Gradient-Based Estimation of Air Flow and Geometry Configurations in a Building Using Fluid Dynamic Adjoint Equations

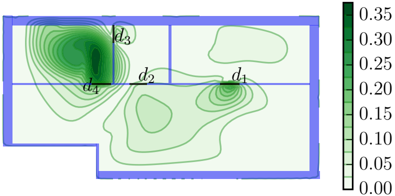
Runxin He, Humberto Gonzalez
runxinhe@email.wustl.edu

Electrical & Systems Engineering
Washington University in St. Louis

July 13, 2016

Motivations

1. Apartment's indoor configuration is one of the important factors to influence the indoor climate distribution.



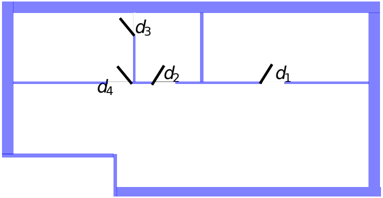
Relative temperature difference with two different indoor configurations

2. HVAC system's model prediction controls are based on accurate indoor climate estimations and configurations.
3. Usually extra sensors are installed inside buildings to sample buildings' indoor configuration changes.

Contributions

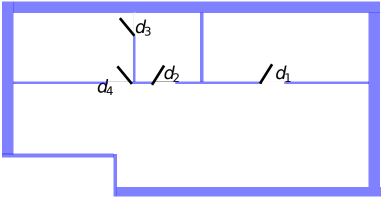
1. Develop an estimation algorithm to indoor temperature and building's configuration based on fluid dynamic model.
2. The estimation algorithm is based on the data from only thermostats, no need for extra sensors.
3. Apply a relaxation method to make our algorithm easy for computation and with efficient memory usage.

Geometry Configuration



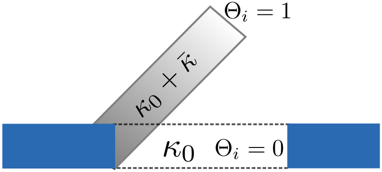
1. $\theta_i \in \{0, 1\}$ is defined to represent the i^{th} door's on/off state.
2. κ: the thermal diffusivity;
   α: viscous forces coefficient.
3. κ and α vary between open-flow area and solid materials, i.e. walls and closed doors.

Geometry Configuration



1. $\theta_i \in \{0, 1\}$ is defined to represent the i^{th} door's on/off state.
2. κ: the thermal diffusivity;
   α: viscous forces coefficient.
3. κ and α vary between open-flow area and solid materials, i.e. walls and closed doors.

α and κ Values



The i^{th} door's area

1. If x is inside open flow area or i^{th} door and $\theta_i = 0$,
   $\alpha(x, \theta) = \alpha_0$,
   $\kappa(x, \theta) = \kappa_0$.
2. If x is inside walls or i^{th} door and $\theta_i = 1$,
   $\alpha(x, \theta) = \alpha_0 + \bar{\alpha}$,
   $\kappa(x, \theta) = \kappa_0 + \bar{\kappa}$.

θ Relaxation

Each door's state, θ_i, is relaxed from binary $\{0, 1\}$ into the interval $[0, 1]$.

Reasons

1. Simplify the optimization problem's numerical calculation.
2. Reduce the algorithm's memory usage.

Physical interpretation

Non-integer values can theoretically be interpreted as averaged observations over the optimization horizon.

Fluid Dynamic Model

Heat convection-diffusion equation

$$\frac{\partial T_e}{\partial t}(x,t) - \nabla_x \cdot (\kappa(x,\theta) \nabla_x T_e(x,t)) + \\ + u(x) \cdot \nabla_x T_e(x,t) = g_{T_e}(x,t),$$

