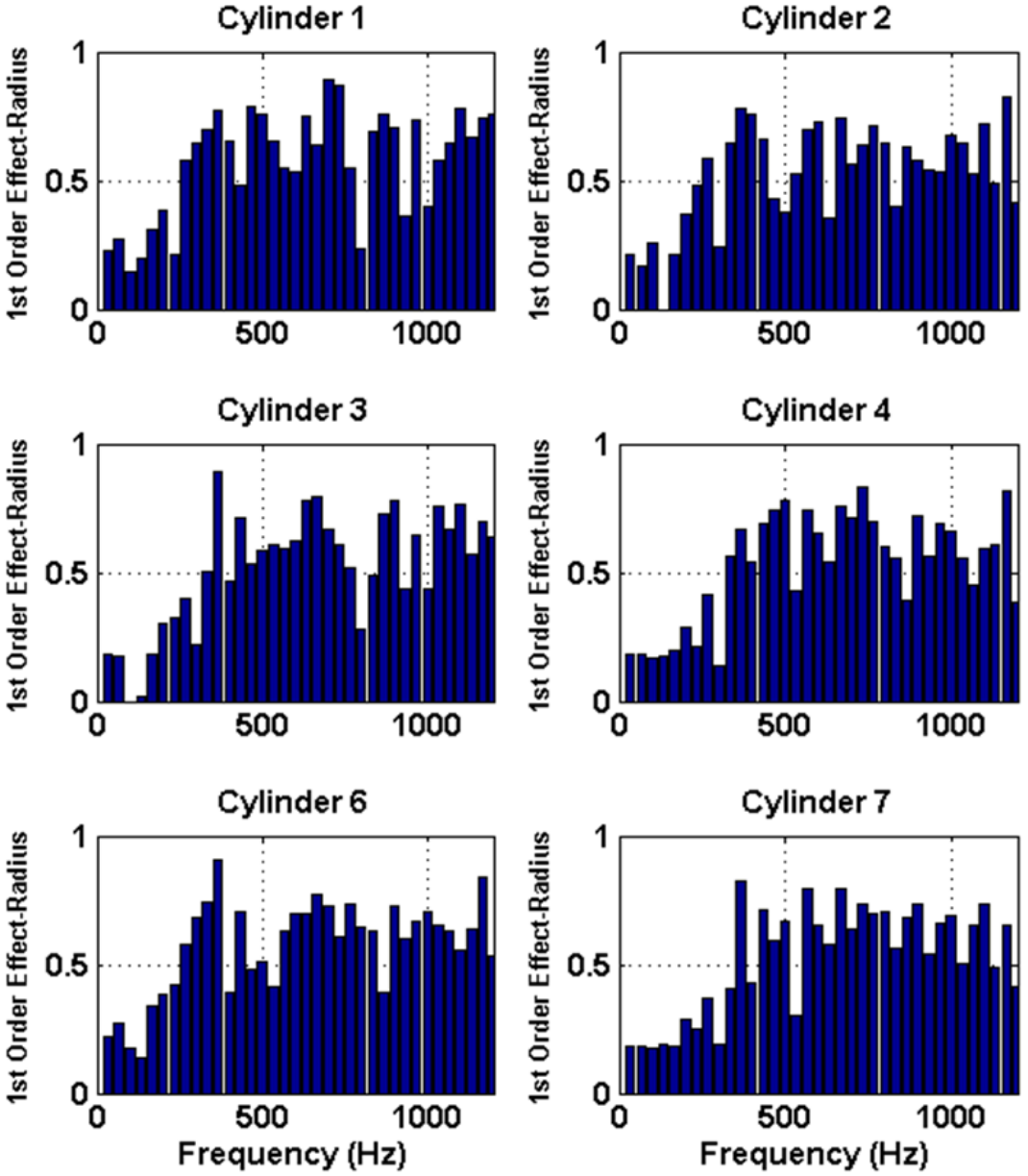
**Table 1:** Numerical values of the variables used in simulation

| Variables | Mean (mm) | Minimum Value (mm) | Maximum Value (mm) |
|---|---|---|---|
| Manifold radius, ***r*** | 50 | 30 | 70 |
| Manifold depth, ***b*** | 27 | 20 | 40 |
| Manifold width, ***ha*** | 14 | 8 | 20 |

