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Mathematical Model of hot-rolled Strip's Camber Formation

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ABSTRACT

A mathematical model of hot-rolled strip's camber formation at non-uniform reduction across the width based on transversal displacements of metal in the hearth of plastic deformation is developed. Transversal displacements of metal reduce unevenness of elongations across strip width, wherein the camber curvature becomes smaller, and its radius is larger than that calculated on the assumption of plane deformation scheme.
A mathematical model is developed with the help of the variational principle of possible changes of strain state (principle Jourdain). It is assumed that the input velocities' unevenness of the metal flow in the hearth of deformation across the strip width passes into an output one proportionally to reductions. Further, velocities of deformation and displacements of metal in the hearth of deformation during rolling and corresponding powers are calculated.
The power of turning of the output cross-section of the hot rolled strip during the formation of camber is separately considered.
Solving Euler-Poisson's equation and submitting input unevenness the row of Fourier, we receive expression for coefficients, which take into account influence of the transversal displacements of metal on decreasing of output unevenness, for every harmonic.

Keywords: hot-rolled Strip's Camber, non-uniform Reduction across the Width, Metal transverse Displacements, Jourdain variational Principle, Euler-Poisson's Equation, output Cross-Section of the hot rolled Strip

1. INTRODUCTION

Hot-rolled strip's camber formation has a great influence on straight moving of a strip when it fills the finishing group of hot-rolling mill. Lateral moving of a hot rolled strip may be reason of emergency situation in the mill. That's why the study of hot-rolled strip's camber formation in the roughing group is very important.

2. FORMATION OF CAMBER OF HOT ROLLED STRIP

Process of the formation of camber of a hot rolled strip is under consideration (Figure1). At the entrance of hearth of deformation the left edge of the rolled strip is thicker, than right, and at the exit - the thickness of strip is constant along a width. At uneven reduction of strip across the width, transverse displacements of metal take place in the hearth of the plastic deformation (Bernsmann,1972; Belskiy, Tret'yakov, Baryshev, & Kudinov, 1998).

2.1 Formulation of the problem

The metal transverse displacements in the hearth of plastic deformation decrease unevenness of the lengthening coefficients and longitudinal stresses across width of the rolled strips at the exit of the hearth of the plastic deformation. This effect it was suggested to take into account with the help of a coefficient ρ :

$$\frac{\Delta\lambda(y)}{\lambda} = \rho\left(\frac{\delta h_0(y)}{h_{0m}} - \frac{\delta h_1(y)}{h_{1m}}\right), \qquad (1)$$

where $\Delta\lambda(y)$ and λ – the value of the current non-uniformity of the elongations and value of average elongation across the strip width, $\delta h_0(y)$ and h_0 – the value of the current transverse thickness variation and average thickness of rolled stock,

