ST.PETERSBURG STATE POLYTECHNICAL UNIVERSITY

St.Peterburg State Polytechnical University

SUPERSONIC AXIAL COMPRESSOR STAGE SIMPLIFIED ANALYSIS

Prof. Y.B. Galerkin
PhD, Ass. Prof. K.V. Soldatova

July, 2014

Supersonic axial stages with big pressure ratio are increasingly in demand. There is a problem to elevate pressure ratio of a stage up to 3 and more. Efficiency of a stage can be limited by shock wave losses at high supersonic speeds. The numerical analysis of losses was made in a plane cascade. Calculated losses in shock waves depend on a velocity coefficient and an angle of a shock wave. Pressure loss calculation in subsonic parts of a stage was made by loss coefficient whose value was based on expert assessment. It is shown that up to velocity coefficient 1.5 shock wave losses are not an obstacle for an acceptable level of stage efficiency.

INTRODUCTION

Application of supersonic axial compressor stages is an effective way to decrease mass and size of gas turbines. It is reported that stages with pressure ratio up to 2,8 and blade velocity about 450 m/s can operate quite satisfactory – Fig. 1.

Euler coefficients were calculated at suppositions $\eta_{t\,ad}$ = 0,87, γ=1,4, c_p=1005 J/kg, T_{0t} = 288 K and presented in the Table 1.

Euler coefficient at an impeller outer diameter:

$$\psi_T = \frac{H}{U^2}. \qquad (1)$$

Total enthalpy:

$$H = \frac{\gamma}{\gamma - 1} R T_{1t} \left(\left(\pi_t \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{t\,ad}}, \qquad (2)$$

Table 1

Estimation of Euler coefficients of high pressure ratio axial stages

π_i	U, m/s	H, J/kg	ψ_T
1,6	370	47810	0,349
1,82	455	62080	0,300
1,9	370	66960	0,489
2,05	455	75850	0,366
2,25	420	86740	0,492
2,55	440	102000	0,527
2,8	450	113780	0,562

$$H = \frac{\gamma}{\gamma - 1} R T_{1i} \left((\pi_i)^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{iad}}, \qquad (2)$$