Stationary incompressible Navier-Stokes

$$-\frac{1}{Re} \triangle_x u(x) + \big(u(x) \cdot \nabla_x\big) u(x) + \\ + \nabla_x p(x) + \alpha\big(x,\theta\big) u(x) = g_u(x); \text{ and}$$

$$\nabla \cdot u(x) = 0$$

Initial condition

For $t = 0$,

$$T_e(x,t) = \pi_0(x),$$

Boundary conditions

For $x \in \partial\Omega$,

$$T_e(x,t) = T_a, \text{ and}$$
$$u(x) = 0.$$

Fluid Dynamic Model

Heat convection-diffusion equation

$$\frac{\partial T_e}{\partial t}(x,t) - \nabla_x \cdot (\kappa(x,\theta)\nabla_x T_e(x,t)) + \\ + u(x) \cdot \nabla_x T_e(x,t) = g_{T_e}(x,t),$$

Stationary incompressible Navier-Stokes

$$-\frac{1}{Re}\triangle_x u(x) + \big(u(x)\cdot\nabla_x\big)u(x) + \\ + \nabla_x p(x) + \alpha(x,\theta)u(x) = g_u(x); \text{ and,}$$

$$\nabla \cdot u(x) = 0$$

Initial condition

For $t = 0$,

$$T_e(x,t) = \pi_0(x),$$

Boundary conditions

For $x \in \partial\Omega$,

$$T_e(x,t) = T_a, \text{ and}$$
$$u(x) = 0.$$

Fluid Dynamic Model

Heat convection-diffusion equation

$$\frac{\partial T_e}{\partial t}(x,t) - \nabla_x \cdot (\kappa(x,\theta)\nabla_x T_e(x,t)) +$$
$$+ u(x)\cdot\nabla_x T_e(x,t) = g_{T_e}(x,t),$$

Stationary incompressible Navier-Stokes

$$-\frac{1}{Re}\triangle_x u(x) + \big(u(x)\cdot\nabla_x\big)u(x) +$$
$$+ \nabla_x p(x) + \alpha\big(x,\theta\big)u(x) = g_u(x);\ \text{and}$$
$$\nabla \cdot u(x) = 0$$

Initial condition

For $t = 0$,

$$T_e(x,t) = \pi_0(x),$$

Boundary conditions

For $x \in \partial\Omega$,

$$T_e(x,t) = T_a,\ \text{and}$$
$$u(x) = 0.$$

Fluid Dynamic Model

Heat convection-diffusion equation

$$\frac{\partial T_e}{\partial t}(x,t) - \nabla_x \cdot (\kappa(x,\theta)\nabla_x T_e(x,t)) + \\ + u(x) \cdot \nabla_x T_e(x,t) = g_{T_e}(x,t),$$

Stationary incompressible Navier-Stokes

$$-\frac{1}{Re}\triangle_x u(x) + (u(x)\cdot\nabla_x)u(x) + \\ + \nabla_x p(x) + \alpha(x,\theta)u(x) = g_u(x); \text{ and,}$$

$$\nabla \cdot u(x) = 0$$

Initial condition

For $t = 0$,

$$T_e(x,t) = \pi_0(x),$$

Boundary conditions

For $x \in \partial\Omega$,

$$T_e(x,t) = T_a, \text{ and}$$
$$u(x) = 0.$$

1. Background
2. Problem Description
3. Methodology
4. Numerical Results

Generate the optimization problem

Optimization problem

$$\min_{\theta,\pi_0} \mathcal{J}(\theta,\pi_0) = \sum_{i=1}^{n_t} \int_{t=0}^{T} \left(\int_{\Omega_i} T_e(\pi_0,\theta)\,\mathrm{d}x - T_{e,i}^{\star} \right)^2 \mathrm{d}t + \\ + \eta_0 \sum_{i=1}^{n_t} \left(\int_{\Omega_i} \pi_0\,\mathrm{d}x - \pi_{0,i}^{\star} \right)^2 + \eta_1 \|\pi_0\|_{\Omega}^2$$

with the fluid dynamics system as the constraints.
Where ${T_e}^{\star}$: the data sampled by thermometers, $\pi_{0,i}^{\star} = T_{e,i}^{\star}(0)$.

