2016

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Liu, Feilong; Xie, Jia; Ziviani, Davide; Feng, Quanke; van den Broek, Martijn; and De Paepe, Michel, "Analysis on Lubricant Film Force for Two Types of Meshing Pair Profile in Single Screw Compressor" (2016). *International Compressor Engineering Conference*. Paper 2486.  
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Analysis on Lubricant Film Force for Two Types of Meshing Pair Profile in Single Screw Compressor

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ABSTRACT

Single screw compressors are widely used in various industrial fields, however the performance usually drops obviously after thousands of hours of operation due to the wear between the star-wheel and the gate rotor. A lot of meshing pair profiles and manufacturing methods are proposed to solve this problem. Thus, good oil lubrication conditions are conducive to improve the wear resistance of the meshing pair so as to increase the compressor’s operation life. In this paper, the geometric models of the clearances for both profiles, a kind of multi-column profile and single-column profile for oil flooded compressors are carried out, and a mathematical lubrication model is used to analyses the oil lubrication. Results show that the torques caused by the oil film force on both sides of the star-wheel tooth are unbalanced. The total torques of the analyzed multi-column profile is smaller than that of the single column profile, which indicates a better lubrication condition. The analysis can also be used to optimize the profile design of the meshing pair of single screw compressors.

1. INTRODUCTION

Since Zimmern (1972) first developed the single screw compressor (SSC) with the profile of straight-line envelope type, it is now widely used in various fields, such as air compression, refrigeration, industrial progress, heat pump and etc.. However, due to the poor wear-resistance performance, the discharge capacity decreases sharply after thousands hours’ operation. This is mainly because of the rapid wear of the star-wheel tooth flank surface while meshing with the screw groove, which can extends the clearance and then results in the leakage. A lot of works have been done by Zimmern (1976), Jing (1986), Boblitt (1987), Jensen (2000), Wu et al. (2009) in order to find a suitable structure profile for prolonging the operation life of the meshing pair, including both profile design and manufacturing methods. Feng et al. (2005) also proposed a multistraight-line envelope profile. Then, on the basis of
column envelope meshing pair (CEMP) and multistraight-line envelope profile, a multicolumn envelope meshing pair (MEMP) was developed by Wu et al. (2009) aimed to reduce the friction.

According to Cameron’s study (1983), a well formed hydrodynamic lubrication between two sliding surfaces can generate a lubricant film and avoid the wear. Similarly to the meshing pair in SSC, a good lubrication between the screw groove and the tooth flank surface can also reduce or even avoid the wear problem. In the study of Post and Zwaans (1986) about the hydrodynamic lubrication, the tooth will move to the smaller force side because of the unbalanced lubricant film forces generated on each side of the tooth flank. This will lead to the clearances’ increasing on one side and decreasing on the other side. According to the research of Jisheng et al. (1997), Huang et al. (2007), Kim et al. (2014) and Larsson (2009), when the change reaches to the critical point, the flow regime will change from hydrodynamic lubrication to boundary lubrication, which will cause the metal contact between the meshing pair and lead to the wear out of the tooth flank. Thus, it is necessary to analyze the lubrication situation between the meshing pair. Huang et al. (2015) created a useful optimization method of the meshing pair profile based on the proposed lubrication simulation analysis about the CEMP.

In this paper, a simulation model is used to analyses the oil lubrication in two types of meshing pair, a kind of multi-column profile and single column profile for oil flooded compressors. Real design parameters of these two types meshing pair for the same single screw compressor are given as calculation examples. The oil lubrication analysis results are then compared and discussed.

2. GEOMETRY MODEL

2.1 Geometric And Kinematic Relations Between The Meshing Pair

Fig.1 shows the relationships between screw rotor and star-wheel while operation. $\phi_2$ is the rotation angle. R1 is the radius of the screw rotor. For any contact points $P_f$ and $P_b$ on each side along the tooth, the related swinging radius $r_{2j}$ is

$$r_{2j} = \sqrt{(r_2 - (b/2))^2 - (L-l)^2 - (b/2)^2}$$ (1)

Where $r_2$ is the radius of the star-wheel, $b$ is the tooth width, $l$ is the distance from the tooth root to the contact point, $L$ is the tooth length.

