Shape Optimization of a Compressor Supporting Plate Based on Vibration Modes

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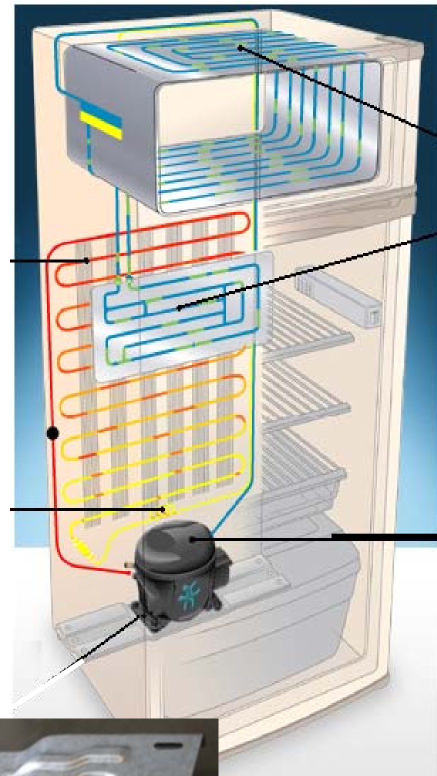
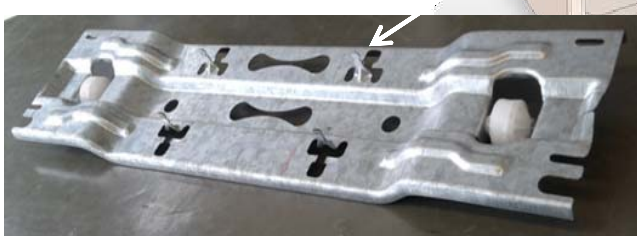
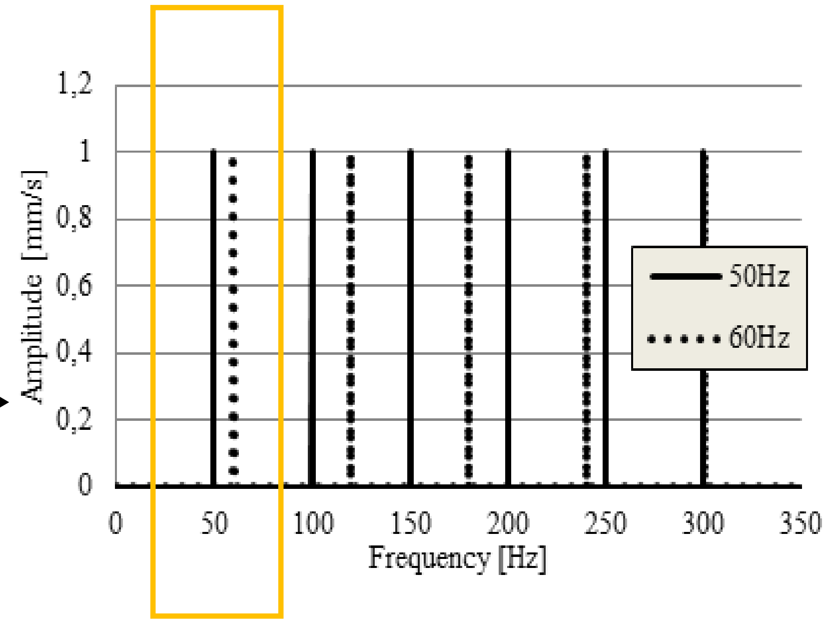
- Supporting plate