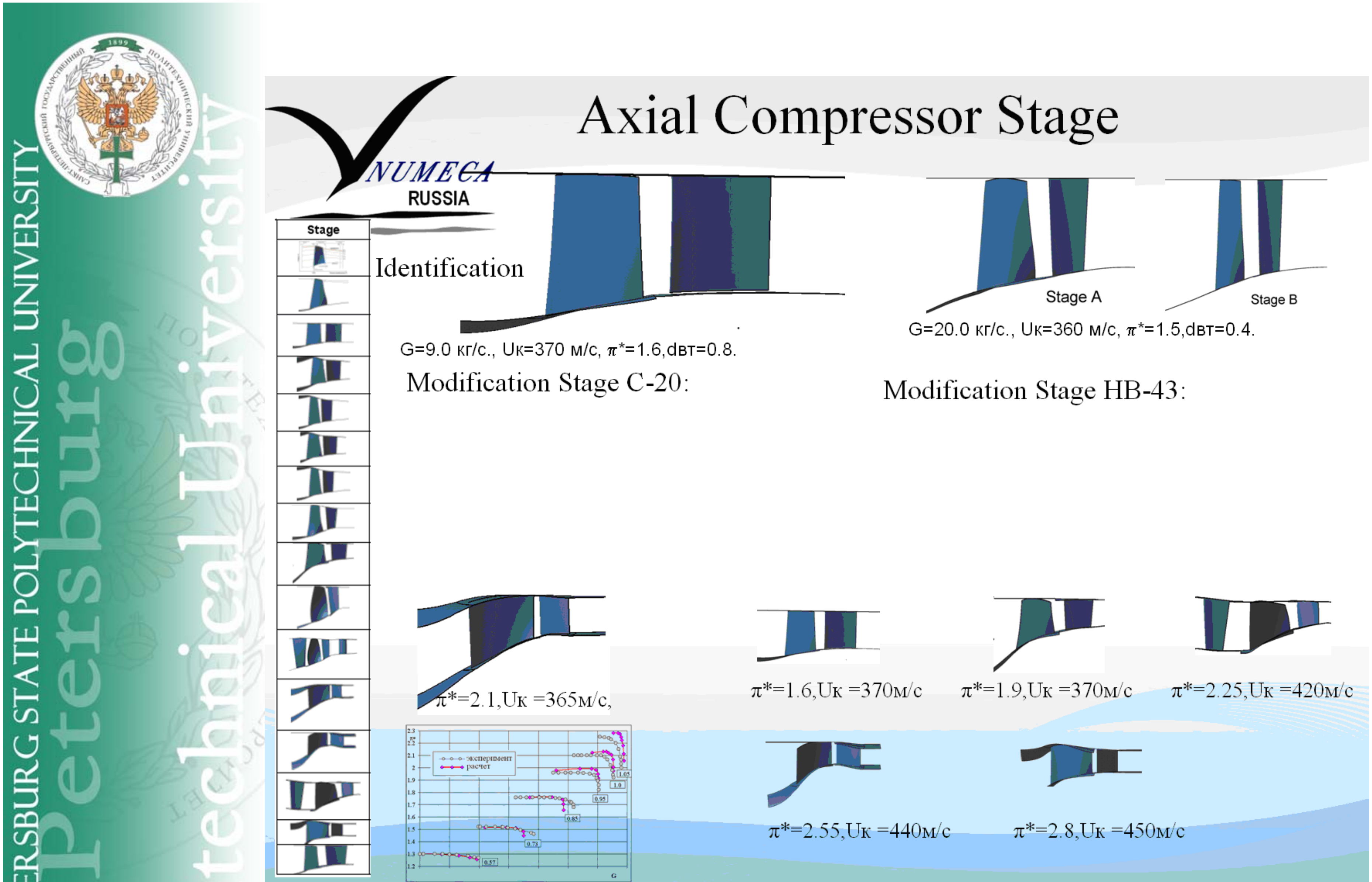

Fig. 1. Information on modern supersonic axial compressor stages

Object

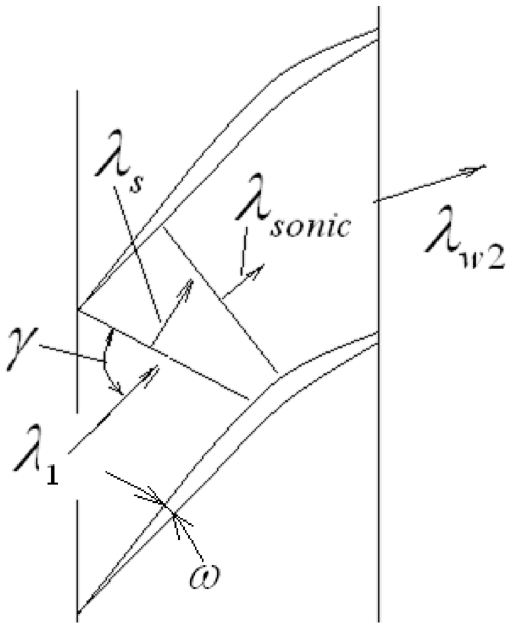

Fig. 2. Elementary supersonic blade cascade and oblique – direct shock wave scheme

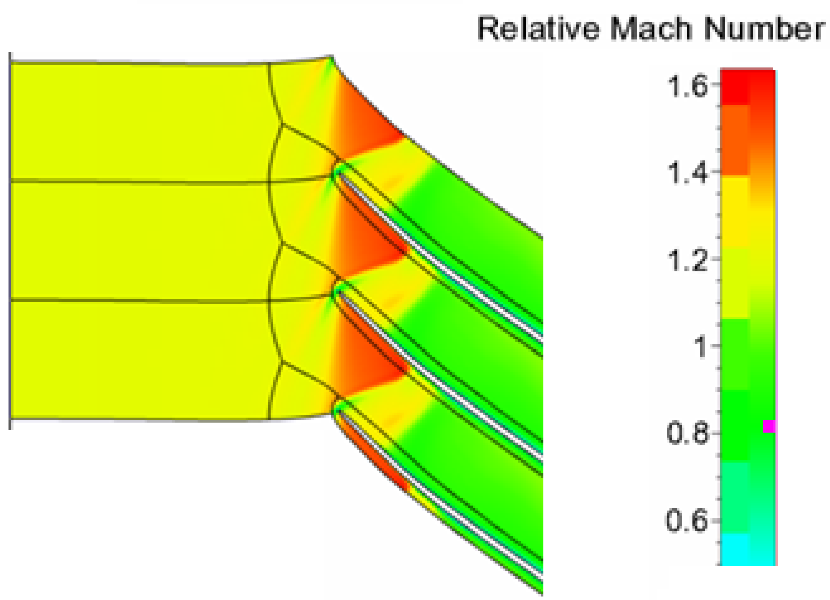

Fig. 3. Typical Mach number field at an impeller inlet (NUMECA Fine/AutoGrid)

Scheme and equations

Oblique shock angle depends on a leading edge angle ω and Mach number:

$$tg\left(\gamma_{os}-0,5\omega\right)=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{\gamma+1}\frac{1}{M_1^2\,sin^2\,\gamma_{os}}\right)tg\gamma_{os}. \qquad (3)$$

Minimal value of an oblique shock wave is function of Mach number:

$$sin\,\gamma_{os0}=\frac{1}{M_1}. \qquad (4)$$

There is no shock but a sound wave only if an angle value is γ_{os0}.