The swinging radius from the contact point to the rotor center, $r_{1fj}$ and $r_{1bj}$ are
\[
\begin{align*}
\begin{cases}
r_{1f} &= a - r_2j \cos(\phi_2 + \delta) \\
r_{1b} &= a - r_2j \cos(\phi_2 - \delta)
\end{cases}
\end{align*}
\]  
Where \(a\) is the center distance between the screw rotor and the star-wheel, \(\delta\) is the space width half angle

\[
\delta = \arccos\left(\frac{b}{2R_z}\right)
\]  

The unengaged length on both sides of the tooth \(l_f\) and \(l_b\) are

\[
\begin{align*}
\begin{cases}
l_f &= L - \frac{R_z \cos(\delta + \phi_j) - a + R_t}{\cos \phi_j} \\
l_b &= L - \frac{R_z \cos(\delta - \phi_j) - a + R_t}{\cos \phi_j}
\end{cases}
\end{align*}
\]  

2.2 Double-column Meshing Pair

The double-column meshing pair profile is established mainly based on the MEMP, shown in Fig.2. The geometric and kinematic relations between the tooth and the groove can be described by three coordinates. Coordinate, \(S(t,n)\) is used to describe the center coordinates of the designed columns, which could be given through the design process. The other two Cartesian coordinates \(S_f(x_f,h_f)\) and \(S_b(x_b,h_b)\) are used for expressing the geometry of the clearances in both sides (front and back) of the tooth. Axes \(x_f\) and \(x_b\) coincide with the front and back screw groove flank, \(h_f\) and \(h_b\) two orthogonal axes to \(x_f\) and \(x_b\), respectively, which can express the clearances between the screw groove and the tooth flank. As shown in Fig.2, the tooth flanks of the double-column profile are consisted of two column segments and three tangent lines for each side. Points 1-8 are the tangent points of the arcs and lines. The groove inclined angles \((\alpha_{f1}, \alpha_{b1})\), equals to the inclined angles of the tangent lines at the meshing points \((c,d)\). These angles can be obtained by the geometric and kinematic relations between the star wheel and the screw shown in Fig.1.

\[\text{Figure 2: Double-column meshing pair}\]

For this double-column meshing pair profile, columns \(O_2\) and \(O_3\) are the basic columns, which are parallel to the tooth center line. \(O_1\) and \(O_4\) are the auxiliary columns with inclined angles \(\beta_f\) and \(\beta_b\). According to the MEMP theory proposed by Wu et al. (2009), the center coordinates of the auxiliary columns at any position along the tooth can be calculated once \(\beta_f\), \(\beta_b\) and the center coordinates on the tooth top are given after design.

The groove inclined angles are expressed as
The clearance functions for the front side of the tooth can be expressed as follows:

\[
\begin{align*}
\tan \alpha_f &= \frac{\omega_f r_2}{\omega_f r_1} \\
\tan \alpha_i &= \frac{\omega_i r_2}{\omega_i r_1}
\end{align*}
\]  

(5)

\[
h_f(t) = \begin{cases} 
\frac{(t_1-t)\sin(\alpha_1-\alpha_i)}{\cos(\alpha)} & t_1 < t \leq u \\
\xi \cos(\alpha_1) + r - r \cos(\alpha_1 - \arcsin(\frac{t_1-t}{r})) & t_1 \leq t \leq t_1 \\
\frac{(t_2-t)\sin(\alpha_2-\alpha_i)}{\cos(\alpha_2)} & t_1 \leq t < t_2 \\
\arcsin\left(\frac{t_3-t}{r}\right) - \alpha_i & t_2 \leq t < t_3 \\
\frac{(t_4-t)\sin(\alpha_4-\alpha_i)}{\cos(\alpha_4)} & 0 \leq t < t_4
\end{cases}
\]

(6)

\[
x(t) = \begin{cases} 
\frac{(u-t)\cos(\alpha_1-\alpha_i)}{\cos(\alpha)} & t_1 \leq t \leq u \\
\frac{(t_1-t)\cos(\alpha_1-\alpha_i)}{2} & t_1 \leq t < t_1 \\
\arcsin\left(\frac{t_1-t}{r}\right) + \alpha_1 & t_1 \leq t < t_1 \\
\frac{(t_2-t)\cos(\alpha_2-\alpha_i)}{2} & t_2 \leq t < t_2 \\
\arcsin\left(\frac{t_2-t}{r}\right) - \alpha_1 & t_2 \leq t < t_2 \\
\frac{(t_4-t)\cos(\alpha_4-\alpha_i)}{2} & 0 \leq t < t_4
\end{cases}
\]