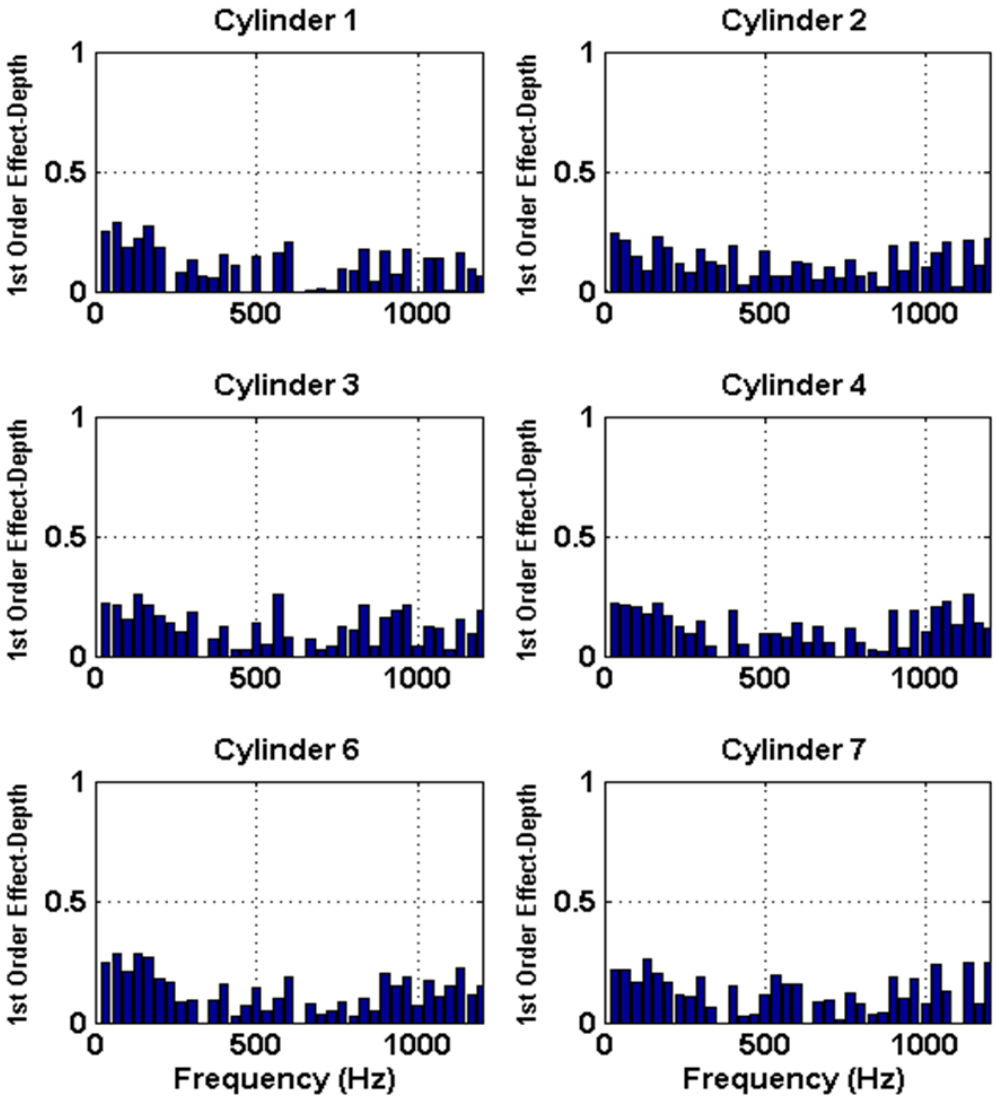
# Results : First Order Effects (Radius)

- If the sensitivity index at particular frequency is low, it shows that at that particular frequency the pressure response is less affected by varying the parameter of interest.
- Figure shows the manifold pressure response is less sensitive to variation in radius at lower frequencies, i.e. up to 300 Hz, because the value of the sensitivity index is low.
- Beyond this range, the variations in manifold radius have a much more prominent effect on gas pulsations.