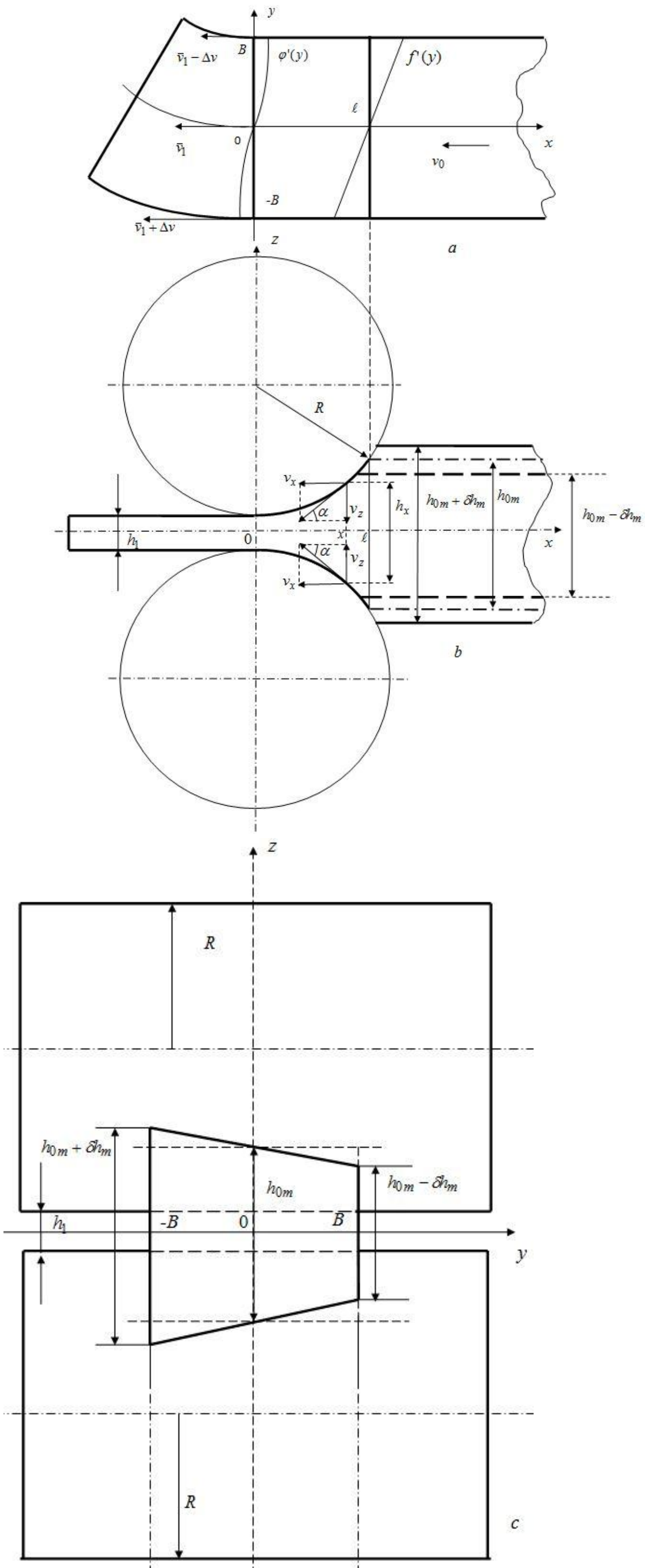

Figure 1 *Scheme of the process*

$\delta h_1(y)$ and h_1 – the value of the current transverse thickness variation and average thickness of the strip, $0 < \rho < 1$ – coefficient taking into account the influence of transverse displacements of the metal in the hearth of plastic deformation.

In the case under consideration output transverse thickness variation $\delta h_1(y) = 0$, while input transverse thickness variation $\delta h_0(y)$ changes linearly from $(+\delta h_m)$ on the left edge (–B) to $(-\delta h_m)$ on the right (+B), B – semiwidth of the strip (Figure1c).

On an entrance to the hearth of plastic deformation, unevenness of high-rise deformation, longitudinal tensions and velocities of metal flow across strip width, is noted. Since the value of deformation and velocities unevenness is significantly small in comparison with their average estimates, generally this unevenness can be expressed as unevenness of metal velocities on an entrance to the hearth of plastic deformation.

Let us describe input unevenness of velocities as $f'(y) \ll 1$, and output – $\varphi'(y) \ll 1$. Let us assume that the spreading when rolling is absent, i.e. $f(0) = f(B) = 0$ and $\varphi(0) = \varphi(B) = 0$, B – semiwidth of a strip; in case of plane deformation scheme $f'(y) = \varphi'(y)$.

For the deformation hearth we will assume model of the rigid-plastic media with elastic external zones, i.e. we consider that a metal, not possessing an elasticity in the deformation hearth, at once acquires it on an exit from the deformation hearth.