The shock parameters are easier analyzed by velocity coefficient $\lambda=w/\sqrt{\frac{2\gamma}{\gamma+1}RT_t}$ as it directly proportional to flow velocity. Mach number and velocity coefficient are connected by Eq. (5):

$$M_1=\lambda_1\sqrt{\frac{\frac{2}{\gamma+1}}{1-\frac{\gamma-1}{\gamma+1}\lambda_1^2}}. \qquad (5)$$

The next equation defines a velocity coefficient after an oblique shock:

$$\lambda_{1_s} = \sqrt{\lambda_1^2 \cos^2 \gamma_\infty + \frac{\left(1 - \frac{\gamma - 1}{\gamma + 1} \lambda_1^2 \cos^2 \gamma_\infty\right)^2}{\lambda_1^2 \left(1 - \cos^2 \gamma_\infty\right)}}. \qquad (6)$$

If velocity coefficient $\lambda_{1_s} > 1$ a normal shock follows with subsonic velocity after it;

$$\lambda_{sonic} = \frac{1}{\lambda_s}. \qquad (7)$$

Isentropic equations connect total and static pressures at a cascade inlet, after oblique and normal shocks. They are:

$$\frac{p_{1t}}{p_1} = \left(\frac{1}{1 - \frac{\gamma - 1}{\gamma + 1} \lambda_1^2}\right)^{\frac{\gamma}{\gamma - 1}} \quad (8), \qquad \frac{p_{st}}{p_s} = \left(\frac{1}{1 - \frac{\gamma - 1}{\gamma + 1} \lambda_s^2}\right)^{\frac{\gamma}{\gamma - 1}} \quad (9),$$

$$\frac{p_{sonict}}{p_{sonic}} = \left(\frac{1}{1 - \frac{\gamma - 1}{\gamma + 1} \lambda_{sonic}^2}\right)^{\frac{\gamma}{\gamma - 1}}. \qquad (10)$$

Static pressure ratios in oblique and normal shocks are given by equatio

$$\frac{p_c}{p_1} = \frac{\lambda_1^2\left[1 - \frac{4k}{(\gamma+1)^2}\cos^2\gamma_w\right] - \frac{\gamma-1}{\gamma+1}}{1 - \frac{\gamma-1}{\gamma+1}\lambda_1^2}. \tag{11}$$

$$\frac{p_{sonic}}{p_2} = \frac{\lambda_2^2 - \frac{\gamma-1}{\gamma+1}}{1 - \frac{\gamma-1}{\gamma+1}\lambda_2^2}. \tag{12}$$

The equations (8 - 12) define total pressure loss in shock waves:

$$\frac{p_{sonic\,t}}{p_{1t}} = \left(\frac{p_2}{p_1}\cdot\frac{p_{sonic}}{p_2}\cdot\frac{p_{sonic\,t}}{p_{sonic}}\right) / \frac{p_{1t}}{p_1}. \tag{13}$$

Losses in a subsonic part are calculated by loss coefficient ζ_{od} :

$$H_{wod} = \zeta_{od}\frac{W_{sonic}^2}{2}. \tag{14}$$

This coefficient is connected with velocities in a subsonic part:

$$\frac{w_{sonic}^2 - w_1^2}{2} = H_{ad} + H_{wad} . \qquad (15)$$

Pressure ratio in a subsonic part could be derived from Eq. (14, 15):

$$\frac{p_1}{p_{sonic}} = \left(1 + \frac{\gamma - 1}{\gamma + 1} \frac{\lambda_{sonic}^2\left[1 - (\lambda_1 / \lambda_{sonic})^2\right] - \zeta_{ad}}{1 - \frac{\gamma - 1}{\gamma + 1}\lambda_{sonic}^2}\right)^{\frac{\gamma}{\gamma - 1}} . \qquad (16)$$

Isentropic equation connects total and static pressures at the exit of the cascade:

$$\frac{p_1^*}{p_1} = \left(\frac{1}{1 - \frac{\gamma - 1}{k + 1}\lambda_1^2}\right)^{\frac{\gamma}{\gamma - 1}} . \qquad (17)$$

Calculated parameters

The following parameters are presented as result of calculations:

- velocity coefficients after oblique and normal shock waves $\lambda_2 = f(\lambda_1, \gamma_\infty)$, $\lambda_{sonic} = f(\lambda_1, \gamma_\infty)$,
- static pressure ratios after oblique and normal shock waves and in model as a whole of a stage, subsonic part included:

$$\pi_2 = \frac{p_2}{p_1} = f(\lambda_1, \gamma_\infty), \pi_{sonic} = \frac{p_{sonic}}{p_2} = f(\lambda_1, \gamma_\infty), \pi = \frac{p_{ex}}{p_1} = f(\lambda_1, \gamma_\infty).$$

