Practical Nonlinear Model Predictive Control with Hammerstein Model Applied to a Test Rig of Refrigeration Compressors

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Practical Nonlinear Model Predictive Control with Hammerstein Model
Applied to a Test Rig for Refrigeration Compressors

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ABSTRACT

This paper presents implementation issues and the results of a suboptimal nonlinear model predictive controller used to control the suction and discharge pressures of a compressor when it is tested in a refrigeration test rig. The objective of this rig is to emulate operational conditions to which refrigeration compressors can be subjected when used in refrigeration systems, such as household refrigerators and freezers, allowing quick measurements of some compressor characteristics under such nominal conditions. There is a coupling between suction and discharge pressures and the behavior of both variables is nonlinear with respect to the valve openings, thus the plant to be controlled can be characterized as multivariable and nonlinear. The controller implemented in this paper is the practical nonlinear model predictive control algorithm, which is a general framework that can be used for the implementation of nonlinear model predictive controllers considering almost any class of nonlinear model. This paper considers an specific nonlinear model architecture, the Hammerstein nonlinear model. This model is composed of a static nonlinear element in series with a linear dynamic part, and is widely used, being a simple but flexible nonlinear model. The dynamics of the real test rig were identified using this nonlinear model structure and the identification results are discussed. The practical nonlinear model predictive controller was implemented in the real test rig, being tested at distinct operating conditions. Proposed controller results are compared with the ones obtained with a classical PID controller. The modeling approach and the controller implementation presented good results, showing that it is possible to use nonlinear model predictive control algorithms in refrigeration test rigs, and that this use can contribute to increasing the productivity and operational efficiency of compressor tests.

1. INTRODUCTION

The technological development of refrigeration compressors has an intrinsic connection to the ability of testing them in all the possible operating conditions. For this purpose, the refrigeration industry needs to build dedicated test rigs, thus resulting in feedback control problems. In general, refrigeration applications that require feedback control use classical techniques, such as PID (Proportional-Integral-Derivative) controllers. Applications such as the test rig for measuring isentropic and volumetric efficiencies in variable speed compressors, discussed in Vetsch et al (2016), and the superheat degree in the outlet of the evaporator, presented by Maia et al (2010), are examples of uses of PID controllers in refrigeration systems. This kind of control solution is used mainly because it is simple and presents good closed-loop behavior for a wide class of processes. However, for processes with significant time delay, nonlinearity or coupling between variables, the performance of PID controllers may not be satisfactory, and advanced control techniques can provide an advantage over them.

Even though in industry it is common to use linear controllers to control nonlinear plants, the use of a nonlinear controller can bring advantages in terms of performance and robustness. Algorithms like nonlinear model predictive...
controllers are harder to be implemented than classical controllers, such as PID, but poses the process control problem in the time domain, so the concepts involved are intuitive and at the same time the tuning is relatively easy, even for the multiple-inputs and multiple-outputs (MIMO) case. In addition, model predictive control allows constraints, such as valve opening limitations and pressure limits, to be handled during the design phase. Many examples can be found in the literature, such as the MIMO control of a calorimeter using a dead time compensator, presented by Flesch et al (2012), and the control of superheat in a refrigeration system using predictive functional control with neural networks, in the work of Pedersen et al (2017).

In this paper, formulation and implementation issues of an advanced control technique applied to a real refrigeration test rig are presented. The proposed control algorithm is a novel formulation of the Practical Nonlinear Model Predictive Control (PNMPC), algorithm introduced by Plucénio et al (2007). The formulation presented in this paper extends the classical PNMPC to use Hammerstein models as the nonlinear model used for the predictions. This technique was implemented in a refrigerating compressor test rig, which is able to impose the suction and discharge pressures of the compressor under test along a wide range of values. The real test rig was identified with a Hammerstein model and the PNMPC with this model architecture was implemented. The results of the proposed controller were compared with the ones of a PID controller, which is the common choice for this kind of rig.