Applying the Jourdain variation principle to such defined deformation we get:

$$\delta(\tau_S \iiint\limits_{\Omega} \mathrm{H} d\Omega - \iint\limits_{S} \vec{\sigma}^n \vec{v} ds + \tau_S \sum_{i=1}^{n} \iint\limits_{S_i} |\Delta v_i| ds) = 0 . \qquad (2)$$

where H – shear deformation rates intensity; $\vec{\sigma}^n$ and $\vec{v}$ – full external stresses operating on the surface of the deformation hearth S, and corresponding to them velocities of displacements; Ω – volume of the deformation hearth; τ_S – shear yield point of strip material; Δv_i – a jump of velocities on i - th shear surface S_i; δ – variation symbol.

The expression enclosed in parentheses is a functional and represents a rolling power N_{roll}. The first integral represents the power of internal resistance, the second – the power of external forces acting on the deformation hearth borders, the third – power of cut forces.

The expression for the longitudinal velocities' distribution across the strip width can be written in the assumption that the input velocities' unevenness of the metal flow in the hearth of deformation across the strip width passes into an output one proportionally to reduction:

$$v_x(y)=\overline{v_x}\left[1+f'(y)\frac{h_x-h_1}{\Delta h}+\varphi'(y)\frac{h_0-h_x}{\Delta h}\right], \qquad (3)$$

where $\Delta h=h_{0m}-h_1$ - absolute reduction,

$h_x=h_1+\Delta h\left(\frac{x}{\ell}\right)^2$ - current strip thickness in the hearth of deformation approximated by quadratic parabola; ℓ - length of the heart of deformation; $v_x(y)$ - distribution of longitudinal velocities of the metal across the strip width in the cross section *x*, divided to the circumferential speed of the roll v_r ; $\bar{v}_x$ - average value of velocities of metal across the strip width in cross-section x, divided to the circumferential speed of the roll v_r ; $\bar{v}_0$ - average value of strip input velocity, divided to the circumferential speed of the roll v_r ; $\bar{v}_1$ - average value of strip output velocity, divided to the circumferential speed of the roll v_r .

2.2 Solution of the problem

Mass flow stability principle (current strip thickness h_x is denoted by h) $h_1\bar{v}_1=h\bar{v}_x$.

Average value of strain rate of metal across the strip width in cross-section x,

$$\bar{\xi}_x=\frac{\partial\bar{v}_x}{\partial x}=-\frac{h_1\bar{v}_1}{h^2}h'=-\frac{\bar{v}_x h'}{h}. \qquad (4)$$

Thus

$$\bar{v}_x=-\bar{\xi}_x\frac{h}{h'}. \qquad (5)$$

On the basis of (3) - (5) we obtain an expression for the strain rate ξ_x :

$$\xi_x=\frac{\partial v_x}{\partial x}=\overline{\xi_x}\left[1+\frac{\varphi' h_0-f' h_1}{\Delta h}\right],$$

where $\bar{\xi}_x=\frac{\partial\bar{v}_x}{\partial x}$ - average value of strain rate of metal across the strip width in cross-section x.

Proceeding from a kinematic admissibility of a velocities' field on a roll surface, we receive $\frac{v_z}{v_x}=tg\alpha=\frac{1}{2}\frac{dh}{dx}=\frac{h'}{2}$. Taking into account that value of high-rise deformation is constant, we receive $\xi_z=\frac{v_z}{z}$. On a strip surface $\xi_z=v_x\frac{h'}{h}$.

Taking into account (2) and (4) we receive

$$\xi_z=-\bar{\xi}_x\left[1+f'\frac{h-h_1}{\Delta h}+\varphi'\frac{h_0-h}{\Delta h}\right]. \qquad (6)$$