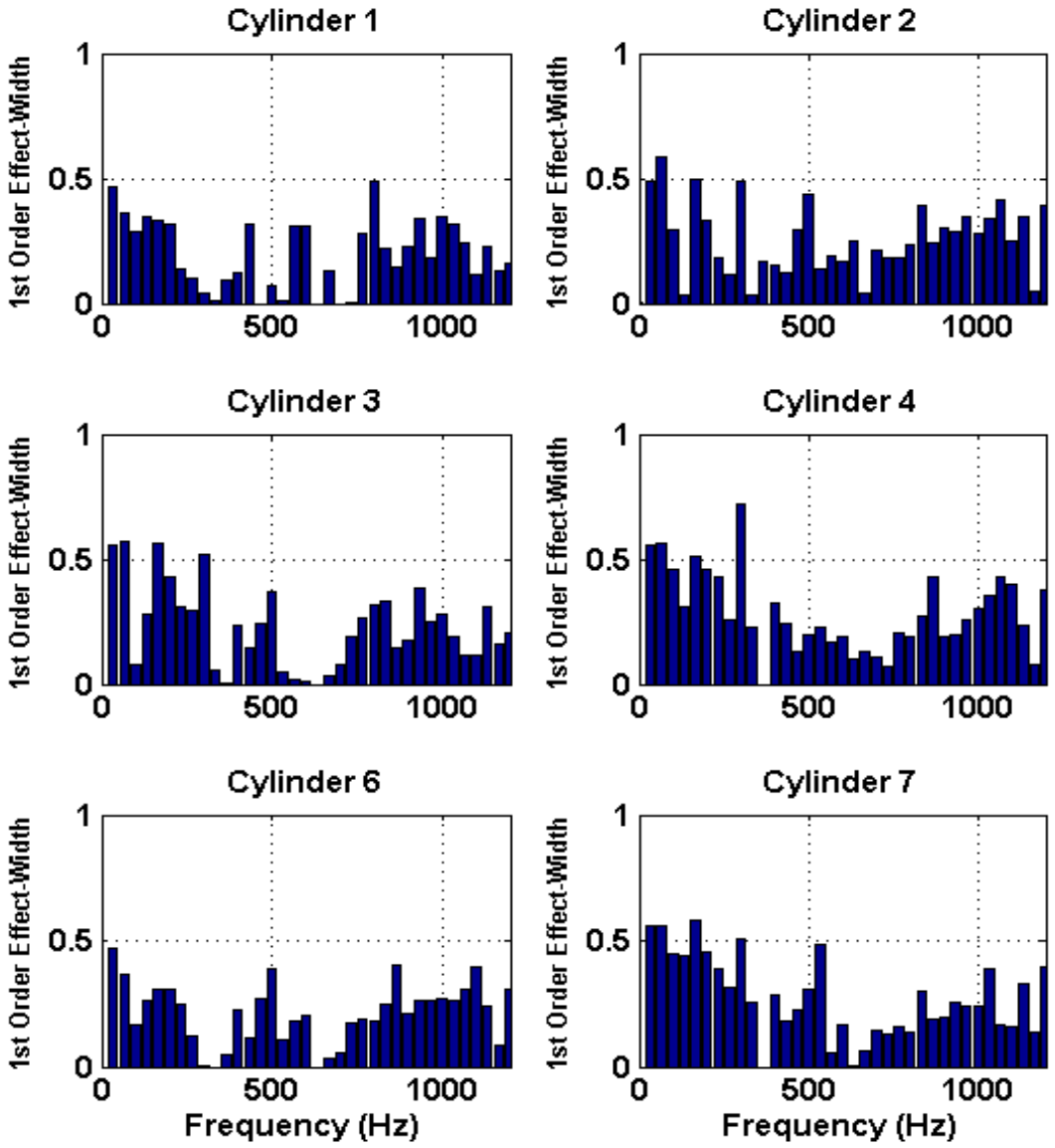
# *Results : First Order Effects (Depth)*

- Figure shows the value of sensitivity indices for each frequency in the range 0-1200 Hz.
- manifold depth does not have much effect on the gas pulsations