Gradient Descent Method in R^n

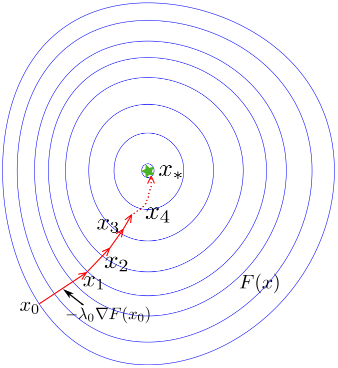


Illustration in R^2

Initial point x_i where $i = 0$.

While has not reached the termination condition:

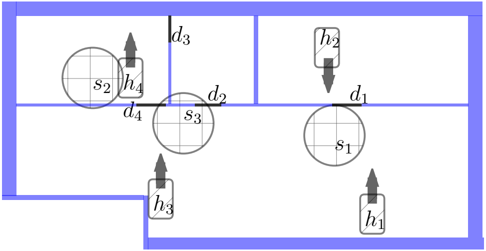
1. Compute $\nabla F(x_i)$ based on cost function $F(x)$.
2. Update λ_i in order to keep $F(x_i - \lambda_i \nabla F(x_i)) \leq F(x_i)$.
3. Update $x_{i+1} \leftarrow x_i - \lambda_i \nabla F(x_i)$ and $i \leftarrow i + 1$.

Gradient Descent Method in Infinite-dimensional Space

The solutions to the fluid dynamic system are in functional spaces, which are infinite-dimensional.

1. The derivatives of the cost function w.r.t. variables are linear maps (*Fréchet derivatives*).
2. In order to compute these derivatives, we compute a new linear PDE system based on the original fluid dynamic system (*Adjoint Equations*).

Apartment Floor Plan for Simulations



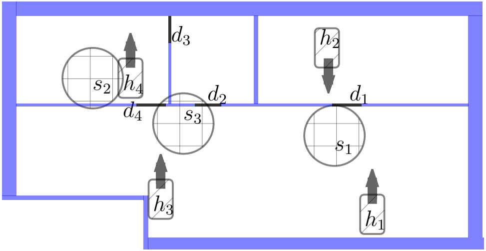
1. $h_{1,2,3,4}$: HVAC units with heaters and fans
2. $d_{1,2,3,4}$: doors inside the apartment
3. $s_{1,2,3}$: thermostats

Comparison: Probabilistic estimation methods*

1. Regard θ and π_0 as random variables with uncertain probability distributions.
2. The estimation problem is formulated to find the optimal distributions that would most likely produce the acquired sensor data in expectation.
3. It generates a mix-integer optimization problem.

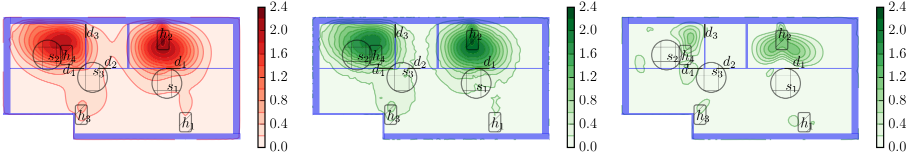
* H. T. Banks and K. L. Bihari, "Modelling and estimating uncertainty in parameter estimation," *Inverse Problems*, vol. 17, no. 1, 2001.

Simulation I



All **three** thermostats work, **two** different initial temperature distributions, π_0, and **six** combinations of doors' configurations, θ.

Temperature's Estimation Error for Simulation I



(a) Initial temperature π_0

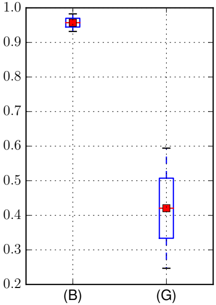
(b) Estimation error by probabilistic estimation method[†].

(c) Estimation error by Gradient-based estimation method.

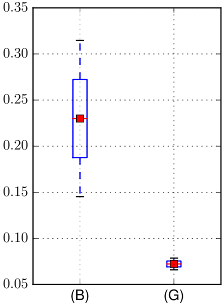
[†] H. T. Banks and K. L. Bihari, “Modelling and estimating uncertainty in parameter estimation,” *Inverse Problems*, vol. 17, no. 1, 2001.