From a condition of medium incompressibility we will receive expression for strain rate alongaxis y :

$$\xi_y=\overline{\xi_x}\left(f'-\varphi'\right)\frac{h}{\Delta h}. \qquad (7)$$

We receive expression for metal displacements across width, having integrated (7) with respect to the variable y :

$$v_y=\frac{-\bar{v}_x h'}{\Delta h}\left(f-\varphi\right).$$

The intensity of the strain rates we calculate, neglecting the influence of shear deformation, using the expression (5), (6) and (7):

$$\mathrm{H}=2\sqrt{\xi_x^2-\xi_y\xi_z}=\left|\bar{\xi}_x\right|\cdot\left\{2+f'\left[\frac{h-2h_1}{\Delta h}\right]+\varphi'\left[\frac{2h_0-h}{\Delta h}\right]+\left(f'\right)^2\left[\frac{h_1^2-hh_1+h^2}{(\Delta h)^2}\right]+\left(\varphi'\right)^2\left[\frac{h_0^2-hh_0+h^2}{(\Delta h)^2}\right]+f'\varphi'\left[\frac{-2h_1h_0+h_0h+h_1h-2h^2}{(\Delta h)^2}\right]\right\}. \qquad (8)$$

To calculate the power of internal resistance is necessary to integrate the expression (8) in accordance with equation (2) and specificity of the chosen model of the medium:

$$\frac{N_{in}}{\tau_s}=\int_{-B}^{B}\int_0^{\ell}\int_0^{h}\mathrm{H}\,dydxdz. \qquad (9)$$

The power of the forces of sliding friction between the rolls and the strip:

$$\frac{N_{sl}}{\tau_s}=4\mu\int_{-B}^{B}dy\int_0^{\ell}\left|\Delta v_{sl}\right|dx=4\mu\int_{-B}^{B}dy\int_0^{\ell}\sqrt{\left|\Delta v_\tau^2+v_y^2\right|}dx,$$

where Δv_{sl} - sliding velocity of metal on the surface of the roll, $\Delta v_\tau=v_x-1$ - sliding velocity of metal on the roll surface in the rolling direction divided by circumferential speed of the roll v_{roll}, μ - friction coefficient.

After simple transformations we get:

$$\frac{N_{sl}}{\tau_s}=4\mu\ell\int_{-B}^{B}dy\int_0^1\frac{\Delta h}{h}\sqrt{\left(t^2-t_n^2\right)^2+\left[\frac{2\bar{v}_1h_1}{\ell\Delta h}\cdot t\cdot(\varphi-f)\right]^2}\,dt, \qquad (10)$$

where $t=\frac{x}{\ell}$; $t_n=\frac{x_n}{\ell}$, x_n - neutral cross section.

At the strip camber formation at the exit of the deformation hearth the strip bends in a plane $x0y$ (Figure 1a). Compute the power that is expended in bending of the strip.

The metal velocities distribution at the exit of the deformation hearth is presented in Figure 2a-2b, where v_{1m} - averaged over strip width, speed of the strip at the exit of the deformation heart; Δv_m - half an amplitude of unevenness of metal velocities at the exit of the deformation heart; $\Delta v_1(y)$ - current unevenness of metal velocities at the exit of the deformation heart.

During one unit time, exit cross-section is moved from the position 1-1 to the position 2-2 with the rotation angle and radius equal to, respectively, φ and R_t (Figure 3). It takes the power of rotation:

$$N_t = M_t \cdot \omega_t , \tag{11}$$

where M_t - strip bending moment; ω_t - angular velocity of rotation of the cross section of the strip at the exit of the deformation hearth.

Define cross-section rotation radius:

$$R_t = B\frac{\bar{v}_1 - \Delta v_m}{\Delta v_m} = B\frac{\bar{v}_1}{\Delta v_m} . \tag{12}$$

