Theoretical Study of Seal Spring in a Wankel Compressor

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ABSTRACT

The successful fabrication of a miniature Wankel compressor relies on its seal and seal spring. The factors that influence the seal of a miniature Wankel compressor mainly include axial double mechanical seal and apex seals. Axial mechanical seal depends on machining precision on the end face. In the compression chamber, the springs for seal flake on the Wankel rotor put pressure on the internal face of the cylinder, which generates the desired amount of pressing force. When springs fail, high-pressure chamber and low-pressure chamber will be connected. Spring performance will directly affect the efficiency of a Wankel compressor. In this paper, we aim to introduce kinematic analysis of apex seals and force analysis of springs in a miniature Wankel compressor.

Keywords: seal; seal spring; Wankel compressor; kinematic analysis; force analysis

1. INTRODUCTION

At present, with the increasing requirement for temperature control of personal air-conditioning system, the development of personal air-conditioning facility has become extremely urgent. Meso- and micro-compressors act as the core parts of a cooling system, and Wankel-type units have become prime choices for micro-cooling systems because of their unique advantages, including simplicity in structure, high efficiency, long life, low vibration, smaller and lighter. Several organizations have obtained some achievements in research on the micro-Wankel engine and Wankel compressor. Heppner et al. analyzed the leakage flow and the friction loss of rotary engine and compressor and established design parameters for micro engine sealing systems. Hsiao et al. presented the simulation of a rotary compressor and its performance comparison with measured results. Prater and William described the methodology and results of an experiment to measure the fundamental undamped natural frequency and damping ratio for the discharge reed valve in a rolling piston rotary compressor.

The Wankel compressor is a kind of rotary machine. It is similar to the Wankel engine and rotary compressor in terms of structure and operation; it consists of a rotor, a cylinder, a crankshaft, a pair of gears and apex seals, compression chambers. As shown in Figure 1, a Wankel compressor consists of three compression chambers, and each chamber is bounded by the internal surface of the cylinder, rotor, two endplates, and apex seals. High-pressure chamber and low-pressure chamber are separated by the apex seals, to satisfy the demand for sealing in the working chamber. From the above discussion, it appears that the material and structure of seal is one of the key problems to research the miniature Wankel compressor at present.

In this paper, we aim at the research of all the seals and seal springs in a Wankel compressor, including the analysis of the motions and forces of seal springs, the design of seals. Finally, seals and springs are manufactured based on the theoretical analyses.

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2. THE PRINCIPLE OF WANKEL ROTARY COMPRESSOR

The cylinder profile of a Wankel rotary compressor is epitrochoid, and the rotor is the inner driving drum envelope of the epitrochoid. The eccentricity $e$ is the distance between cylinder center and rotor center. The operating principle of Wankel compressor is shown in Figure 2. The rotor, which slides around the internal face of the cylinder, spins round the axis of the cylinder, and on its own axis. A pair mesh gear is shown in Figure 2, and the gear ratio is 3:2. The diagram of the complete working process of a Wankel compressor is shown in the Figure 2. Figure a shows the intake process, and the pressure remains unchanged until the volume of the intake chamber reaches its maximum. The compression process begins with the intake valve closing, as illustrated in Figure b. The operation transforms into an exhaust process while the pressure reaches the point of exhaust pressure, as demonstrated in Figure c. In Figure d, the gas pressure is kept stable until the minimum volume of the exhaust chamber is attained, after which the exhaust valve is shut. From the Figure 1, it is not difficult to find that there are three chambers working in parallel. That is to say, intake-compression-exhaust process are achieved in two chambers while the eccentric shaft completes a revolution.

3. KINEMATIC ANALYSIS OF THE SEAL IN A WANKEL COMPRESSOR

3.1 Dynamics of a point on a body with translation and rotation

Figure 3 illustrates the seal and the spring in a Wankel compressor, and the motion diagram of seals in the compression chamber is shown in Figure 4. With the effect of springs, the seals lean against the internal surface of the cylinder, which generates the desired amount of force. Once the spring fails, the performance of a Wankel compressor may be worse or lost.
As is shown in Figure 4, there are three apex seals equispaced on the rotor, 120° apart. A Plane-Rectangular coordinate system will be established to describe a point at the apex of one of the rotor locations, including its acceleration, velocity, movement.

