Herrick Laboratories
PURDUE UNIVERSITY

2016 Purdue Conferences
Compressor Engineering
Refrigeration and Air Conditioning
High Performance Buildings

Analysis of Dynamic Stability of Ejector Expansion Refrigeration System

Jianqiang Deng, Lixing Zheng, Fei Wang

Xi’an Jiaotong Universtiy, China

Email: dengjq@mail.xjtu.edu.cn

13-7-2016

PURDUE UNIVERSITY

Center for High Performance Buildings at Purdue

CONTENS

Background

Stability is one of the basic conditions for the system oper of systems can work normally and reliably under unst

Oscillatory motion of mixture-vapor transition point

Reflects an internal matching degree among and between structure parameters and operation parameters.

Instability between throttling device and evaporator

Instabilities of refrigeration system*

instability

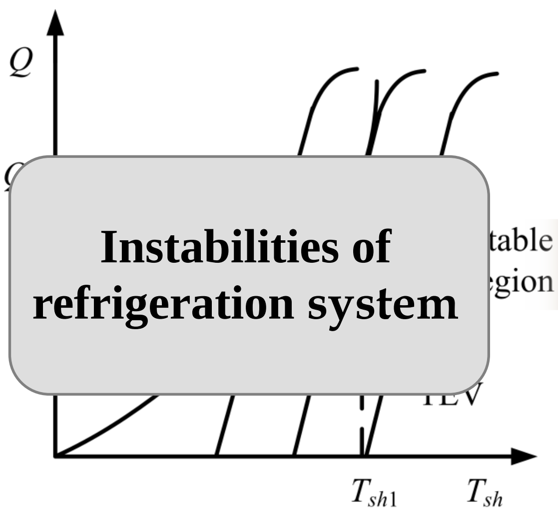
The Lyapunov Stability Theorem

Stability margin

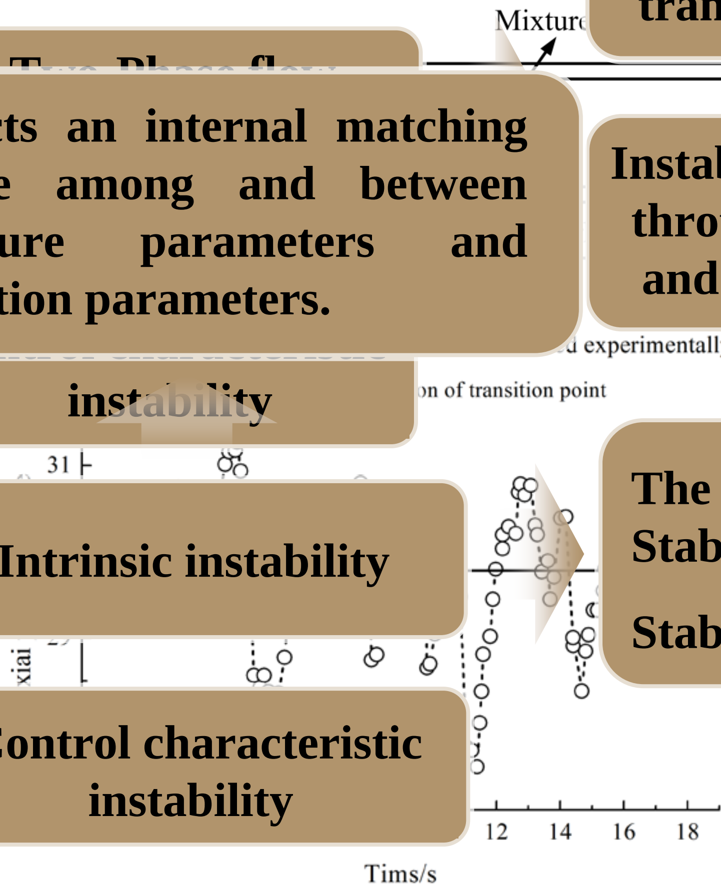
Intrinsic instability