Problem Definition

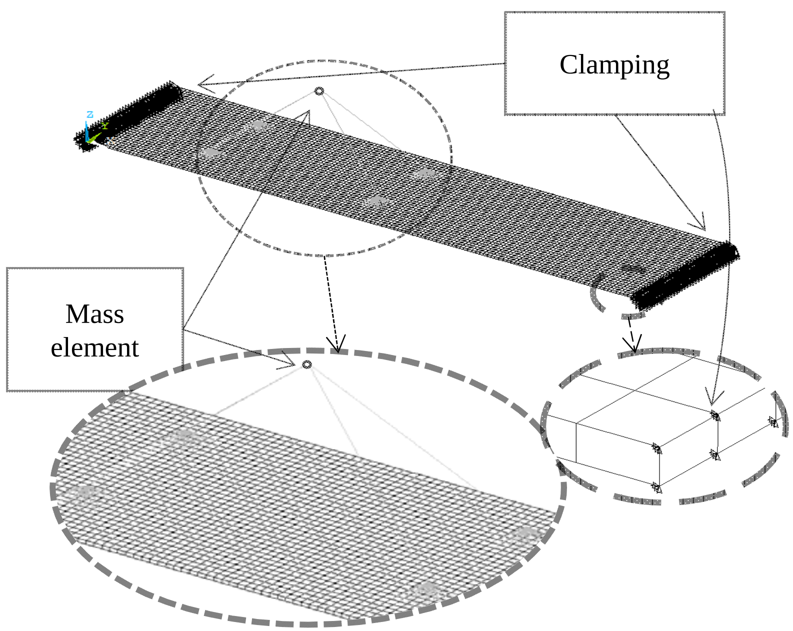
- The support base-plate of a refrigerator is responsible for the structural connection between the cabinet and the compressor, which is its primary function.

- However, the plate must attend some requirements, among them, to have low dynamic response during operation of the compressor. The correct dynamic response of the plate is important for filtering the vibrations of the compressor, reducing the energy transmitted to the cabinet, as well as for not allowing excessive noise radiation.

- The project of a base-plate with desirable vibration characteristics consists on allocating its structural modes as far as possible from the operational frequency and first harmonics.

Problem Definition

Simplified FE model



$$(\boldsymbol{K} - \lambda_i \boldsymbol{M})\boldsymbol{\Phi}_i = 0$$

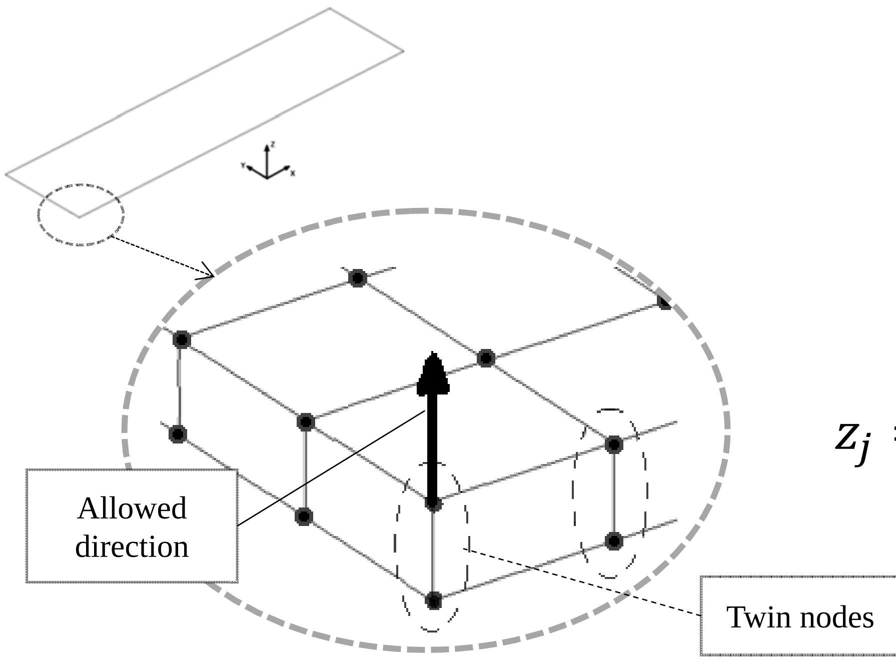
Model Validation (base-plate only)