**Figure 3:** The three-dimensional diagram of the Apex seal

As one of the mainly research subjects, the point P represents the location of the apex seals and coordinate system S₁ is the fixed frame of reference and coordinate system S₂ rotates with the rotor. O and O' are the centers for coordinate systems S₁ and S₂. In coordinate systems S₁ and S₂, θ₁ and θ₂ signifies the rotation angles of the eccentric shaft and rotor, respectively. ̇θ₁ and ̇θ₂ are the rotation velocities of the eccentric shaft and rotor, respectively. The position vector from coordinate system O to coordinate system O' is ̅R · ̅ρ is the position vector from O' to the apex seal.

From the Figure 5, ̅r, the position vector, for point P is given as follows

\[ \vec{r} = \vec{R} + \vec{\rho} \]  

(1)

Differentialing Equation (1) with respect to time results in the velocity with respect to the fixed coordinate system O as

\[ \vec{V} = \vec{\dot{r}} = \vec{\dot{R}} + \vec{\dot{\rho}} \]  

(2)

where
$$\ddot{\rho} = (\dot{\rho})_r + \dot{\omega} \times \ddot{\rho}$$  \hspace{1cm} (3)

where $\omega$ is the absolute rotation rate of coordinate system O'.

Figure 5: Coordinate system used for kinematic analysis

Therefore, the apex velocity $\ddot{V}$ of the spring is given by

$$\ddot{V} = \ddot{R} + (\dot{\rho})_r + \dot{\omega} \times \ddot{\rho}$$  \hspace{1cm} (4)

Point P' is coincident with P in coordinate systems O' but fixed in this coordinate system.

$\ddot{R}$ illustrates the absolute velocity of O' and $\dot{\omega} \times \ddot{\rho}$ is the velocity of P' relative to O' as viewed by a non-rotating observer. That is to say, $\ddot{R} + \dot{\omega} \times \ddot{\rho}$ is the absolute velocity of P'. The term $(\ddot{\rho})_r$ is the velocity of P relative to O' as seen by an observer rotating with the O' coordinate system.

Differentialing Equation (4) with respect to time can be expressed as

$$\frac{\partial}{\partial t} (\ddot{V}) = \frac{\partial}{\partial t} (\ddot{R}) + \frac{\partial}{\partial t} ((\ddot{\rho})_r) + \frac{\partial}{\partial t} (\dot{\omega} \times \ddot{\rho})$$  \hspace{1cm} (5)

where the terms of equation become

$$\frac{\partial}{\partial t} (\ddot{R}) = \ddot{R}$$

$$\frac{\partial}{\partial t} ((\ddot{\rho})_r) = (\ddot{\rho})_r + \dot{\omega} \times ((\ddot{\rho})_r)$$

$$\frac{\partial}{\partial t} (\dot{\omega} \times \ddot{R}) = \dot{\omega} \times \ddot{R} + \ddot{\omega} \times \ddot{R} = \ddot{\omega} \times \ddot{\rho} + \ddot{\omega} \times ((\ddot{\rho})_r) + \dot{\omega} \times (\dot{\omega} \times \ddot{\rho})$$

taking these terms into Equation (5) results in the acceleration $\ddot{a}$ being given as follows:

$$\ddot{a} = \ddot{R} + \dot{\omega} \times \ddot{R} + \ddot{\omega} \times (\dot{\omega} \times \ddot{\rho}) + ((\ddot{\rho})_r) + 2 \dot{\omega} \times ((\ddot{\rho})_r)$$  \hspace{1cm} (6)

where $\ddot{R}$ represents the absolute acceleration of O', $\dot{\omega} \times \ddot{R}$ and $\dot{\omega} \times (\dot{\omega} \times \ddot{\rho})$ is the acceleration of P relative to O' as viewed from a non-rotating observer. In other words with $\dot{\omega} \times \ddot{\rho}$ is the tangential acceleration and $\dot{\omega} \times (\dot{\omega} \times \ddot{\rho})$ is the centripetal acceleration of P towards the axis of rotation through O'. Vector $(\ddot{\rho})_r$ is the acceleration of point P.
relative to the O' coordinate system as viewed by an observer moving with O'. Vector $\vec{2}\vec{\omega} \times \left(\vec{\rho},\right)$ is the Coriolis acceleration.