Angular velocity of rotation of the cross section

$$\omega_t = \frac{\Delta v_m}{B} . \tag{13}$$

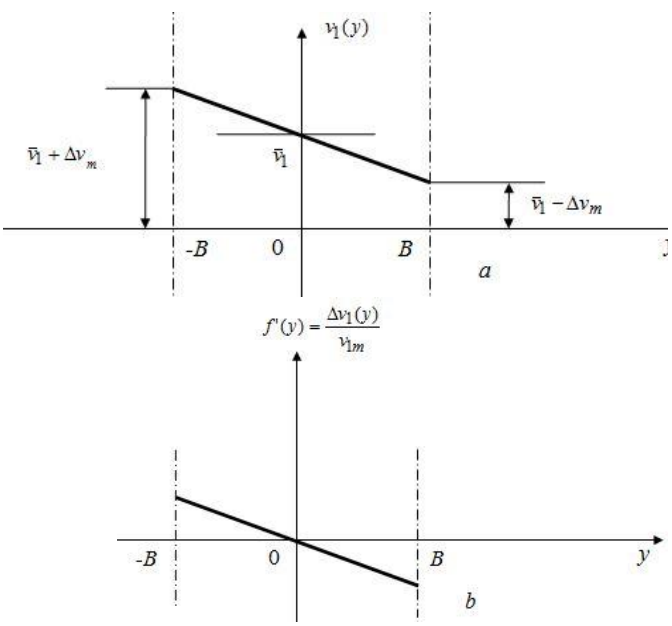

Figure 2 *Metal velocities' distribution at the exit of the deformation hearth*

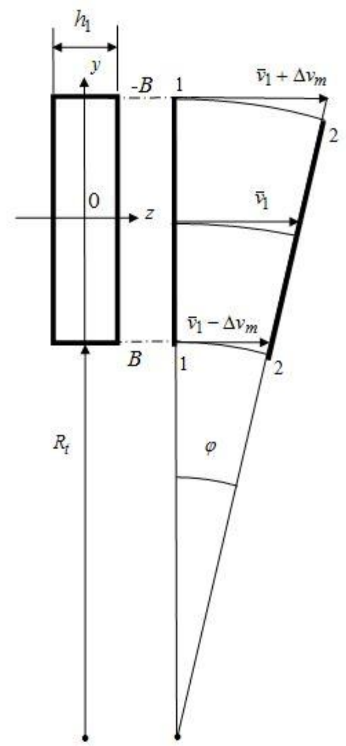

Figure 3 *Scheme of rotation of the cross section of the strip*

Strip bending moment:

$$M_t = \sigma_{max} \cdot W , \tag{14}$$

where σ_{max} - maximum tensile stress in the strip at the exit of the hearth of deformation; W - the resistant moment of the cross- section of the strip.

$$W = \frac{J_z}{y_{max}} , \tag{15}$$

where J_z - the moment of inertia of the cross-section about the axis of rotation; y_{max} - the distance from the axis of rotation to the most stretched fibers of the strip.

$$J_z = 4R_t B^2 h_1 + 2R_t^2 B h_1 + \frac{8}{3} h_1 B^3 ; \tag{16}$$

$$y_{max} = R_t + 2B . \tag{17}$$

As σ_{max} we take its upper bound (yield point):

$$\sigma_{max} = \sqrt{3} \cdot \tau_s . \tag{18}$$

Substituting (16) and (17) in (15) in view of (12) and neglecting small values, we get

$$W=\frac{4\frac{B\bar{v}_1}{\Delta v_m}B^2h_1+2Bh_1\frac{B^2\bar{v}_1^2}{(\Delta v_m)^2}+\frac{8}{3}h_1B^3}{B\frac{\bar{v}_m}{\Delta v_m}}=\frac{8}{3}h_1B^2\frac{\Delta v_m}{\bar{v}_1}+$$

$$+2B^2h_1\frac{\bar{v}_{1m}}{\Delta v_m}+4B^2h_1. \qquad (19)$$

From (19), (14), (13) and (11) we get:

$$N_t=\sqrt{3}\cdot\tau_s\cdot\left(\frac{8}{3}h_1B^2\frac{\Delta v_m}{\bar{v}_1}+2B^2h_1\frac{\bar{v}_1}{\Delta v_m}+4B^2h_1\right)\frac{\Delta v_m}{B}=$$

$$=\sqrt{3}\cdot\tau_s\cdot\left[\frac{8}{3}Bh_1\cdot\bar{v}_1\left(\frac{\Delta v_m}{\bar{v}_1}\right)^2+4Bh_1\cdot\bar{v}_1\left(\frac{\Delta v_m}{\bar{v}_1}\right)+2Bh_1\bar{v}_1\right] \qquad (20)$$