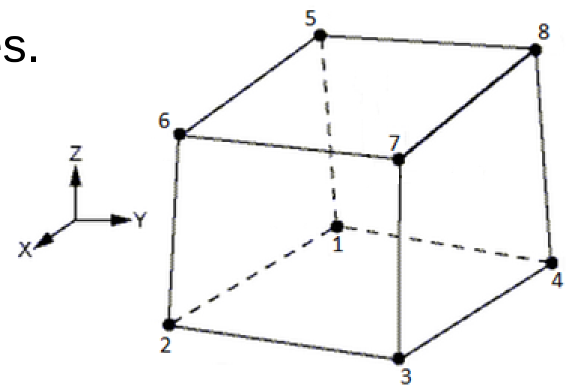
Natural frequency order	Measured	SHELL181	SHELL281	SOLID45	SOLID95	SOLID45 non-conforming
1	34,1	34,1	34,1	56,5	34,1	34,1
2	88,7	87,4	87,2	88,6	87,3	87,4
3	94,3	94,5	94,5	155,9	94,5	94,5
4	182,8	180,6	180,2	190,4	180,4	180,7
5	185,6	186,3	186,1	306,0	186,2	186,3
6	288,1	285,1	284,4	317,2	284,7	285,2
7	307,6	309,2	308,8	479,4	308,9	309,3
8	407,8	405,6	404,6	506,4	405,1	405,9
9	460,0	462,7	461,9	684,3	462,0	462,8
10	547,3	546,6	545,2	756,5	545,8	547,0

Problem Definition

Parameterization of the problem

- Mesh parameters = optimization variables.





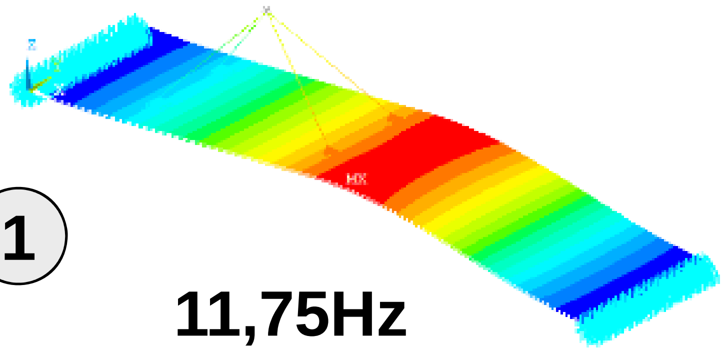
$$z_j = \begin{cases} z_j, & for\ \ j = 1,2,3,4 \\ z_{j-4} + thk, & for\ j = 5,6,7,8 \end{cases}$$

$$z_{low} \leq z_j \leq z_{upper}$$

- Number of variables = number of the nodes on the plate bottom face.

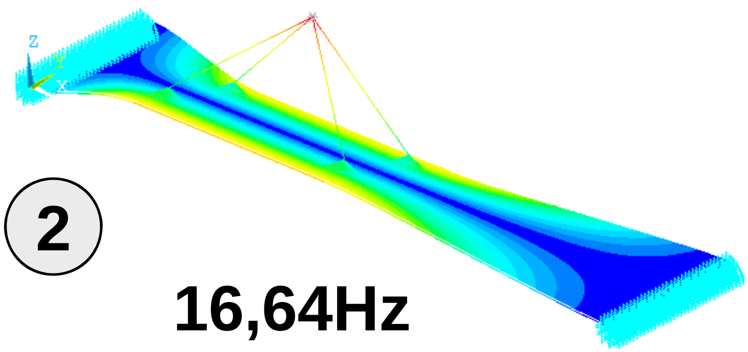
Problem Definition

First modes of the simplified FE model.