### 3.2 Application to a rotary Wankel compressor

For the Wankel rotary compressor (Table 1), the following formulas are given as known due to the timing mechanism among the crankshaft, the motor and the internal-external gearing. In the micro-compressor, a 20-tooth external gear is fixed to the rear cover. A 30-tooth internal gear rotates about the fixed external gear with the crankshaft offset equal to the center distance difference of the meshing external-internal gears. This results in a 3:1 angular and rotational relationship between the crankshaft and the motor.

<table>
<thead>
<tr>
<th>Table 1: Parameters of Wankel compressor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>generation radius $R$</td>
</tr>
<tr>
<td>$n_2$</td>
</tr>
<tr>
<td>height of apex seal $h$</td>
</tr>
<tr>
<td>mass of apex seal $m$</td>
</tr>
<tr>
<td>displacement</td>
</tr>
</tbody>
</table>

So the speed of the crankshaft and rotation relationships are given by

$$\theta_1 = 3\theta_2, \quad \omega_1 = 3\omega_2$$

Since $\left(\vec{\rho},\right)=0$, point P does not move relative to O' for a rotating observer. The equation for the apex seal velocity can be described as

$$\vec{V} = \vec{R} + \vec{\omega} \times \vec{\rho}$$  \hspace{1cm} (7)

The acceleration $a$ can be simplified as follows (since $\vec{\rho} = 0$, then $\vec{\dot{\rho}} = 0$):

$$\vec{a} = \vec{R} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})$$ \hspace{1cm} (8)

According to the Figure 5, the position vector for the apex seal is expressed as

$$\vec{r} = \begin{bmatrix} R \cos \theta_1 + |\rho| \cos \theta_2 \\ R \sin \theta_1 + |\rho| \cos \theta_2 \\ 0 \end{bmatrix}$$ \hspace{1cm} (9)

So the velocity vector is given as

$$\vec{\dot{V}} = \begin{bmatrix} -\omega_1 R \sin \theta_1 - \omega_2 |\rho| \sin \theta_2 \\ \omega_1 R \cos \theta_1 + \omega_2 |\rho| \cos \theta_2 \\ 0 \end{bmatrix}$$ \hspace{1cm} (10)

The acceleration vector can be obtained by

$$\vec{\ddot{a}} = \begin{bmatrix} -\omega_1^2 R \cos \theta_1 - \omega_2^2 |\rho| \cos \theta_2 \\ -\omega_1^2 R \sin \theta_1 - \omega_2^2 |\rho| \sin \theta_2 \\ 0 \end{bmatrix}$$ \hspace{1cm} (11)
In a Wankel compressor, the Equation (9) can be used to develop for the path of the apex seal for a given crankshaft offset and the distance from the center of the motor to the apex seal location. The path the seal takes for one revolution is shown in Figure 6. This is approximately the same geometry as the static cylinder that the seal is in contact with during operation and will be assumed to be the path of the apex seal in this paper.

![Figure 6: Apex seal path](image)

Utilizing the MATLAB software, the velocity of the seal as a function of crankshaft rotation angle found from Equation (10) is shown in Figure 5. And the seal velocity is presented as that in the fixed coordinate system. The velocity is given for the speed of crankshaft equal to 1500rpm. Meanwhile, the acceleration of the apex seal is shown in the fixed coordinate directions. As a function of crankshaft rotation angle at 1500rpm, the acceleration is presented by evaluating Equation (11) in Figure 8.