Consider the design $\frac{\Delta v_m}{\bar{v}_1}$:

$$\frac{\Delta v_m}{\bar{v}_1}=\frac{v_1(B)-v_1(-B)}{\bar{v}_1}=\varphi'(B)-\varphi'(-B)=\int_{-B}^{B}\varphi''(y)dy. \qquad (21)$$

Rewrite the expression (20):

$$N_t=\sqrt{3}\cdot\tau_s\cdot\left[\frac{8}{3}Bh_1\cdot\bar{v}_1\left(\int_{-B}^{B}\varphi''(y)dy\right)^2+\right.$$

$$\left.+4Bh_1\cdot\bar{v}_1\left(\int_{-B}^{B}\varphi''(y)dy\right)+2Bh_1\bar{v}_1\right]. \qquad (22)$$

According to Cauchy-Bunyakovsky $\left[\int_{-B}^{B}\varphi''(y)dy\right]^2\le 2B\cdot\int_{-B}^{B}[\varphi''(y)]^2dy$ inequality we will write down:

$$\frac{N_t}{\tau_s}=\sqrt{3}\cdot\left[\frac{16}{3}B^2h_1\cdot\bar{v}_1\cdot\int_{-B}^{B}[\varphi''(y)]^2dy+\right.$$

$$\left.+4Bh_1\cdot\bar{v}_1\int_{-B}^{B}\varphi''(y)dy+2Bh_1\bar{v}_1\right]. \qquad (23)$$

Because Jourdain variational principle can be applied only to the mechanical systems in equilibrium, then considered the hearth of deformation should be balanced. For this purpose at the exit of the hearth of deformation it is necessary to put the moment which provides the rectilinear movement of a strip and which power is numerically equal to the power of strip rotation. Thus the strip will carry away the power spent for accumulation of potential energy:

$$\frac{N_{pot}}{\tau_s}=\bar{v}_1h_1\int_{-B}^{B}\frac{\left(\sigma_x^*\right)^2}{2E^*}dy=$$

$$=\bar{v}_1h_1\int_{-B}^{B}\frac{\left(\sigma_1^*\right)^2-2\varphi'(y)E^*\sigma_1^*+\left[\varphi'(y)E^*\right]^2}{2E^*}dy=$$

$$=\frac{\bar{v}_1h_1\left(\sigma_1^*\right)^2}{2E^*}B+\bar{v}_1h_1\int_{-B}^{B}\frac{[\varphi'(y)]^2E^*}{2}dy, \qquad (24)$$

where $\sigma_x^*(y)=\sigma_1^*-\varphi'(y)E^*$, $\sigma_1^*=\frac{\bar{\sigma}_1}{\tau_s}$, $\bar{\sigma}_1$ - the averaged over strip width the front specific tension. Neglecting powers of shear forces, we write down expression for rolling power according to (2):

$$\frac{N_{roll}}{\tau_s}=\frac{1}{\tau_s}\left(N_{in}+N_{sl}+N_t+N_{pot}\right)=$$

$$=\int_{-B}^{B}\left(\int_0^{\ell}\int_0^{h}\mathrm{H}dzdx+4\mu\ell\int_0^1\frac{\Delta h}{h}\sqrt{\left(t^2-t_n^2\right)^2+\left[\frac{2\bar{v}_1h_1}{\ell\Delta h}\cdot t\cdot(\varphi-f)\right]^2}dt+\right.$$

$$+\sqrt{3}\frac{16}{3}B^2h_1\cdot\bar{v}_1\cdot[\varphi''(y)]^2+4\sqrt{3}Bh_1\cdot\bar{v}_1\cdot\varphi''(y)+ \qquad (25)$$

$$\left.+\bar{v}_1h_1\frac{[\varphi'(y)]^2E^*}{2}\right)dy+2Bh_1\bar{v}+\frac{\bar{v}_1h_1\left(\sigma_1^*\right)^2}{2E^*}B.$$