Instabilities of refrigeration system

Control characteristic instability



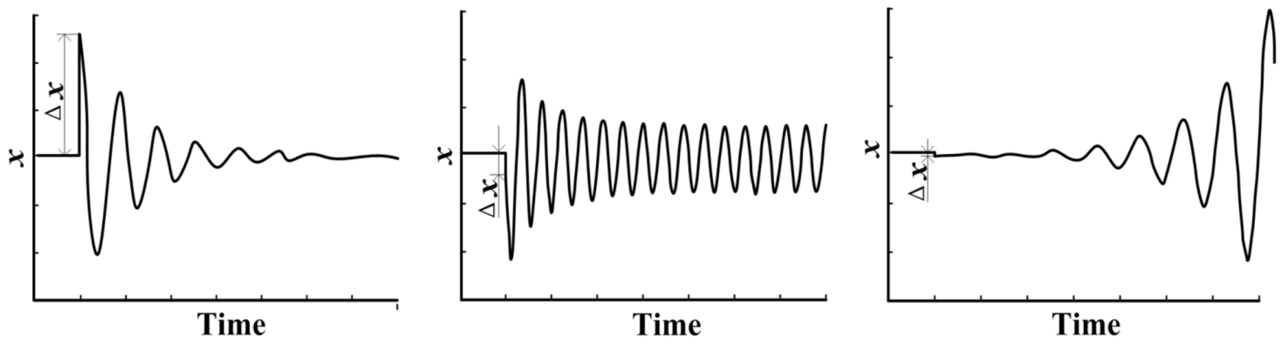
MSS line and TEV lines



Two-phase flow instability

Lyapunov Stability Theorem

Lyapunov stability theory is usually regarded as classical theoretical foundation for stability analysis. The theory mainly involves the *V* function method and the First Approximation Theorem. The stability analysis can be attributed to the stability criterion of zero solution of the linear differential equations.

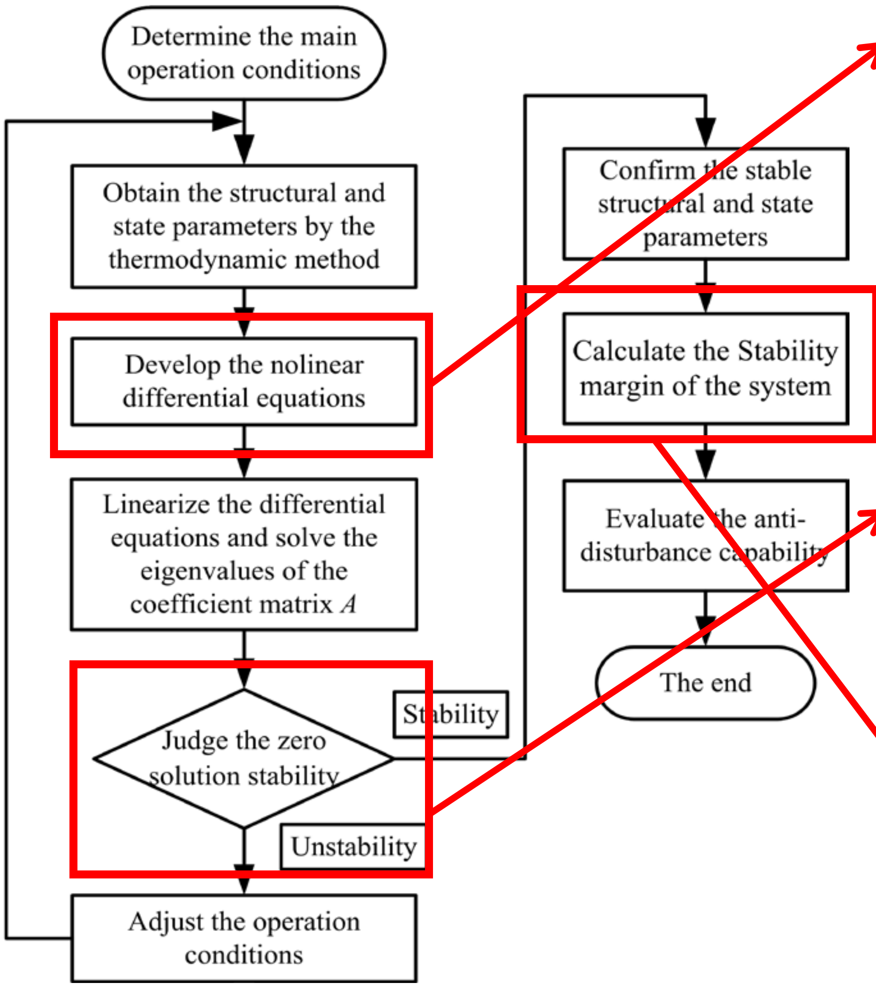


(a) Asymptotic stability of zero solution (b) Stability of zero solution (c) Instability of zero solution

Motion forms of system state parameters with the disturbance

In this study, we combine the First Approximation Theorem of Lyapunov Stability Theorem and the calculation of the stability margin to propose a method of dynamic stability to evaluate the matching degree in a system.