Statistical Results of Simulation I

Columns: (B) Probabilistic estimation method[‡], (G) Gradient-based estimation method.



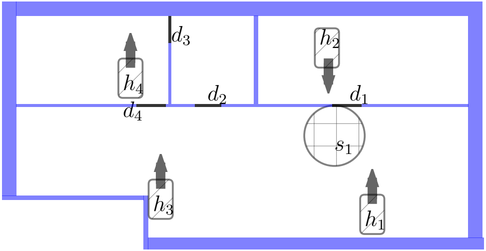
Relative estimation error of π_0



Average estimation error of θ

‡ H. T. Banks and K. L. Bihari, "Modelling and estimating uncertainty in parameter estimation," *Inverse Problems*, vol. 17, no. 1, 2001.

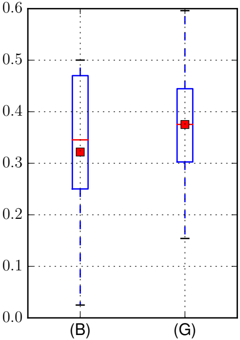
Simulation II



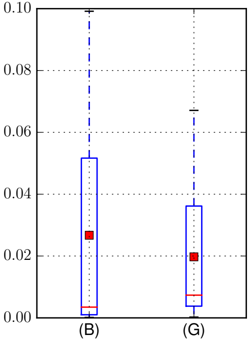
Only sensor S_1 works in this simulation.
One different initial temperature distributions, π_0, and six combinations of doors' configurations, θ.

Statistical Results of Simulation II

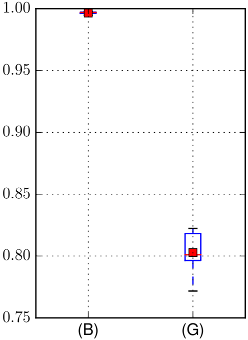
Columns: (B) Probabilistic estimation method[§], (G) Gradient-based method.



(a) Average estimation error of θ.



(b) Estimation error of θ_1.

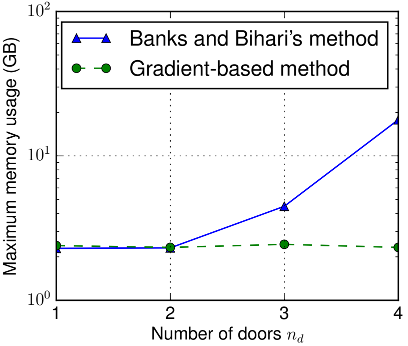


(c) Relative estimation error of π_0.

[§] H. T. Banks and K. L. Bihari, “Modelling and estimating uncertainty in parameter estimation,” *Inverse Problems*, vol. 17, no. 1, 2001.

Memory Usage Result

Memory usages in (GB) of the experiments v.s. different numbers of estimated doors.



Conclusions and Future Work

Conclusions

1. Develop an optimization algorithm to estimate indoor temperature accurately.
2. Show the possibility to estimate indoor geometry configurations with the data only from thermostats.
3. Show the efficient memory usage of our algorithm.

Future work

We are developing a model predicted control system using this estimation algorithm coupled with our optimal control algorithm[a].

[a] R. He and H. Gonzalez, "Zoned HVAC control via PDE-constrained optimization," To appear in Proceedings of the 2016 American Control Conference. arXiv: 1504.04680, 2016.

Conclusions and Future Work

Conclusions

1. Develop an optimization algorithm to estimate indoor temperature accurately.
2. Show the possibility to estimate indoor geometry configurations with the data only from thermostats.
3. Show the efficient memory usage of our algorithm.

Future work

We are developing a model predicted control system using this estimation algorithm coupled with our optimal control algorithm[a].

[a] R. He and H. Gonzalez, "Zoned HVAC control via PDE-constrained optimization," To appear in Proceedings of the 2016 American Control Conference. arXiv: 1504.04680, 2016.