- polytropic efficiency and loss coefficient of shock waves or a stage in a whole if subsonic part is taken into account:

$$\eta = \frac{\lg\left(\frac{p_2}{p_1}\right)}{\frac{\gamma}{\gamma-1}\lg\left(\frac{1-\frac{\gamma-1}{\gamma+1}\lambda_2^2}{1-\frac{\gamma-1}{\gamma+1}\lambda_1^2}\right)}, \quad (18)$$

$$\zeta = (1-\eta)\left(1-\frac{\lambda_2^2}{\lambda_1^2}\right). \quad (19)$$

ST.PETERSBURG STATE POLYTECHNICAL UNIVERSITY

Petersburg Politechnical University

Efficiency and shock waves parameters

Graphics in the Fig. 4 show two zones of a cascade possible operation:
- subsonic flow after an oblique shock wave with angles bigger than 720-620 (the bigger value corresponds to a smaller velocity coefficient);
- supersonic flow after an oblique shock wave with angles smaller than 720-620 and a normal shock wave after.

The next Fig. 5 demonstrates velocity coefficients after normal shock if a flow is supersonic after an oblique shock.

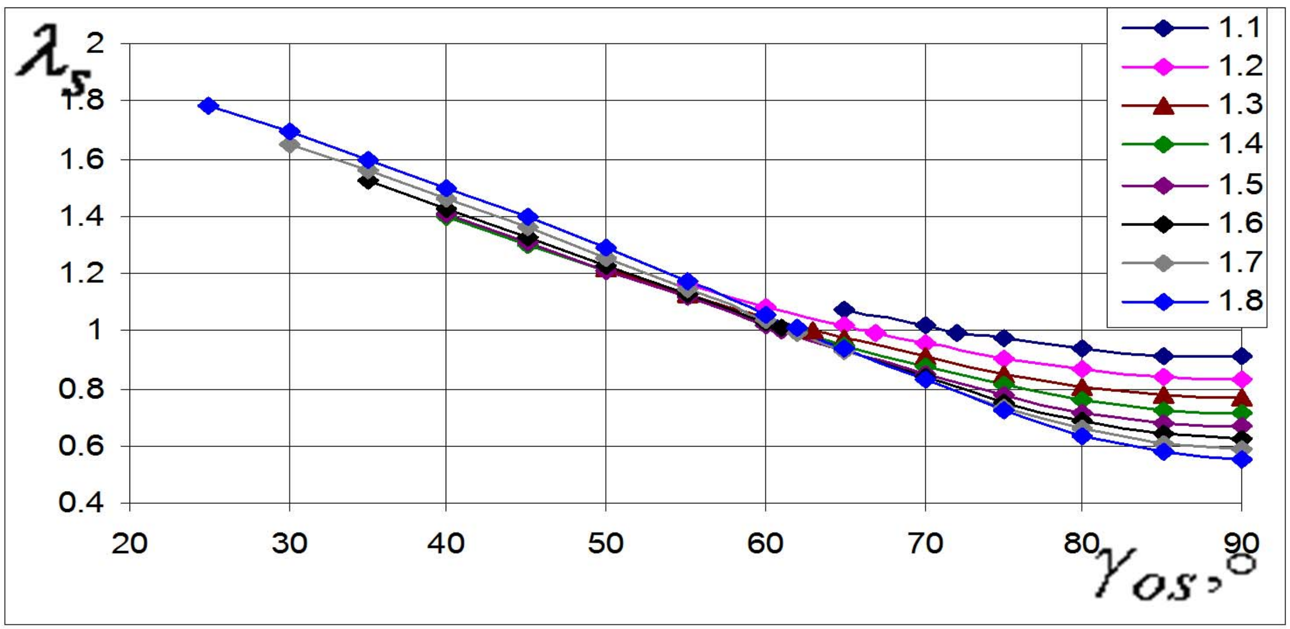

Fig. 4. Velocity coefficients after an oblique shock wave

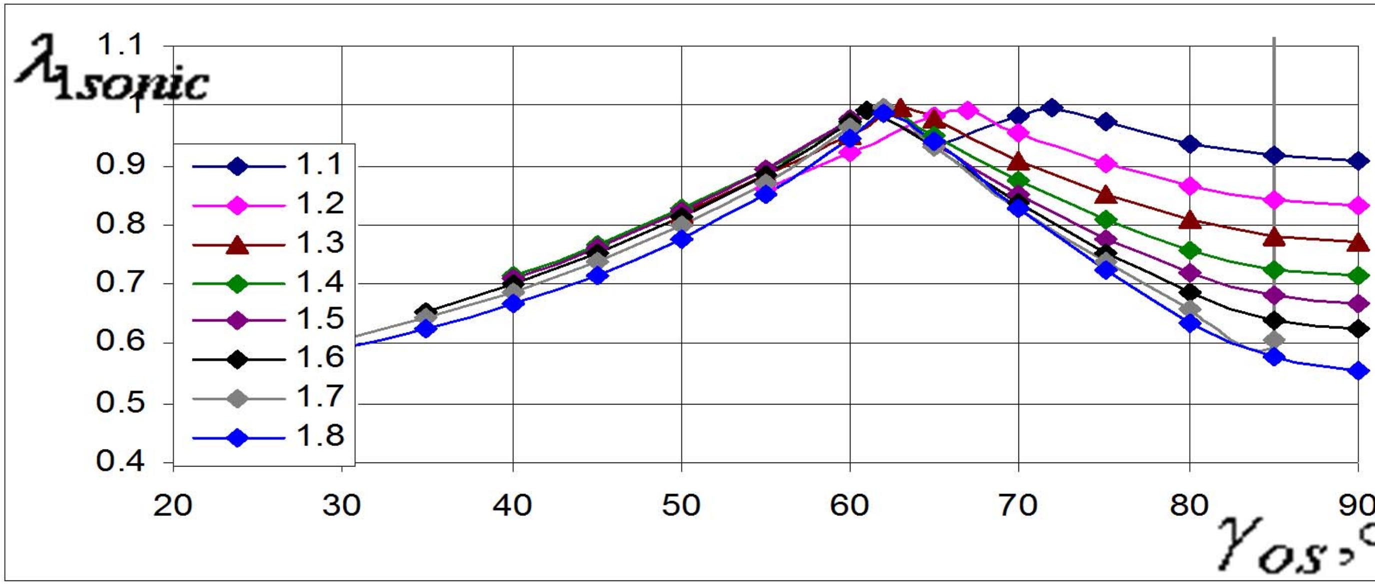

Fig 5. Velocity coefficients in a subsonic part of a cascade

The highest pressure ratio at given velocity coefficient take place when front angle is about 40^0. Extremely high pressure ratios at high velocity coefficients $\lambda_1 > 1,5$ hardly are realistic now as values $\lambda_1 = 1,6\text{-}1,8$ correspond to a blade speed U >600 m/s. The maximum pressure ratio 2,8-8,9 for $\lambda_1 = 1,4\text{-}1,8$ corresponds to an angle $\gamma_\infty \approx 40^0$. Corresponding efficiency values (Fig. 6) are 0,89 – 0,925.