Analysis method



Flowchart of dynamic stability analysis method

STEP1: Develop governing equations based on the three conservation laws .

Continuity: $$\frac{\partial \rho}{\partial t} = \frac{\dot{m}_{in} - \dot{m}_{ou}}{A\Delta z}$$

Momentum: $$\frac{\partial \dot{m}}{\partial t} = \frac{P_{in}A_{in} - P_{ou}A_{ou}}{\Delta z} + \frac{\dot{m}_{in}u_{in} - \dot{m}_{ou}u_{ou}}{\Delta z}$$

Energy: $$\frac{d(\dot{m}u)}{dt} = \dot{m}_{in}h_{in} - \dot{m}_{ou}h_{ou}$$

STEP2: Discuss the eigenvalue structure of the linear equations.

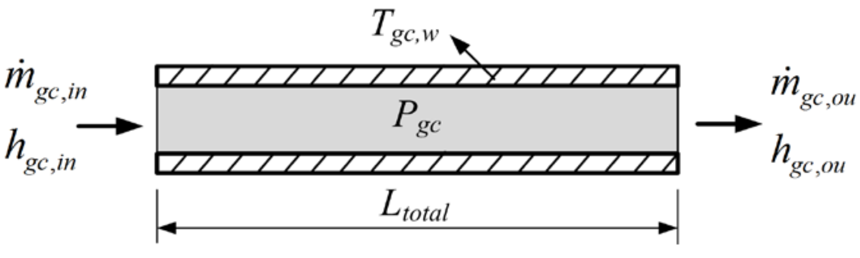
$$Z \cdot \dot{x} = f(x,u) \quad \Longrightarrow \quad \dot{x} = A\cdot\delta x + B\cdot\delta u \qquad A = Z^{-1} f_x$$

STEP3: Calculate the stability margin and evaluate the anti-disturbance capability.

Assuming the j order eigenvalue is $\lambda_j=-\tau_j+i\omega_j$, the logarithmic decrement is $\delta_j=2\pi\tau_j/\omega_j$. The stability margin is represented by the minimum logarithmic decrement, $\delta_{min}=\min(\delta_j)$.

Case study 1

Dynamic stability of gas cooler



Physical model of gas cooler

The right side of the governing equation is dealt with by Taylor series expansion and its first order partial derivative f_x is obtained. The coefficient matrix A is

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$\lambda_1 = 0$

Governing equations

$$\frac{\partial(\rho_{gc} A_{cs})}{\partial t} + \frac{\partial(\dot{m}_{gc})}{\partial z} = 0$$

$$\frac{\partial(A_{cs}\rho_{gc} h_{gc} - A_{cs} P_{gc})}{\partial t} + \frac{\partial(\dot{m}_{gc} h_{gc})}{\partial z} = \alpha_i A_i (T_{gc,w} - T_{gc,r})$$

$$(C_p \rho A)_{gc,w} \frac{\partial T_{gc,w}}{\partial t} = \alpha_i A_i (T_{gc,r} - T_{gc,w}) + \alpha_o A_o (T_{gc,wa} - T_{gc,w})$$

$$\lambda_2 \cdot \lambda_3 = \frac{1}{\dot{m}_{gc} (C_{p,gc} \rho V)_{gc,w} (\alpha_i A_i \alpha_o A_o)_{gc}} \left[2(\alpha_i A_i)_{gc} \left(\frac{\partial h_{gc}}{\partial u_{gc}}\bigg|_{\rho_{gc}} \right) + 2(\alpha_o A_o)_{gc} \left(\frac{\partial h_{gc}}{\partial u_{gc}}\bigg|_{\rho_{gc}} \right) \left(1 - \frac{\partial T_{gc,wa}}{\partial T_{gc,w}} \right) + \alpha_i A_i \left(\frac{\partial T_{gc,r}}{\partial u_{gc}}\bigg|_{\rho_{gc}} \right) \left(1 - \frac{\partial T_{gc,wa}}{\partial T_{gc,w}} \right) \right] > 0$$