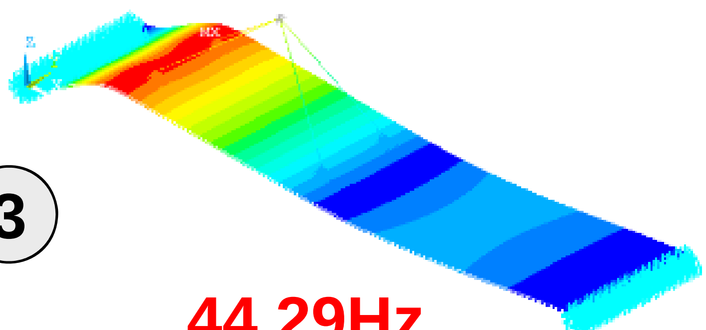
1

11,75Hz



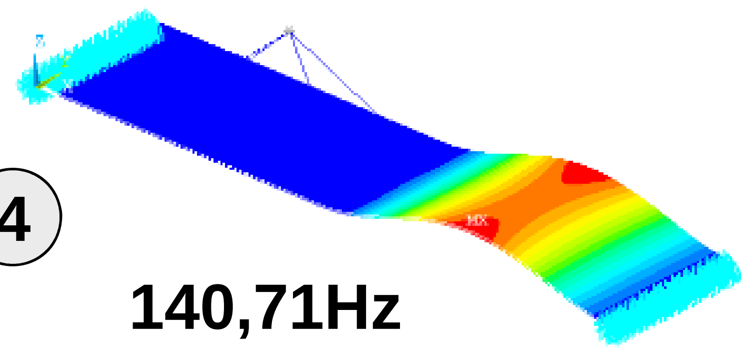
2

16,64Hz



3

44,29Hz



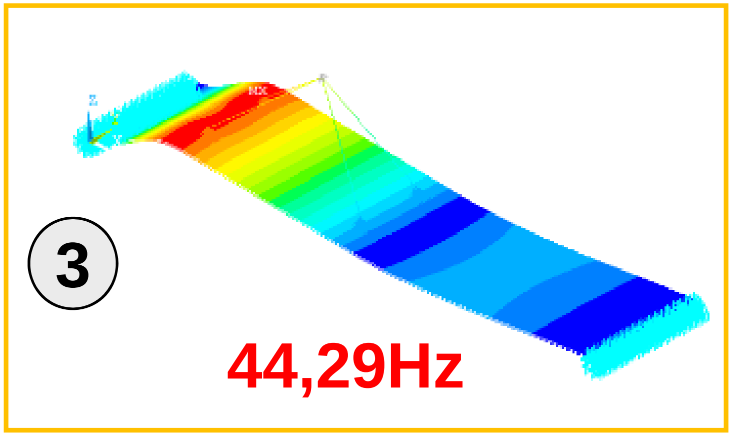
4

140,71Hz

Problem Definition

Optimization problem: objective function

$$\min f(\boldsymbol{z})$$
$$subject\ to\ g(\boldsymbol{z}) \leq 0$$
$$z_{low} \leq z_i \leq z_{upper}$$



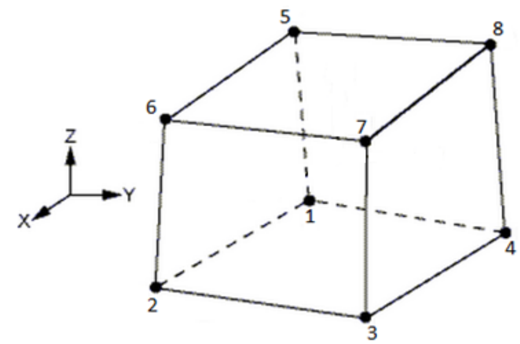
The 3rd natural frequency of the system is 44,3Hz, very close to the fundamental frequency of excitation. The authors suggest tuning this structural mode to 75Hz, **avoiding the resonances near 50Hz**. Consequently, due to the target value of this tuning, the 3rd mode will be also far from 100Hz, the first excitation harmonic.

$$f(\boldsymbol{z}) = (\lambda_3 - 2\pi 75)^2$$

Problem Definition

Optimization problem: constraint

- modification of twin nodes coordinates are enabled only for Z-direction and the plate thickness is considered constant;
- elementary volumes do not change along the process. So, a mass constraint, widely used in optimization procedures, cannot be used;
- this work proposes a measure of mesh distortion as a constraint for the optimization problem, defined as:



Average edge of element bottom face

$$g(\boldsymbol{z}) = \frac{1}{N_e}\sum_{e=1}^{N_e}\overline{L_e}$$

$$\overline{L_e} = \frac{|n_2 - n_1| + |n_3 - n_2| + |n_4 - n_3| + |n_1 - n_4|}{4}$$

Problem Definition

Optimization problem: sensitivity analysis

- Gradient Method: due to large number of variables.

$$\frac{\partial \lambda_i}{\partial z_j} = \boldsymbol{\Phi}_i^{\mathrm{T}} \left(\frac{\partial \boldsymbol{K}}{\partial z_j} - \lambda_i \frac{\partial \boldsymbol{M}}{\partial z_j} \right) \boldsymbol{\Phi}_i,$$ (Haftka and Gurdal, 1992)