The structure of the paper is organized as follows. Section 2 presents the Hammerstein model formulation. Section 3 introduces the main ideas of MPC and presents the formulation of PNMPC. Section 4 presents the refrigerating compressor test rig used in this paper and the nonlinear identification procedure. Section 5 presents the development of the PNMPC controller in the test rig and the controller results compared with the ones of a PID controller. Section 6 presents the conclusions of the paper.

2. HAMMERSTEIN NONLINEAR MODEL

Hammerstein is a class of models defined as a structured block with static nonlinearities associated to dynamic linear blocks. The main structure of the Hammerstein model is illustrated in Figure 1, where a linear dynamic block is connected to the output of a nonlinear static block. A similar model is known as Wiener model and has the order of the blocks exchanged, i.e., the nonlinear static block appears on the output of the dynamic linear block (Eskinat, Johnson, & Luyben, 1991).

![Figure 1: Block diagram of the Hammerstein model structure.](image)

The static input nonlinearity block is defined as a nonlinear function \( f(\cdot) \), while the dynamic linear block is built as a linear transfer function. The equations which define the discrete-time version of Hammerstein model are:

\[
  w(k) = f(u(k)),
\]

\[
  y(k) = \frac{B(z^{-1})}{A(z^{-1})} w(t),
\]

with \( u(k) \) and \( y(k) \) representing the inputs and outputs, respectively; \( A(z^{-1}) \) and \( B(z^{-1}) \) representing polynomials in \( z^{-1} \); and \( x(k) \) being an internal variable.

Saturation, deadzone, sigmoidal functions, piecewise-linear functions, and polynomial functions are the nonlinear functions typically used in this kind of model. The later, in special, is used when the nonlinear input-output relationship involves powers and products of the system inputs and is the most widely used configuration of a Hammerstein model. The formulation of a polynomial function \( f(u(k)) = P(u(k)) \) is:

\[
  x(k) = P(u(k)) = c_n u^n(k) + c_{n-1} u^{n-1}(k) + \cdots + c_1 u(k) + c_0,
\]
with \( c_i, i = 0...n \), representing the polynomial coefficients, and \( n \) the order of the polynomial. For a given set of input and output data, these parameters can be estimated using iterative methods, such as Gauss-Newton and Levenberg-Marquardt. More details about iterative methods for nonlinear least squares problems are presented in Golub and Pereyra (2003).

### 3. PRACTICAL NONLINEAR MODEL PREDICTIVE CONTROL

The model predictive control paradigm encompasses several algorithms that use an explicit model of the process to predict its future behavior along a horizon, aiming to find an optimal sequence of future control actions. The general operation of these algorithms consists in the use of a the process model to obtain an output prediction, stated as \( \hat{y}(k+i | k) \), which generates predictions along a prediction horizon \( i = N_1 \ldots N_2 \), taken at time instant \( k \). With this output prediction an optimization problem is set, used to obtain a sequence of control actions \( u(k+j) \), taken along a control horizon \( j = 0 \ldots N_u - 1 \). This optimization problem generally minimizes a cost functional of the predicted setpoint tracking error and the control effort. The first control action is applied to the system and the receding horizon rolls forward for the next algorithm iteration.

The output prediction is part of the optimization problem and it directly affects the complexity of the optimization task. The most common choice is a linear model, which results in a quadratic programming optimization problem, since there are algorithms that solve this task efficiently. On the other hand, when a nonlinear model is used to obtain the output prediction, the optimization problem becomes nonlinear, so there is no guarantee that the solution is the global minimum of the problem and the computational burden is considerably increased.

For the sake of simplicity, the algorithm is first presented in its SISO (Single-Input, Single-Output) formulation. The extension to the MIMO (Multiple-Input, Multiple-Output) case is presented at the end of this section. The cost functional in the SISO case has the same form as in GPC (Generalized Predictive Control), introduced by Clarke, Mohtadi and Tuffs (1987), and is defined as:

\[
J = \sum_{i=N_1}^{N_2} \delta(i)[\hat{y}(k+i | k) - w(k+i | k)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\Delta u(k+j-1)]^2, \tag{4}
\]