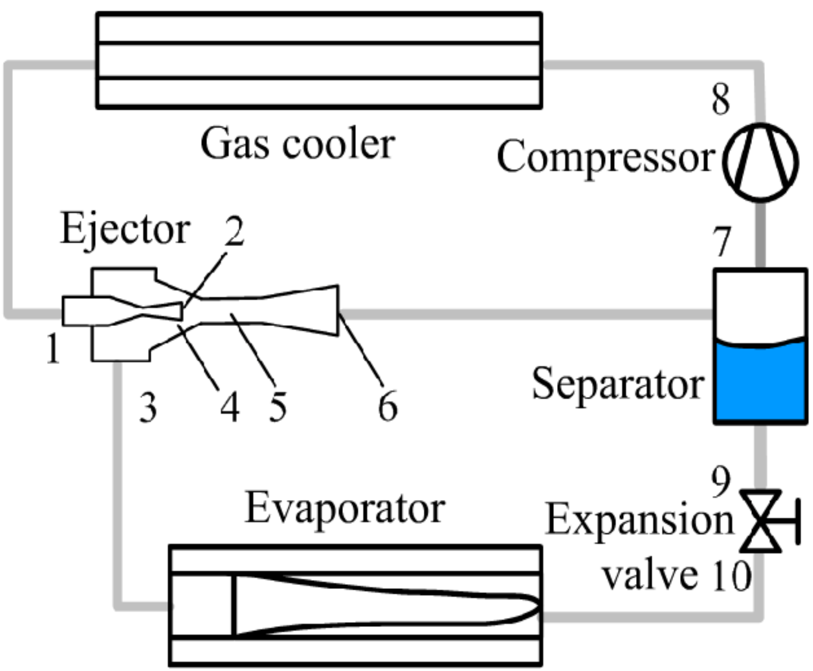
$\lambda_2 < 0, \ \lambda_3 < 0$

The gas cooler is in zero solution stability

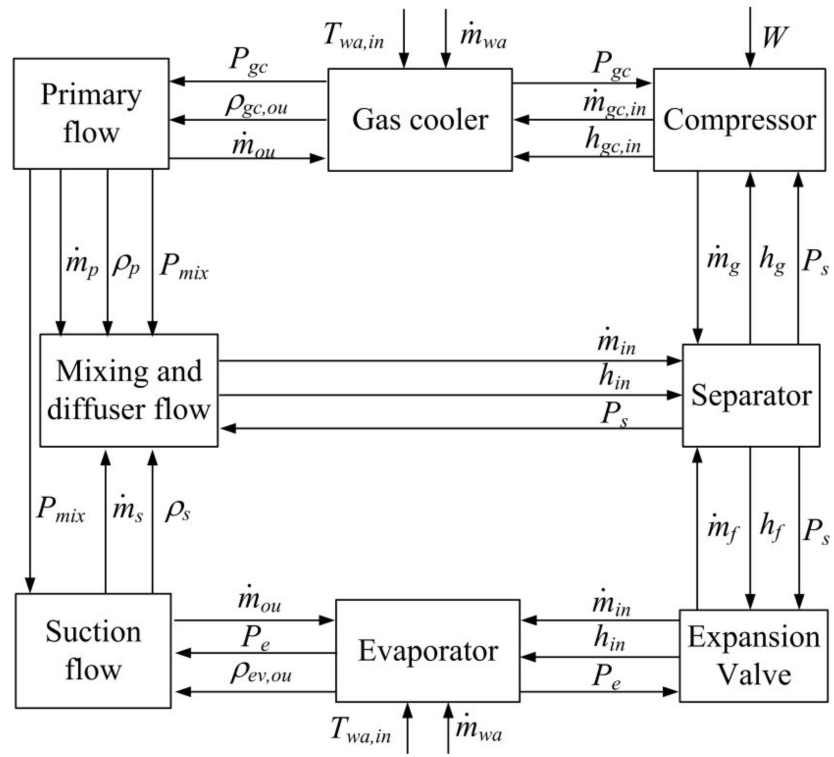
In actual thermal process of the gas cooler, when the input parameters are changed because of the small disturbance, there are variations in the output parameters, and finally it will achieve stability. The result shows the consistency between the mathematical stability and the actual stability

Case study 2

Dynamic stability of EERS



Transcritical CO_2 EERS (Ejector Expansion Refrigeration System)



Coupling relationship diagram for physical parameters of each components in EERS

The EERS has a special equilibrium stability relationship relative to the traditional vapor compression system (VCS), and the system performance and stability are more sensitive to the operating parameter and the match of component.