- Node coordinates modification: only elements in the vicinity are distorted.

$$\frac{\partial \lambda_i}{\partial z_j} = \sum_{e=1}^{N_v} \left(\boldsymbol{\Phi}_{ie}^{\mathrm{T}} \left(\frac{\partial \boldsymbol{K}_e}{\partial z_j} - \lambda_i \frac{\partial \boldsymbol{M}_e}{\partial z_j} \right) \boldsymbol{\Phi}_{ie} \right)$$

- For SOLID45 non conforming:

$$\partial \boldsymbol{M_e} / \partial z_j = 0$$

$$\frac{\partial \boldsymbol{K}_e}{\partial z_j} = \sum_{p=1}^{N_p} \left(\frac{\partial \boldsymbol{B}_e}{\partial\, z_j}^{\mathrm{T}} \boldsymbol{C} \boldsymbol{B}_e(p) \det(\boldsymbol{J}_e) + \boldsymbol{B}_e{}^{\mathrm{T}} \boldsymbol{C} \frac{\partial \boldsymbol{B}_e(p)}{\partial\, z_j} \det(\boldsymbol{J}_e) + \boldsymbol{B}_e(p)^{\mathrm{T}} \boldsymbol{C} \boldsymbol{B}_e(p) \frac{\det(\boldsymbol{J}_e)}{\partial\, z_j} \right) \alpha_p$$

Problem Definition

Optimization problem: sensitivity analysis

Objective function:

$$\frac{\partial f(\boldsymbol{z})}{\partial z_j} = 2(\lambda_3 - 2\pi 75)\frac{\partial \lambda_3}{\partial z_j}$$

Constraint:

$$\frac{\partial g(\boldsymbol{z})}{\partial z_j} = \frac{1}{N_e}\sum_{\mathrm{e}=1}^{N_e}\frac{\partial \overline{L_e}}{\partial z_j}$$

$$\frac{\partial \overline{L_e}}{\partial z_j} = \frac{1}{4}\left(\frac{1}{|n_2 - n_j|}(z_2 - z_j) + \frac{1}{|n_j - n_4|}(z_j - z_4)\right)$$

Numerical Procedure

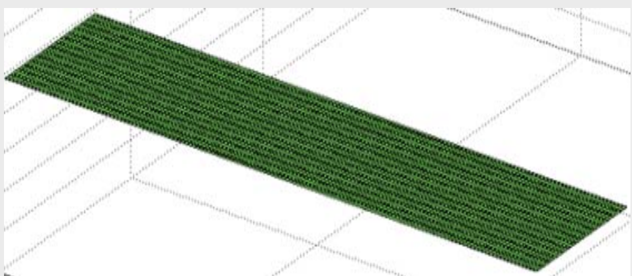
Large number of variables: gradient-based optimization algorithm (*Method of Moving Asymptotes* – MMA from Svanberg, 1987)

i) input initial design values;
ii) enter the optimization loop. It calls the FEM program (after writing the current iteration values of design variables to a file) to run a modal analysis in batch mode and return to optimization procedure the eigenvalues and eigenvectors;
iii) calculate the sensitivities in the interface module and send these information to MMA;
iv) if attended the stopping criteria, stop the optimization and plot the optimized design. Otherwise, repeat from (ii).

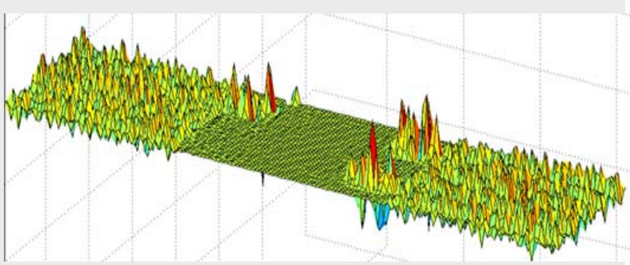
Numerical Procedure

Numerical instabilities: this problem occurs due to the symmetry of the geometry in Z-direction, since a node movement in negative or positive sense leads to the same values on derivatives of the objective function.