with \( \hat{y}(k+i | k) \) representing the predicted process outputs trajectory taken at time instant \( k \); \( w(k+i | k) \) representing the future setpoint sequence known at time instant \( k \); \( \Delta u(k+j-1) \) representing the control increments; \( N_1 \) and \( N_2 \) representing the maximum and minimum prediction horizons, with \( N = N_2 - N_1 + 1 \); and \( N_u \) representing the control horizon. The weighting sequences \( \delta(j) \) and \( \lambda(j) \), associated to setpoint tracking and control effort, respectively, are used to tune the response speed and control smoothness. A large \( \delta(j)/\lambda(j) \) ratio prioritizes the transient response, while a smaller ratio penalizes the variations of the control action, resulting in smoother control signals (Camacho & Bordons, 2007).

The control signal to be applied to the plant at time instant \( k \), defined as \( u(k) = u(k-1) + \Delta u(k) \), is obtained through the minimization of the cost function of Equation (4). The optimization problem is formulated as:

\[
\min_{\Delta u} J \quad \text{subject to} \quad A\Delta u \leq b, \tag{5}
\]

with matrix \( A \) and vector \( b \) defining the optimization constraints. Equation (5) has an analytical solution if the model is linear and constraints are not used, otherwise the problem requires iterative methods. A detailed explanation about the typical constraints and how to structure \( A \) and \( b \) can be found in Camacho and Bordons (2007).

The Practical Nonlinear MPC (PNMPC) algorithm, illustrated in Figure 2, uses a nonlinear model of the process to predict its future behavior, but a linearization is done at each sampling instant to guarantee that the cost function, Equation (4), is affine in the control increment, so that Equation (5) can be solved by computationally efficient methods (Plucenio et al., 2007). As a nonlinear problem is transformed into a quadratic programming problem, the MPC algorithm is suboptimal when compared with the original nonlinear formulation, but the new formulation can use...
smaller sampling times. In addition, if the sampling time is small when compared to the system dynamics, the results of both formulations are equivalent. The original formulation of PNMPC proposed the use of a process simulator to obtain the nonlinear predictions, but several nonlinear model structures can be used.

The main idea of PNMPC, independently of the model structure chosen for the predictions, is to write a general prediction equation, which is affine in the control:

$$\hat{y}(k) = G(k)\Delta u(k) + f(k),$$

where the output prediction array is

$$\hat{y}(k) = [\hat{y}(k + N_1 | k) \ldots \hat{y}(k + N_2 | k)]^T,$$

and the array of control increments array is defined as

$$\Delta u(k) = [\Delta u(k | k) \ldots \Delta u(k + N_u - 1 | k)].$$

The dynamic matrix $G(k) \in R^{N \times N_u}$ contains the step-response coefficients of the local linear approximation of the nonlinear model, and for the case when $N_1 = d + 1$, with $d$ representing the process dead-time, its structure is

$$G(k) = \begin{bmatrix} s_{11}(k) & 0 & \ldots & 0 \\ s_{21}(k) & s_{12}(k) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1}(k) & s_{(N-1)2}(k) & \ldots & s_{(N-N_u+1)(N_u)}(k) \end{bmatrix},$$

where the vector $[s_{11} \ldots s_{N1}]^T$ corresponds to the first $N$ step-response coefficients, obtained applying a step around the current operating point at time instant $k$. The remaining columns are obtained with the same approach, applying steps around the current operating point at time instants $k + 1 \ldots k + N_u - 1$.

The free trajectory vector $f(k)$, which depends only on the past, is obtained from the free nonlinear trajectory of the model. To obtain $f(k)$, the model is simulated with the past inputs and the future control increments set as zero, i.e. $\Delta u(k + j) = 0, \forall j \in [0, N_u - 1]$, and the whole free response is then corrected by adding to each of its elements the modeling error for the current instant, defined as

$$e(k | k) = y(k) - \hat{y}(k | k).$$
The cost functional, detailed in Equation (4), is built using the prediction array $y(k)$ and the array of control increments. As matrix $G$ is the local linear approximation of the nonlinear model, the optimization problem defined in (5) is affine, thus the unconstrained problem has an analytical solution for each local linear approximation and the constrained problem can be solved using quadratic programming algorithms.