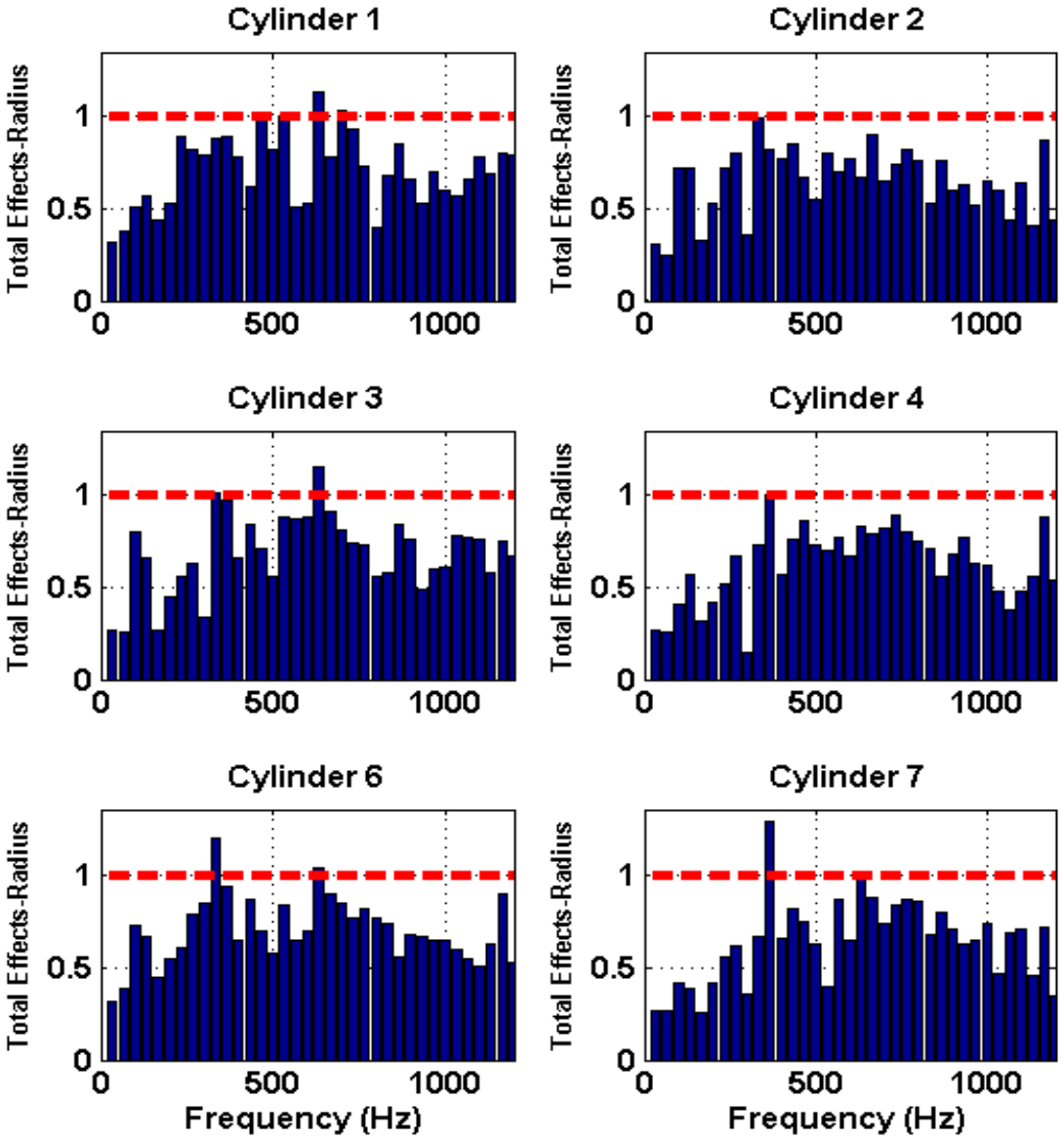
# *Results : First Order Effects (Width)*

- The manifold width is more influential than the depth of the manifold, but less influential compared to the radius