Initial design: flat plate

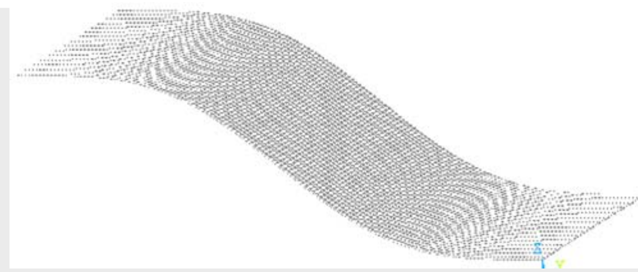


Zero initial deformation

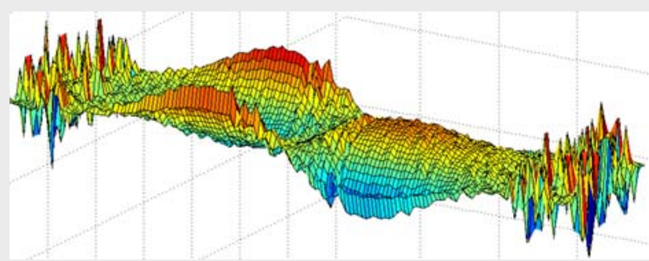


Calculated derivatives of objective function

Initial design: proportional to a mode shape



Slight initial deformation based on a mode shape (graphically amplified)

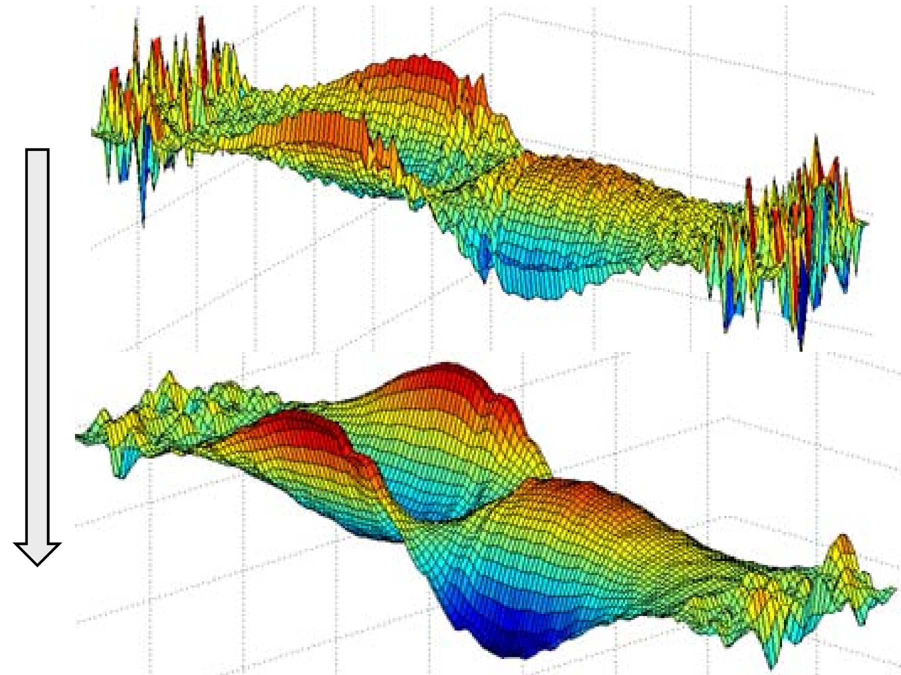


Calculated derivatives of objective function

Numerical Procedure

Numerical instabilities: in order to avoid abrupt variation on the objective function derivatives along the design domain, this work suggests the use of a filter applied to obtain a weighted local average of the derivative values:

$$\overline{\frac{\partial f(\boldsymbol{z})}{\partial z_j}} = \frac{1}{\sum_{i=1}^{N_{filt}} H_i} \sum_{i=1}^{N_{filt}} H_i \frac{\partial f(\boldsymbol{z})}{\partial z_j}$$