Varying expression (25), we get Euler-Poisson's equation to find an extremal $\varphi(y)$:

$$F_\varphi-\frac{d}{dy}F_{\varphi'}+\frac{d^2}{dy^2}F_{\varphi''}=0,$$

where

$$F=4\mu\ell\int_0^1\frac{\Delta h}{h}\sqrt{\left(t^2-t_n^2\right)^2+\left[\frac{2\bar{v}_1h_1}{\ell\Delta h}\cdot t\cdot(\varphi-f)\right]^2}dt+$$

$$+\sqrt{3}\frac{16}{3}B^2h_1\cdot\bar{v}_1\cdot[\varphi''(y)]^2+4\sqrt{3}Bh_1\cdot\bar{v}_1\cdot\varphi''(y)+$$

$$+\bar{v}_1h_1\frac{[\varphi'(y)]^2E^*}{2}. \qquad (26)$$

Calculate the components of equation (26):

$$F_\varphi=4\mu\ell\int_0^1\frac{\frac{\Delta h}{h}\left(\frac{2\bar{v}_1h_1}{\ell\Delta h}\right)^2t^2(\varphi-f)}{\sqrt{\left[\left(t^2-t_н^2\right)\right]^2+\left[\frac{2\bar{v}_1h_1}{\ell\Delta h}t(\varphi-f)\right]^2}}dt, \qquad (27)$$

$$\frac{d}{dy}F_{\varphi'}=\bar{v}_1h_1E^*\cdot\varphi''. \qquad (28)$$

$$\frac{d^2}{dy^2}F_{\varphi''}=\frac{32}{3}\sqrt{3}\cdot h_1\bar{v}_1B^2\cdot\varphi^{IV}. \qquad (29)$$

After simple transformation and simplification, we get the Euler-Poisson equation in the following form:

$$\varphi - \frac{1}{K^2} \cdot \varphi'' + \frac{32\sqrt{3}}{3} \frac{B^2}{E^*} \frac{1}{K^2} \cdot \varphi^{IV} = f \ , \qquad (30)$$

where $\frac{1}{K^2} = \frac{E^* \ell h_m \Delta h}{16 \mu h_1}$, $h_m = \frac{h_{0m} + h_1}{2}$.

Present unevenness of output velocities of metal without taking into account the transverse movements in the hearth of deformation $f'(y)$ and taking into account those $\varphi'(y)$ in the following form:

$$f'(y) = \sum_{i=1}^{n} A_i Sin\left(i\pi \frac{y}{B} \right) \ , \qquad (31)$$

$$\varphi'(y) = \sum_{i=1}^{n} B_i Sin\left(i\pi \frac{y}{B} \right). \qquad (32)$$

Substituting (29) and (30) in (28), we get

$$B_i = \rho_i \cdot A_i \ , \qquad (33)$$

where ρ_i - the coefficient which takes into account the influence of transverse displacements of the metal in the hearth of plastic deformation on the reduction of unevenness of the *i*-th harmonic of the output speed of the metal during the formation of a camber;

$$\rho_i = \frac{1}{1 + \frac{1}{(KB)^2} (i\pi)^2 \left[1 + \frac{32\sqrt{3}}{3E^*} (i\pi)^2 \right]} \ . \qquad (34)$$

Thus, to estimate the actual camber of the rolled strip it is necessary to take into account the value of reducing of the unevenness of the lengthening in the hearth of plastic deformation across the width in accordance with the expression (34).

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