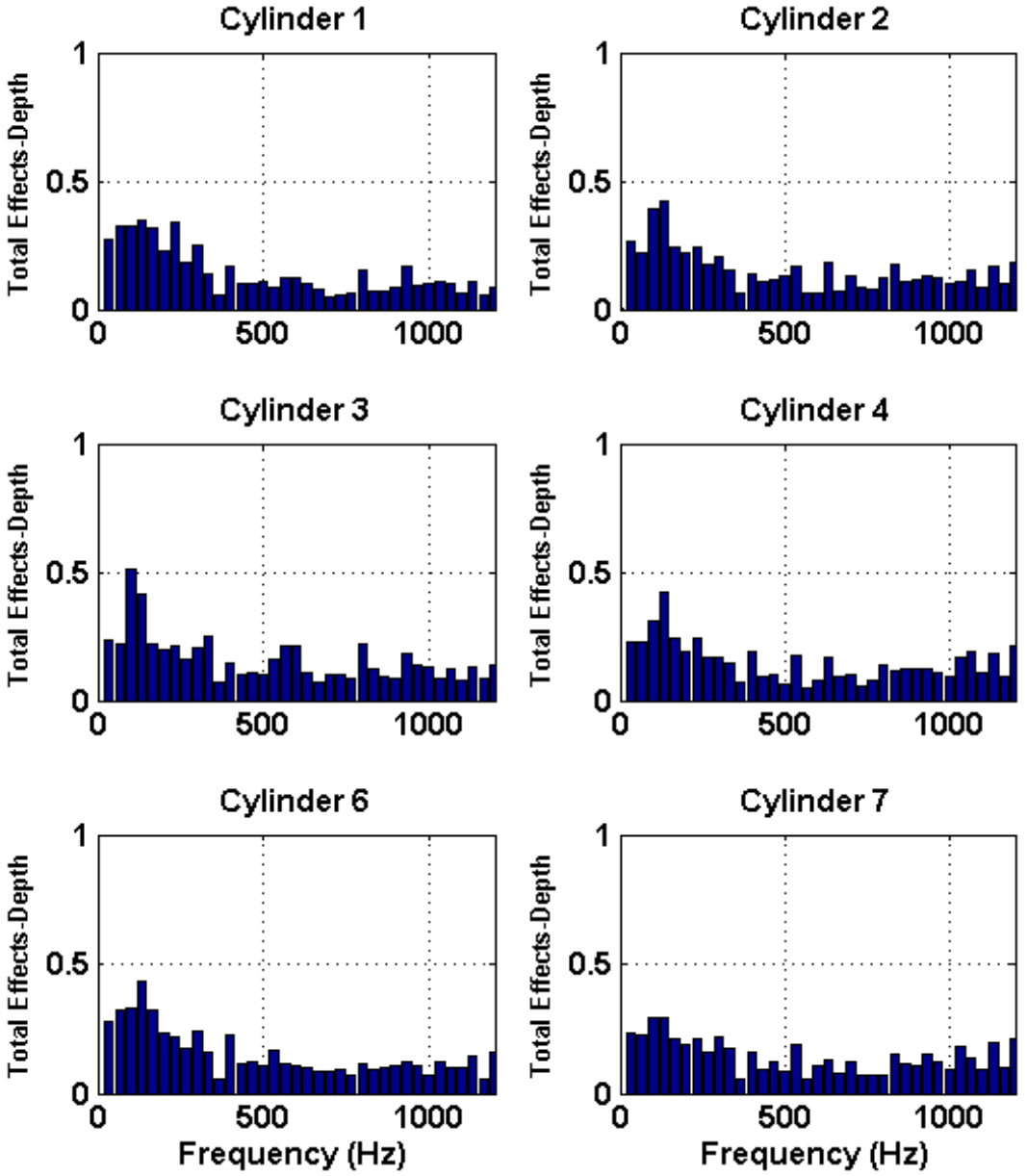
# Results : Total Effects (Radius)

- For a linear system, the total effect should add up to unity.
- radius has much more impact across the entire frequency range
- the value of radius at certain frequencies is more than one, which indicates that there are nonlinear interactions taking place by changing the parameter radius.