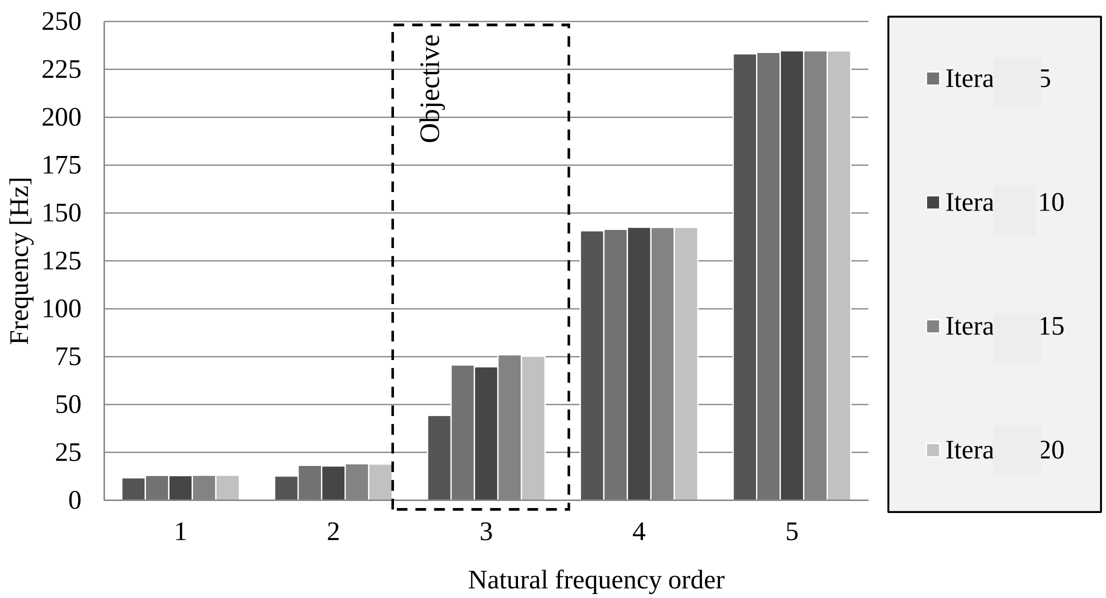


Results

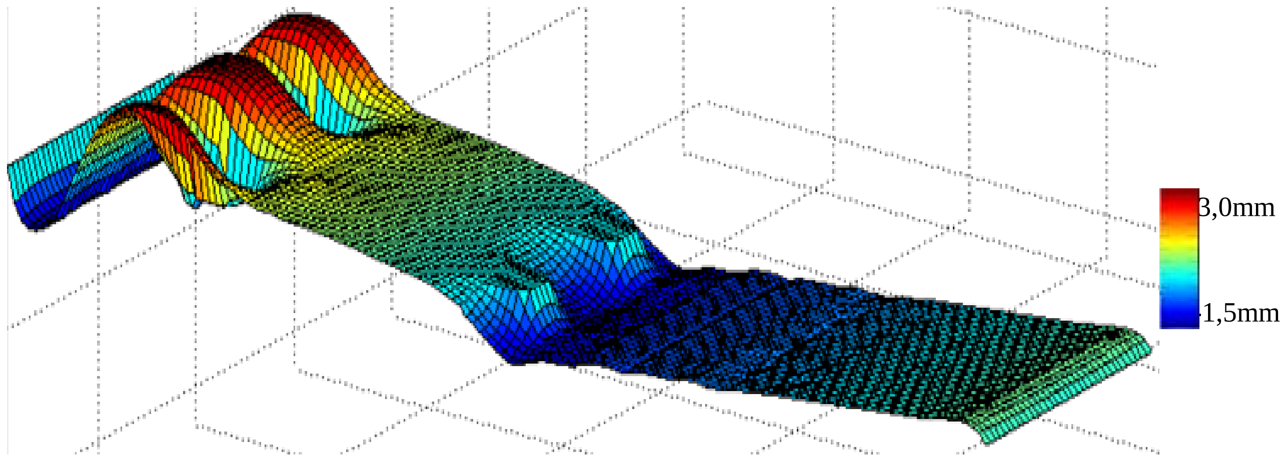
$$z_{lower} = -10mm$$
$$z_{upper} = 10mm$$