The extension to the MIMO case is straightforward, with matrix $G$ and vector $f$ built with the SISO equivalents for each input-output pair.

The dynamic matrix $G$ for the MIMO case is defined as:

$$G_{\text{MIMO}} = \begin{bmatrix} G_{11} & \cdots & G_{1n} \\ \vdots & \ddots & \vdots \\ G_{m1} & \cdots & G_{mn} \end{bmatrix},$$

with each block $G_{ij}$ representing the matrix $G$ of the SISO case for the pair output $i$ and input $j$, with $i \in [1,m]$ e $j \in [1,n]$.

The free response vector for the MIMO case is defined as:

$$f_{\text{MIMO}} = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix},$$

with each block $f_i$ representing the free response of the output $i$.

In the MIMO case, distinct weighting sequences $\delta_j$ and $\lambda_i$, for the each output $i$ and input $j$ can be used. As in the SISO case, the resulting optimization problem is equivalent to the one used in the MIMO formulation of GPC, and constraints are formulated in the same manner described in Camacho and Bordons (2007).

The control algorithm can be summarized as:

1. Measure the current process outputs.
2. for $i = 1 : m$ do
3. Calculate the SISO nonlinear free trajectories $f_i(k)$ for the current operating point.
4. end for
5. Build the MIMO nonlinear free trajectory matrix $f_{\text{MIMO}}$
6. for $i = 1 : m$ do
7. for $j = 1 : n$ do
8. Obtain the SISO dynamic matrices $G_{ij}(k)$ with the step-response coefficients calculated for the current operating point.
9. end for
10. end for
11. Build the MIMO dynamic matrix $G_{\text{MIMO}}(k)$.
12. Solve the PNMPC quadratic programming problem to find the future control increments $\Delta u(k)$.
13. for $i = 1 : m$ do
14. Apply the first element of the determined array of control increments to the process, i.e. $u_i(k) = u_i(k-1) + \Delta u_i(k)$.
15. end for
16. Return to step 1 with $k = k + 1$.

4. IDENTIFICATION OF THE REFRIGERATION COMPRESSOR TEST RIG MODEL

The experimental evaluation of this paper is done considering a test rig used in the refrigeration industry to evaluate refrigeration compressors under different operating conditions. These conditions are the ones faced in field applications and two of the most important variables to be controlled are the pressures at the compressor inlet and outlet, defined respectively as suction and discharge pressures. The test rig contains two valves and a buffer tank, which is used...
Table 1: Comparison of the results of a Hammerstein nonlinear model and a first-order linear model with time delay.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>MSE</th>
<th>$R^2$</th>
<th>SMAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suction</td>
<td>Linear</td>
<td>0.11</td>
<td>0.89</td>
<td>2.80 %</td>
</tr>
<tr>
<td>Pressure</td>
<td>Hammerstein</td>
<td>0.01</td>
<td>0.97</td>
<td>0.02 %</td>
</tr>
<tr>
<td>Discharge</td>
<td>Linear</td>
<td>11.65</td>
<td>0.65</td>
<td>11.40 %</td>
</tr>
<tr>
<td>Pressure</td>
<td>Hammerstein</td>
<td>2.55</td>
<td>0.90</td>
<td>0.06 %</td>
</tr>
</tbody>
</table>

to partially decouple discharge and suction pressure lines. A process and instrumentation diagram of the test rig is presented in Figure 3.

Figure 3: P&ID diagram of the refrigerant compressor test rig

Despite its simplicity, the control problem found in this architecture presents a high generality, being similar to the ones found in more complex rigs used for different purposes. Some examples are the test rigs for performance evaluation of compressors (ASHRAE, 2005), and for the rating of sound and vibration of compressors (AHRI, 2005). In addition, many rigs used for research and development purposes are based on the configuration considered in this paper.

In order to perform the system identification, the test rig was excited with an amplitude modulated pseudo-random binary sequence (APRBS), as described in Nelles (2001), and the dataset of the system dynamics was acquired. The data series was divided in three distinct parts, used to train, validate and test the models. Each data series contains 2000 samples, which correspond to 400 s of data. The performance metric minimized during the model training phase was the Mean of Squared Errors (MSE). Other performance metrics were used to evaluate the quality of the models.