![Figure 7: Velocity of the apex seal in the coordinate axis](image)

At this point, it would be useful to put the same calculation methodology into a coordinate system that is radial and transverse to that of the apex seal itself. This will be done via the coordinate system shown in Figure 9. The radial and transverse directions are along vector $\hat{\rho}$ and perpendicular to this vector, respectively (From Figure 9). In order to express the acceleration in terms of a radial and transverse direction, the acceleration vector should be transformed into these directions. The acceleration can be expressed as:

$$
\begin{bmatrix}
    a_r \\
    a_\theta
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta_2 & \sin \theta_2 \\
    -\sin \theta_2 & \cos \theta_2
\end{bmatrix}
\begin{bmatrix}
    a_x \\
    a_y
\end{bmatrix}
$$

(12)

Using Equation (12) the acceleration shown in Figure after transformation is shown in Figure 10.
Figure 8: Acceleration of the apex seal in the coordinate axis

Figure 9: Coordinate system for radial and transverse components for the apex seal

Figure 10: Acceleration components in the radial and transverse apex seal directions

4. FORCE ANALYSIS OF THE SEAL SPRING
The apex spring plays an vital role in compacting internal surface of the cylinder, which may lead to its plastic deformation and abrasion. It is important to research the force condition and motion of the springs. A schematic drawing of a plate-spring used in the Wankel rotary compressor, is shown in Figure 11. The black line represents the original state, and the red one is operating state of the apex spring.

![Diagram](image)

**Figure 11:** The free-body diagram of apex spring

The stress condition of the spring is the worst, while the seal is along the short axis. As shown in Figure 12, the component force of spring is zero in the horizontal direction, and the resultant force $F_r$ is worked out as follows:

$$ F_r = F_r + F_{gr} + F_{sl} + F_m $$  

(13)

where, $F_r$--the radial inertial force of seal; $F_{gr}$--the back pressure; $F_{sl}$--the force of spring; $F_m$--the friction between the seal and the internal face of the cylinder.

![Diagram](image)

**Figure 12:** The free-body diagram of apex spring in short-axis direction

$$ F_r = -ma_r = \frac{W}{g} \alpha^2 \left( \frac{R}{9} + e \cos \frac{2\alpha}{3} \right) $$  

(14)

where, $W$-- the weight of the seal; $g$--acceleration of gravity; $\alpha$--the angular velocity of the eccentric shaft; $R$--the distance between the center of gravity of seal and the rotor center; $e$--eccentric distance; $\alpha$--the angle of the eccentric shaft.

$$ F_{gr} = lBP_2 - lP_1 \left( \frac{B}{2} - \alpha \sin \phi \right) - lP_1 \left( \frac{B}{2} + \alpha \sin \phi \right) $$  

(15)

where, $l$--the length of the seal; $B$--the width of the seal; $\phi$--the swinging angle of the seal; $P_2$--the pressure of high-pressure chamber; $P_1$--the pressure of low-pressure chamber.

When the seal is along the short axis ($\alpha = 270^\circ$ or $810^\circ$), the radial inertial force of seal $F_r$ has its minimum value:

$$ F_{r_{\text{min}}} = \frac{W}{g} \alpha^2 \left( \frac{R}{9} - e \right) $$  

(16)

At different speed of the motor, the $F_{r_{\text{min}}}$ are shown in the following table 2:
Table 2: $F_{\text{min}}$ at different motor speed

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>300</th>
<th>500</th>
<th>800</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{\text{min}}$</td>
<td>0.003</td>
<td>0.008</td>
<td>0.02</td>
<td>0.031</td>
<td>0.072</td>
<td>0.125</td>
<td>0.196</td>
<td>0.288</td>
</tr>
</tbody>
</table>

At the same time, $F_{gr}$ is simplified as follows:

$$F_{gr} = l \frac{B}{2} (P_2 - P_1) = 0.075 (P_2 - P_1)$$

(17)

The relationship between $F_{\text{min}}$ and $F_{gr}$ will be discussed in this section:

(1) While the Wankel compressor is in the startup, the speed of the motor is about 300 rpm:

$P_r = P_1$

Due to its low speed, the pressure in the high-pressure chamber is equal to that in the low-pressure chamber. The back pressure $F_{gr}$ is zero.