$g(\boldsymbol{z})$<100,1% of its initial value

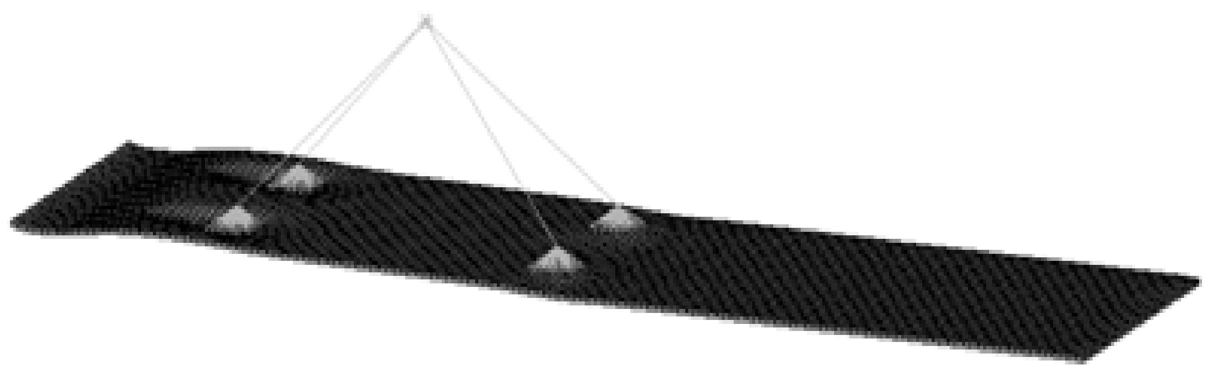
The 3[th] mode is used to create the initial deformation.



Results



Final design of the base-plate (amplified scale).

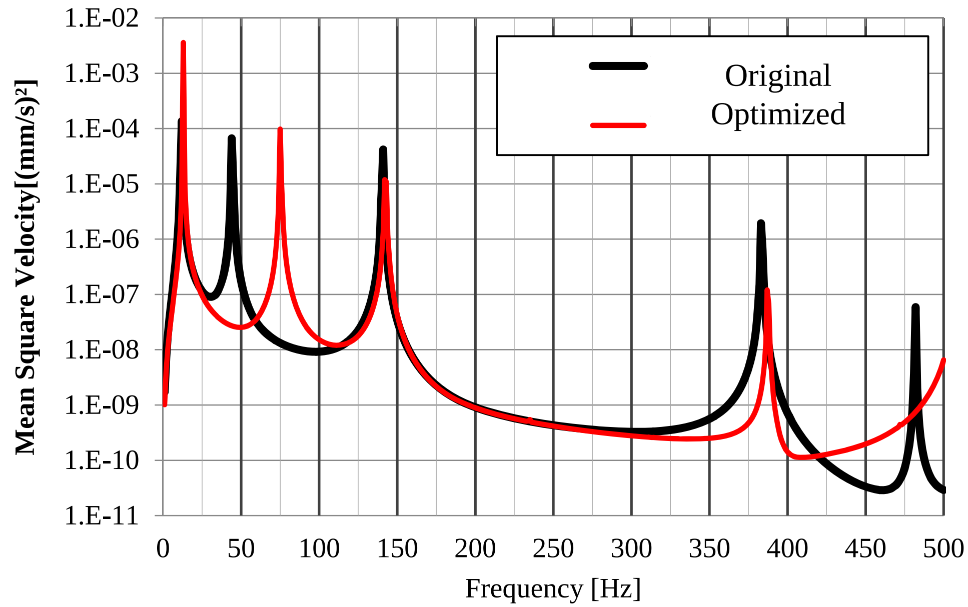


Final design of the base-plate (FE model).

Results and Conclusion

Harmonic Analysis

F = 1N on the mass
ξ = 0.03%
Material: Steel



The great advantage of methods based on the movement of nodes is the possibility of creating unconventional geometries. The application of this method, however, is not a simple task, and varies depending on the problem studied.

ACKNOWLEDGEMENT

The authors are grateful to Prof. Krister Svanberg from Royal Institute of Technology, Stockholm, for providing his Matlab version of MMA.

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