Case study 2

Literatures list of component models of transcritical CO_2 EERS

Components	Modeling approaches	literatures
Compressor	Lumped parameter	Sarkar *et al*. (2006)
Expansion valve	Lumped parameter	Ma *et al*. (2005)
Ejector	Thermodynamics method	Nehdi *et al*. (2007)
Evaporator	Moving-boundary method	Zhang and Zhang (2006)
Gas cooler	Lumped parameter	Rasmussen (2002)
Separator	Lumped parameter	Eldredge *et al*. (2008)

Governing equation after linearization

$$\begin{bmatrix} Z_E & & & \\ & Z_{GC} & & \\ & & Z_{SE} & \\ & & & Z_{EJ} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{X}_{GC} \\ \dot{X}_{SE} \\ \dot{X}_{EJ} \end{bmatrix} = \begin{bmatrix} F_{VXV} & F_{VXG} & F_{VXS} & F_{VXE} \\ F_{GXV} & F_{GXG} & F_{GXS} & F_{GXE} \\ F_{SXV} & F_{SXG} & F_{SXS} & F_{SXE} \\ F_{EXV} & F_{EXG} & F_{EXS} & F_{EXE} \end{bmatrix} \cdot \begin{bmatrix} X_E \\ X_{GC} \\ X_{SE} \\ X_{EJ} \end{bmatrix}$$

where Z_E、Z_{GC}、Z_{SE}、Z_{EJ}、X_{EV}、X_{GC}、X_{SE} and X_{EJ} are Z matrix and state vector matrix of evaporator, gas cooler, separator and ejector, respectively. F is The coefficient matrix.

Case study 2

The initial operation parameters of EERS

Parameters	Unit	Values	Parameters	Unit	Values
Compressor			flow rate	$kg\cdot s^{-1}$	0.956
inlet pressure	MPa	4.29	inlet temperature of cooling water	℃	25.0
outlet pressure	MPa	9.50	outlet temperature of cooling water	℃	55.0
displacement	$m^3\cdot h^{-1}$	1.46	flow rate of cooling water	$kg\cdot s^{-1}$	0.056
rotate speed	$r\cdot min^{-1}$	1450	**Evaporator**		
Expansion valve			flow rate	$kg\cdot s^{-1}$	0.557
outlet pressure	MPa	3.90	superheat	℃	5.0
area	mm^2	1.7485	inlet temperature of chilled water	℃	20.0
Ejector			outlet temperature of chilled water	℃	10.0
primary temperature	℃	36.0	flow rate of chilled water	$kg\cdot s^{-1}$	0.127
area ratio		10.0	**Separator**		
entrainment ratio		0.583	diameter	m	0.1
Gas cooler			high	m	0.6

Eigenvalue of evaporator：

$$\lambda_e = \{-21.82,\ -17.18,\ -2.55,\ -0.60,\ 0\}$$

Eigenvalue of gas cooler：

$$\lambda_{gc} = \{-0.61,\ -0.0042,\ -0.00041\}$$

Individual components is stability

Eigenvalue of separator：

$$\lambda_{se} = \{-1090.56,\ 0\}$$

Eigenvalue of ejector：

$$\lambda_{ej} = \{-5113.77 \pm 14848.09j,\ -1457.31 \pm 4371.96,\ -1647.41,\ 0\}$$

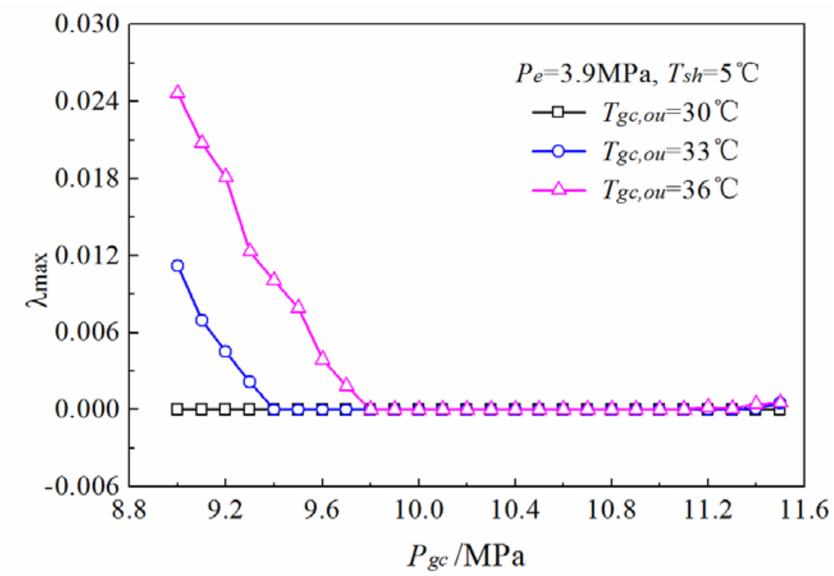
Eigenvalue of system：

$$\lambda_{sys} = \left\{ \begin{array}{l} -5133.76 \pm 14878.09j,\ -1511.85 \pm 4596.42j,\ -552.95 \pm 2880.39j, \\ -1544.41, -17.18,\ -2.48,\ -0.90,\ -0.61,\ -0.16,\ 0.0079,\ -0.0016,\ 0,\ 0 \end{array} \right\}$$