The nonlinear Hammerstein model was built with an $3^{rd}$-order polynomial as the static nonlinearity, for both outputs, and a $1^{st}$-order discrete-time transfer function for each input-output pair as the dynamic linear block. The results of this model are compared with the ones obtained using a $1^{st}$-order linear model with time delay, both illustrated in Figure 4, where the test portion of the dataset is displayed. This linear structure is the most widely used for this type of dynamic system, and was chosen to make the PID tuning, which is described in section 5, easier. The first and third subplots compare the identified models with the measured system outputs for suction and discharge pressures, respectively. The error residuals are displayed in the second and fourth figure portion.

Table 1 compares the results presented by the Hammerstein and linear models. The model identified with the Hammerstein architecture presented the best results, with smaller MSE, which was reflected in larger values of $R^2$. The symmetric mean absolute percentage error (SMAPE), which is an accuracy metric based on relative errors, shows, as well, that the Hammerstein model outperforms the linear model.

5. CONTROLLER DEVELOPMENT AND CLOSED-LOOP RESULTS

One of the main applications of the test rig used in this paper is to put the compressor to operate under several operating conditions within each test. Thus, reference tracking is the main objective of the controller. Taking this into account, the
PNMPC with Hammerstein model was implemented and tuned aiming to obtain fast setpoint transitions for all possible operating points. The prediction horizons were set as $N_s = 30$ and $N_d = 100$, for suction and discharge pressures, respectively. These values were chosen to provide information of the whole transient of the output to the controller. The control horizon was set the same for both variables, $N_u = 5$, aiming to maintain the computational effort within an acceptable bound for this specific application. Taking into account the selected horizons, the weighting parameters $\lambda(j)$ and $\delta(i)$ were tuned to balance the trade off between the speed of reference tracking and the control effort. For simplicity, the values were assumed constant for the whole horizon and the selected values were $\lambda_s = 100$, $\lambda_d = 100$, $\delta_s = 5$, and $\delta_d = 1$, where indexes $s$ stand for suction and $d$ for discharge.

Additionally, a classical PID controller was implemented for each process variable of the rig, as a baseline for the results obtained with the proposed PNMPC controller. To obtain the controller, the linear model identified for each process output was used. The PID parameters were tuned using the method proposed by Skogestad (2003) and the linear process models detailed in Section 4. In this case, two PID controllers with anti-windup were implemented, one for the suction pair of variables (valve and pressure) and another for the discharge pair.

A test scenario was developed considering the typical behavior during a test, consisting in a series of step reference changes for each controlled variable. The controller test results are shown in Figure 5, where it is possible to observe the closed-loop response at distinct operating points for both controllers.

Both controllers presented good results for the controlled variables. The PNMPC with Hammerstein model approach presented some noticeable differences when compared with the PID controllers. The most important is in the dynamic response after suction pressure setpoint changes. For PID, the inlet valve voltage presents strong efforts in these setpoint changes, resulting, due to the coupling effects, in strong perturbation of the discharge pressure. The PNMPC with Hammerstein model is able to attenuate this effect, due to its MIMO formulation and predictive nature. The nonlinear approach presented, also, similar control efforts at different operating points, while the PID controller presented aggressive responses at certain operating points and smooth responses in other conditions, for the same tuning.
6. CONCLUSIONS

This paper presented an investigation about the application of the practical nonlinear model predictive controller (PNMPC) with a polynomial Hammerstein model in a refrigeration compressor test rig. PNMPC is a general framework that can be used for the implementation of nonlinear model predictive controllers considering almost any class of nonlinear model. This kind of control algorithm is harder to be implemented than classical controllers, such as PID, but presents certain benefits, such as an intuitive tuning even for the MIMO case. In addition, model predictive control allows constraints, such as valve opening limitations and pressure limits, to be handled during the design phase.