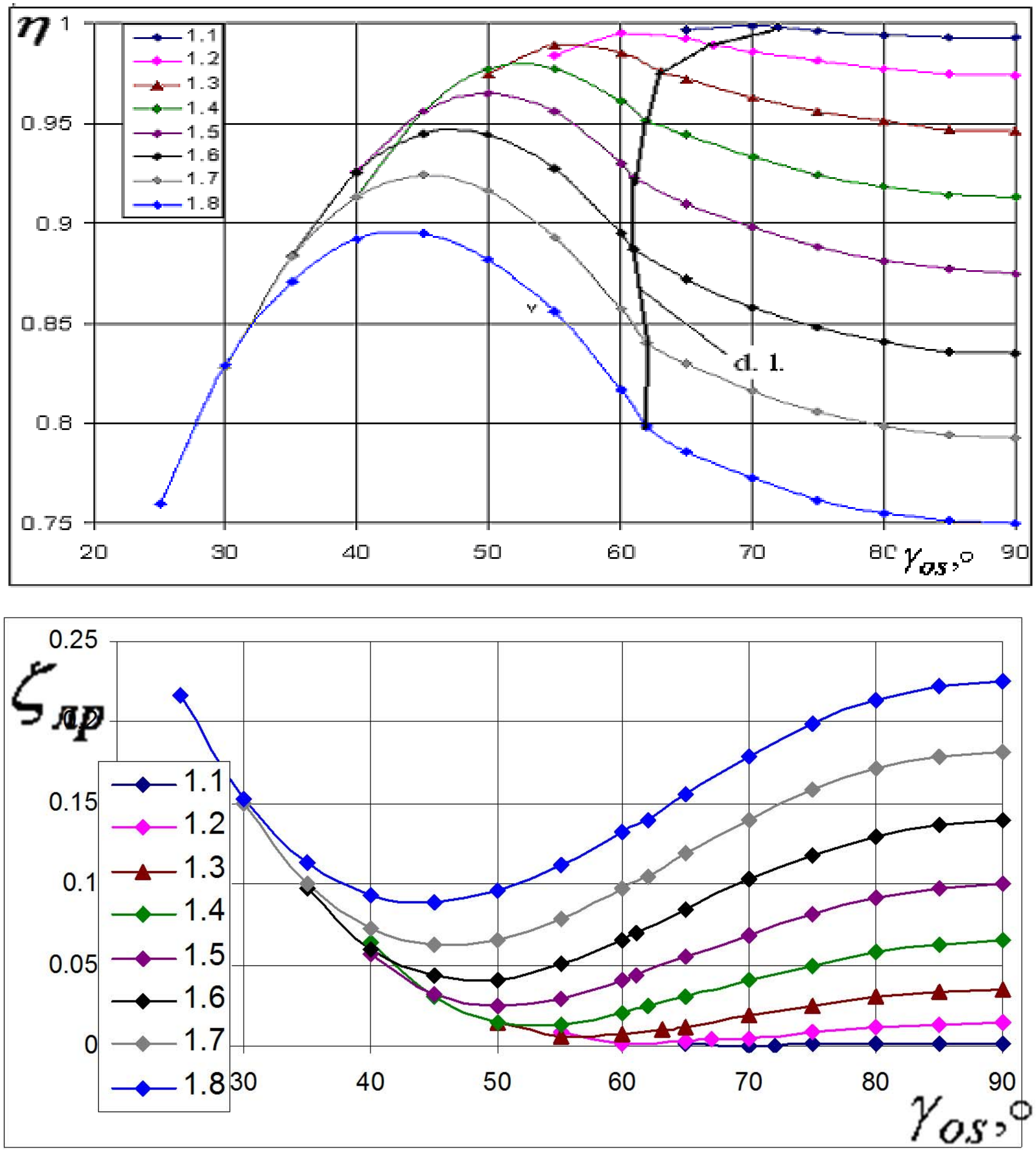

Fig. 6. Efficiency and loss coefficient of shock waves as diffusers

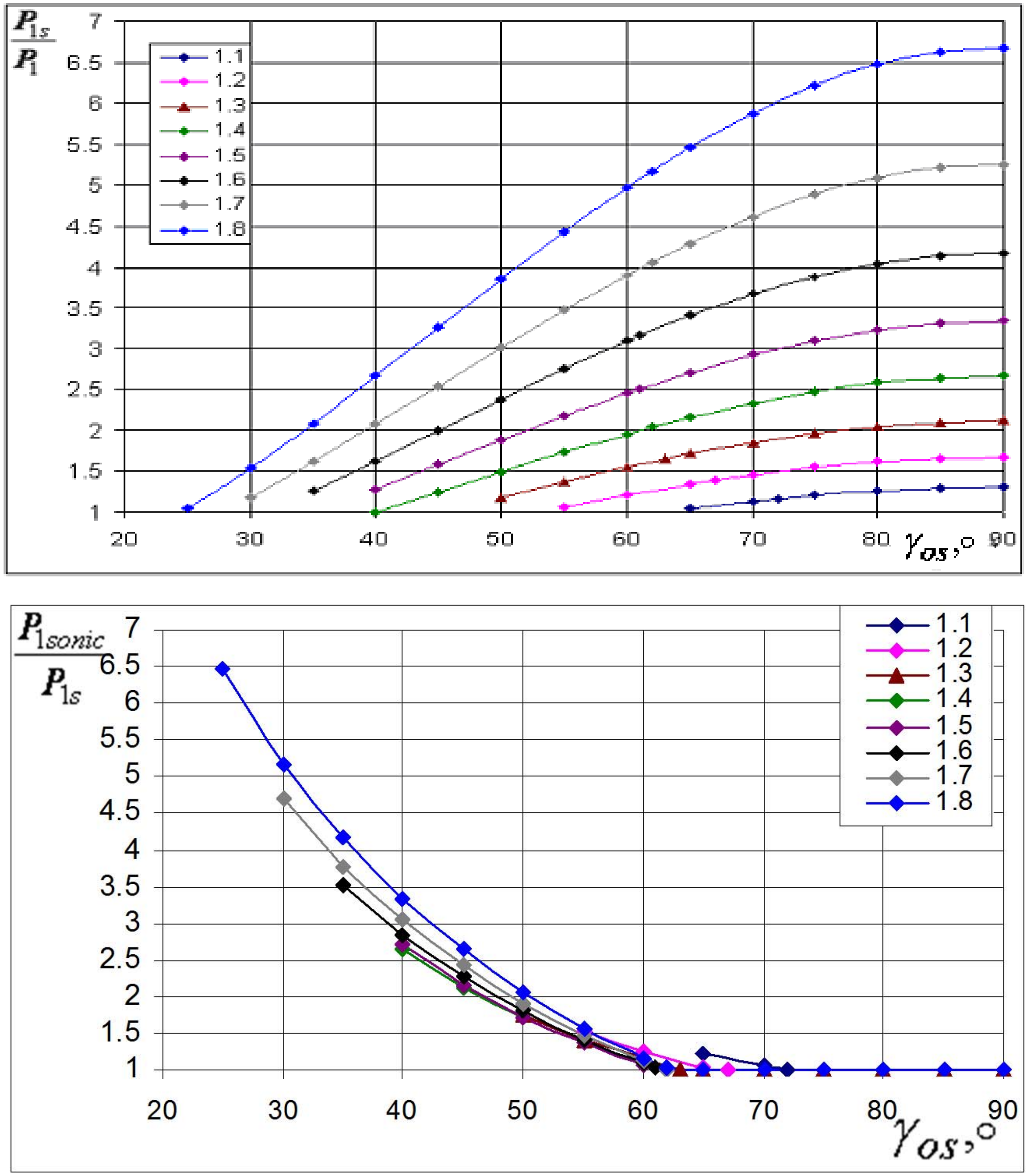

Fig. 7. Pressure ratios in an oblique shock, in a following normal shock and in their combination