(2) After the Wankel compressor start:

a. $P_2 > P_1$

The high-pressure chamber is in the compression process. The pressure in the high-pressure chamber is much greater than that in the low-pressure chamber. The absolute value of the back pressure is greater than the absolute value of the radial inertial force of seal.

b. $P_2 > 0, P_1 < 0$.

The high-pressure chamber is in the exhaust process, and the pressure is positive pressure ($P_2 > 0$). At the same time, the gas pressure is negative pressure during the intake process in the low-pressure chamber ($P_1 < 0$). The absolute value of the back pressure is greater than the absolute value of the radial inertial force of seal.

Moreover, the condition of contacting the seal with the internal surface of the cylinder is as follows:

$$F_R = F_r + F_{gr} + F_{sl} + F_m > 0$$

(18)

In summary, only if the absolute value of the force of spring is greater than the radial inertial force of seal ($F_{sl} > F_{\text{min}} = 0.0003 N$), the above relationship can be satisfied. And the back pressure $F_{gr}$ plays a key role in achieving the effect of the seal, after which the Wankel compressor turns into the operation stage. The force of spring $F_{sl}$ is so small that can be ignored.

In conclusion, in order to ensure the seal to press stably against the internal surface of the cylinder, the spring was designed when the Wankel compressor stops running. Meanwhile, the spring was used to counteract the centripetal force of the seal. Many factors should be concerned on the determination of the force of spring $F_{sl}$, including the force of friction, the abrasion of the seal, we have increased the safety margin of the force.

The deflection of spring (initial amount of compression) is given by:

$$f_1 = \frac{F_{sl} l^3}{4 E b \delta^3}$$

(19)

So, the thickness of spring is worked out as follows:

$$\delta = \sqrt[3]{\frac{F_{sl} l^3}{4 E b f_1}}$$

(20)

In this study, 65Mn is used to manufacture the spring, and the parameters of the spring is:

$$b = 0.8 \text{mm}, \quad h = 2.5 \text{mm}, \quad l = 9.2 \text{mm}$$
the thickness of the spring is achieved:

$$\delta = 0.2 \text{mm}$$

the maximum stress of the spring is expressed by:

$$\sigma_{\text{max}} = \frac{3F_0 l}{2b\delta^2} \leq [\sigma_{0.01}]$$

(21)

where: $f_0$--the deflection of spring(initial amount of compression); $F_0$--the initial force; $l$--the span of spring; $b$--the width of spring; $E$--the elastic modulus(65Mn); $[\sigma_{0.01}]$--elastic limit(65Mn).

From the formula (21), the maximum stress of the spring is less than the limiting stress at given operational conditions. That is to say, the requirements of sealing system in our Wankelcompressor will be satisfied. As shown in Figure 13, the seal and seal spring have been applied in our Wankel compressor.

![Figure 13: Seal and Seal spring in our laboratory](image)

5. CONCLUSIONS

This paper reviewed the research work of Wankel engine and Wankel compressor in recent years, provided a simple mathematical model of rotor, and discussed the influence of seal spring. Through those works, three conclusions are obtained:

1) The dynamics of the rotor of a Wankel compressor have been described in this paper. The analytical treatment developed can be utilized for any rotary Wankel compressor given the crankshaft, rotor design parameters. The apex seal path (the cylinder profile), velocity and acceleration were determined.

2) The seal spring plays a decisive role in seal system and in the performance of the Wankel compressor. If the seal spring failed, the low and high pressure chambers would be connected. And then the performance of the Wankel compressor will weaken or be lost.

3) According to the analysis above, the factors causing seal spring in a Wankel compressor include its thickness ($\delta$), span ($l$), material, and operational conditions. Among these parameters, the thickness and material are the most important factors that influence the performance of a Wankel compressor.

REFERENCES


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