The dynamic behaviors of the suction and discharge pressures of the test rig were identified using a MIMO Hammerstein model and the results are compared with the ones obtained by a MIMO linear model. The Hammerstein nonlinear model outperformed the linear model results, obtaining higher $R^2$ and lower mean squared errors. This result indicates that the dynamic behavior of the test rig is nonlinear, so a nonlinear controller was evaluated and compared with traditional PID controllers.

The practical nonlinear model predictive controller has relatively easy tuning, but its implementation is not as straightforward as the implementation of PID controllers. For operating points near to the nominal condition of the linear model, both controller architectures presented good results. In this situation, the main advantages of PNMPC over PID controllers are its ability to reject efficiently the coupling effects between suction and discharge pressures, since it is naturally a MIMO approach. In addition, the PNMPC presented better overall control results, with a smoother control effort and better regulatory performance. Another advantage of the PNMPC approach is its ability to present similar dynamic behaviors in almost all operating conditions, which does not occur in the PID case. For PID, certain operating conditions presented smooth control responses, while other ones presented aggressive control responses for the same PID tuning. This happens because the plant has a nonlinear behavior, which is not compensated by PID controllers.

This study showed that it is possible to use nonlinear model predictive control algorithms in refrigeration test rigs, and
that this use can result in feedback control loops with better performance, contributing to increases in productivity and operational efficiency of compressor tests. The results of this paper also indicate that regular refrigeration systems, especially for commercial applications, which have more flexible actuators, can benefit from nonlinear controllers if they are designed to be used at many distinct operating points.

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tr>
<td>A</td>
<td>matrix of inequality constraints</td>
<td>(-)</td>
</tr>
<tr>
<td>A(z⁻¹)</td>
<td>denominator of the linear model</td>
<td>(-)</td>
</tr>
<tr>
<td>b</td>
<td>vector of inequality constraints</td>
<td>(-)</td>
</tr>
<tr>
<td>B(z⁻¹)</td>
<td>numerator of the linear model</td>
<td>(-)</td>
</tr>
<tr>
<td>c</td>
<td>polynomial coefficients</td>
<td>(-)</td>
</tr>
<tr>
<td>d</td>
<td>dead time</td>
<td>(-)</td>
</tr>
<tr>
<td>e</td>
<td>modeling error</td>
<td>(-)</td>
</tr>
<tr>
<td>f(·)</td>
<td>nonlinear transfer function</td>
<td>(-)</td>
</tr>
<tr>
<td>f</td>
<td>free trajectory vector</td>
<td>(-)</td>
</tr>
<tr>
<td>G</td>
<td>dynamic matrix</td>
<td>(-)</td>
</tr>
<tr>
<td>m</td>
<td>number of outputs of a model</td>
<td>(-)</td>
</tr>
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<td>n</td>
<td>number of inputs of a model</td>
<td>(-)</td>
</tr>
<tr>
<td>N</td>
<td>prediction horizon</td>
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</tr>
<tr>
<td>N₁</td>
<td>starting point of prediction horizon</td>
<td>(-)</td>
</tr>
<tr>
<td>N₂</td>
<td>end point of prediction horizon</td>
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<tr>
<td>Nᵤ</td>
<td>control horizon</td>
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</tr>
<tr>
<td>P(·)</td>
<td>polynomial</td>
<td>(-)</td>
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<tr>
<td>s</td>
<td>step response coefficient</td>
<td>(-)</td>
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<td>u</td>
<td>control signal</td>
<td>(-)</td>
</tr>
<tr>
<td>Δu</td>
<td>array of control increments</td>
<td>(-)</td>
</tr>
<tr>
<td>x</td>
<td>model internal variable</td>
<td>(-)</td>
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<tr>
<td>y</td>
<td>process output</td>
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<td>ӯ</td>
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<tr>
<td>w</td>
<td>setpoint</td>
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<tr>
<td>λ</td>
<td>setpoint tracking weighting coefficient</td>
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<tr>
<td>δ</td>
<td>control effort weighting coefficient</td>
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**Subscript**

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<th>Description</th>
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<td>d</td>
<td>discharge</td>
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<td>suction</td>
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**ACKNOWLEDGMENT**

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