Stage efficiency simplified analysis

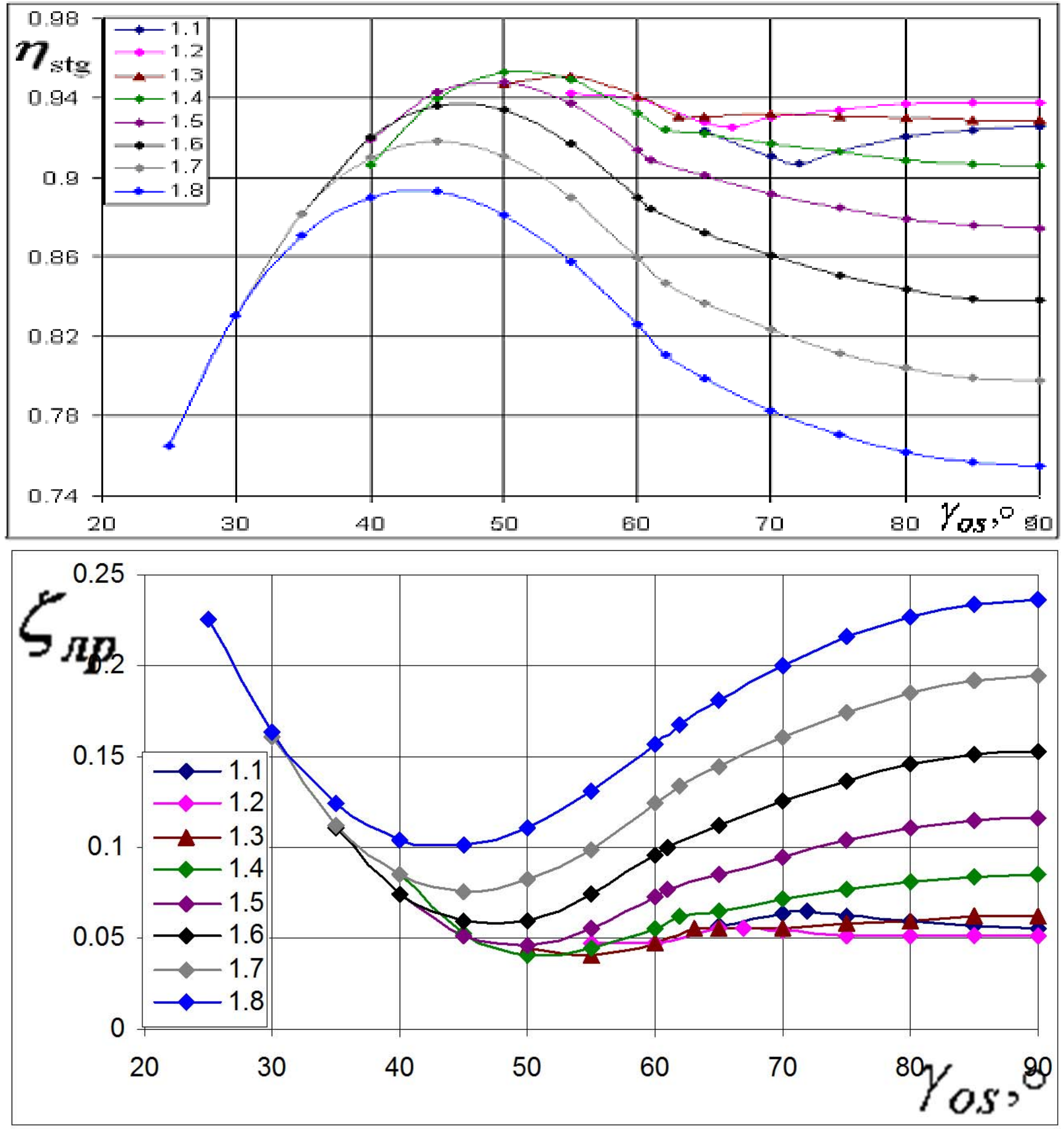

Fig. 8. Stage efficiency (static parameters) and loss coefficient versus γ_{os} and λ_1. Constant values ζ_{def} = 0,085, $\lambda_{w1}/\lambda_{sonic}$ = 0,60