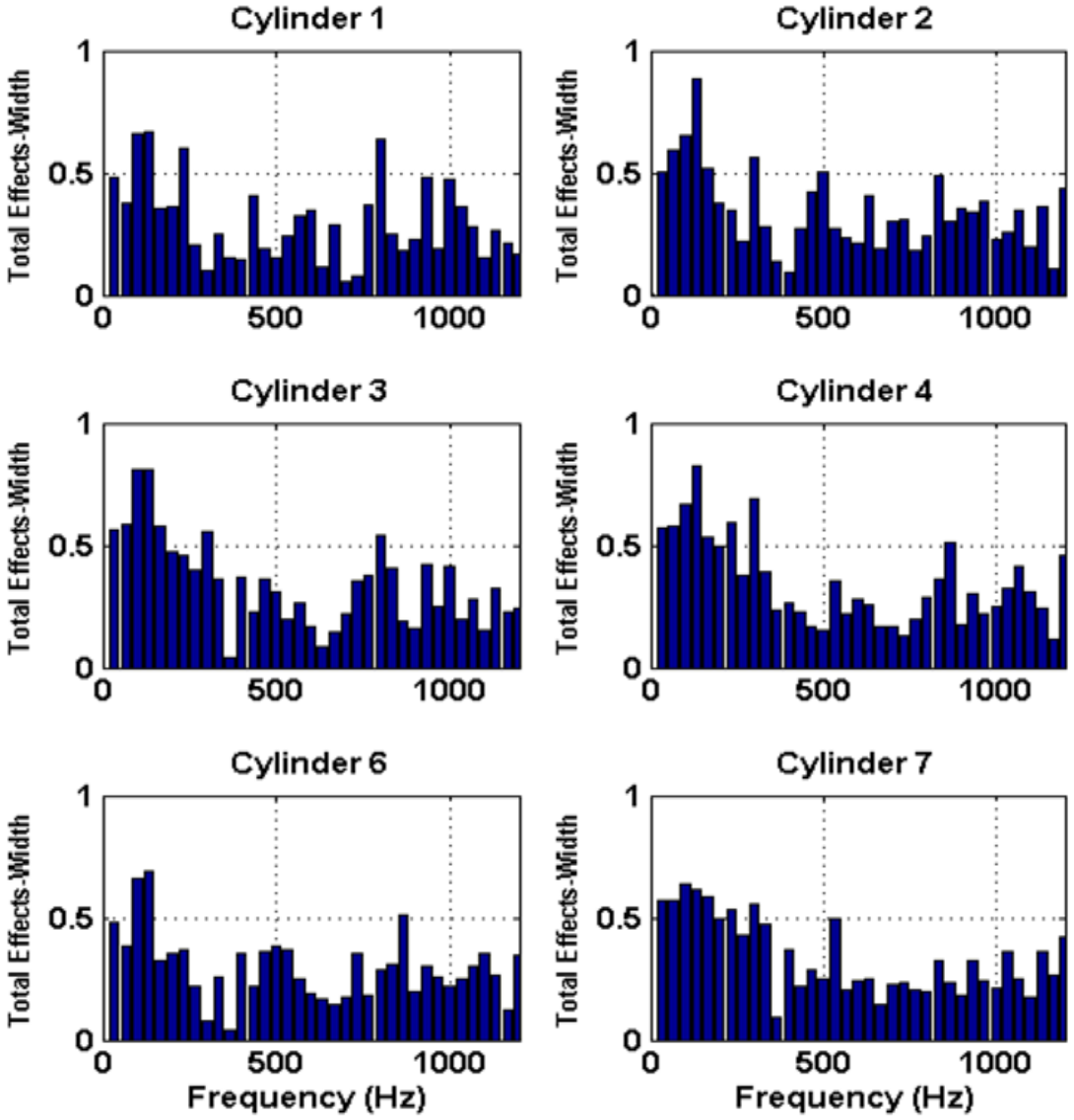
# Results : Total Effects (Depth)

- The sensitivity of manifold depth is quite low, and compared to other factors, the value of depth variation index never goes above 0.5 indicating that there are not many interactions and no nonlinear effects



# *Results : Total Effects (Width)*

- manifold width has more impact at lower frequencies where it is quite influential



# CONCLUSIONS

- Sobol's method of global sensitivity analysis was used to calculate the first order effects and total effects of the gas pulsation due to three design parameter, namely, suction manifold radius, width and depth.
- The uncertainty and sensitivity of individual parameters and also the interaction between these parameters.
- It was also showed that suction manifold pressure response was most sensitive to changes in manifold radius, followed by manifold width and depth.
- It was also seen that at certain frequencies the total effects were greater than one which indicates that there is interaction between parameters.
- This method of sensitivity analysis can be readily extended to any compressor simulation model.

**Thank you for your attention!**

**Questions?**

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