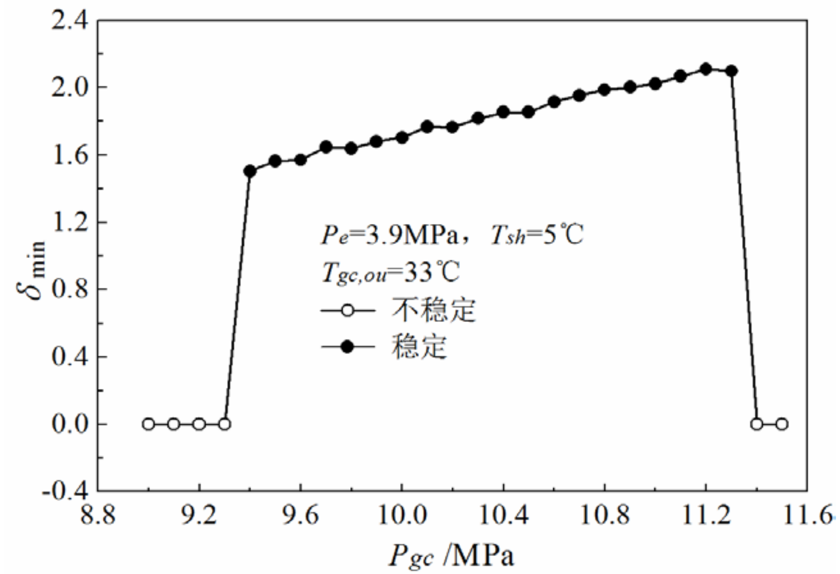
There are two zero value and a positive value , which means the system instability. The results show the stability of individual components cannot guarantee the stability of the whole system.

Case study 2

Analysis of EERS under different working conditions



Effect of the state parameters of gas cooler on system stability: Maximum eigenvalue



Effect of the state parameters of gas cooler on system stability: Minimum logarithmic decrement

The maximal eigenvalue λ_{max} with the change of gas cooler pressure at different gas cooler outlet temperature were obtained.

The minimum logarithmic with the change of gas cooler pressure were investigated.

Conclusions

A dynamic stability analysis method is put forward for the refrigeration system based on the First Approximation Theory of Lyapunov Stability Theorem and the evaluation of stability margin.

The instability of a refrigeration system is divided into the intrinsic instability and the control characteristic instability.

The case study of stability analysis of gas cooler confirms the consistency between the mathematical stability and the actual one.

The dynamic stability analysis on a transcritical CO_2 ejector expansion refrigeration system (EERS) is conducted and the present results show that, even each component of the system is in the stable state, it cannot guarantee the dynamic stability of the whole system.

Discussions

Question1: In this paper, we put forward the concept of intrinsic stability, which refers to the harmonious matching among and between structure parameters and operation parameters. While the Lyapunov Stability Theorem is directed against to motion stability under the small disturbance. What is the relationship between the matching degree and the disturbance?

Response: The change of structure parameters or operation parameters is deem to be the disturbance, and the motion stability analysis can reflect the matching of system under this condition. Moreover, the large change of structure size and system parameters are accumulated by the small disturbance.

Question2: The Lyapunov Stability Theorem is always applied to analyze the control problem, in this paper, we used the theorem to investigate the thermodynamic problem, whether it is reasonable or not?

Response: In this paper, the governing equations were established based on the common conservation laws, and the Lyapunov Stability Theorem is applied to linearize the governing equations. Moreover, this approach has been used to the development of controller of refrigeration in previous literature[1].

[1]Alleyne A. G., Rasmussen B. P. (2007). Advances in energy systems modeling and control. Proceedings of the American Control Conference, New York, pp. 4363–4373.

Question3: Where is the error sources of the presented method used to conduct the stability analysis?

Response: The linearization of the governing equations is core step, moreover, the higher order terms are omitted and only leave the first order term, which is the main error sources. In addition, during the ejector model, the determination of the input parameters would also result in the error .

Thank you for your attention!