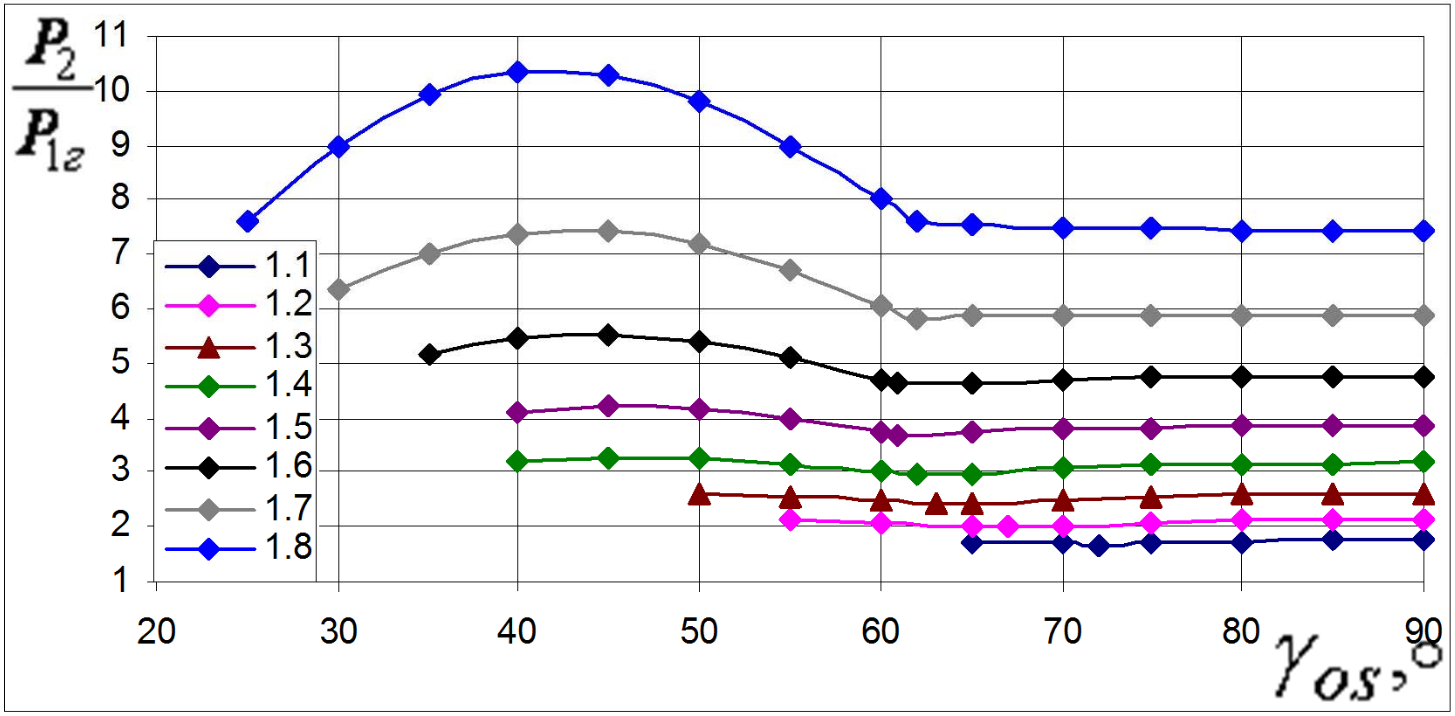

Fig. 9. Pressure ratios in a model of a stage coefficient versus γ_{os} and λ_1

ST.PETERSBURG STATE POLYTECHNICAL UNIVERSITY

Petersburg Politechnical University

CONCLUSION

The above calculations have demonstrated that despite a head loss in shock waves the efficiency can reach more than 90% for sophisticated stages with $\pi_t \approx 3{,}0$. There are several serious problems to be solved though to reach the goal.

Eq. (1) shows that an oblique shock with any necessary angle γ_∞ can be made at a leading edge of a profile. It is not clear if an optimal angle γ_∞ can be always made at a cascade entrance. A profiles interaction and blade load influence on a flow structure.

Shock wave on a surface provokes flow separation. It can reduce efficiency of a subsonic part of a cascade more seriously than it was predicted by loss coefficient choice at presented calculations.

The problem of effective 3-D design of impellers and stators for high supersonic stages is still unsolved too. Effective elementary cascade is the first necessary step for final decision of the problem but not the last one.