(7)
Where \( r \) is the radius of the designed columns, \( \xi \) is the design clearance, \( u \) is the thickness of the tooth, \( \alpha_1 \) and \( \alpha_4 \) are angles of the tangent lines when points 1 and 4 act as the meshing points, which can be got as follows,

\[
\begin{align*}
\tan \alpha_1 &= \frac{\omega R_1}{\omega R_i} \\
\tan \alpha_4 &= \frac{\omega R_4}{\omega R_i} \frac{a - r_2}{a - r_1}
\end{align*}
\]  \( \tag{8} \)

Line 34 is the common tangent line of the two columns whose angle can be calculated according to the geometric relationships between the columns, therefore, \( \alpha_i = \alpha_f, i = 1, 2 \) are the related height of the points.

\[
\begin{align*}
\ell_1 &= t_i - r \sin \alpha_i \quad i = 1, 2 \\
\ell_2 &= t_i - r \sin \alpha_i \quad i = 3, 4
\end{align*}
\]  \( \tag{9} \)

Similarly to the front side, the clearance functions of the back side can be obtained as well.

### 2.3 Single-column Meshing Pair

The single-column meshing pair is introduced and studied by Wu and Feng (2009), shown in Fig.3.

![Figure 3: Single-column meshing pair](image)

Three coordinates are also set similarly to the double-column meshing pair to describe the clearances between the crew groove and the tooth flank. The tooth flanks of the single-column profile are consisted of one column segments and two tangent lines for each side. Points 9-12 are the tangent points of the arcs and lines. The groove inclined angles (\( \alpha_f, \alpha_b \)), which are related to the meshing points (\( g, e \)), can be calculated by

\[
\begin{align*}
\tan \alpha_f &= \frac{\omega R_2}{\omega R_2 g} \\
\tan \alpha_b &= \frac{\omega R_3}{\omega R_3 b}
\end{align*}
\]  \( \tag{10} \)

Then the clearance functions for the front side are

\[
h_f(t) = \begin{cases} 
\xi \cos \alpha_f + r - r \cos(\alpha_f - \arcsin(\frac{t - t_9}{r})) & 1 \leq t \leq t_9 \\
\xi \cos \alpha_f + (t_9 - t) \frac{ \sin(\alpha_v - \alpha_f) }{ \cos(\alpha_v) } & 0 \leq t < t_9
\end{cases}
\]  \( \tag{11} \)
\[ x(t) = \begin{cases} 
\frac{(u-t)\cos(\alpha_j - \alpha_0)}{\cos(\alpha_0)} & t_0 \leq t \leq u \\
\frac{(t_0-t)\cos(\alpha_j - \frac{\arcsin\left(\frac{r_1 - t}{r}\right) + \alpha_0}{2})}{\arcsin\left(\frac{r_1}{r}\right) + \alpha_0} & t_0 \leq t < t_0 \\
\frac{(t_1-t)\cos(\alpha_j - \frac{\arcsin\left(\frac{r_1 - t}{r}\right) - \alpha_j}{2})}{\arcsin\left(\frac{r_1}{r}\right) - \alpha_j} & t_0 \leq t < t_0 \\
\frac{(t_1-t_0)\cos(\alpha_{9,10} - \alpha_j)}{\cos(\alpha_{9,10})} & 0 \leq t < t_0 
\end{cases} \] (12)

Where \( \alpha_9, \alpha_{10} \) are the angles of the tangent lines when points 9 and 10 act as the meshing points, which can be got by:

\[ \tan \alpha_j = \frac{\omega_j r_2}{\omega_j R} \]
\[ \tan \alpha_{10} = \frac{\omega_{10} r_2}{\omega_{10} (a-r_2)} \] (13)

\( t_9 \) and \( t_{10} \) are the related height of the meshing points

\[ t = t_9 - r \sin(\alpha) \quad i = 9,10 \] (14)

The clearance functions for the back side can also be obtained through the same way.

### 2.4 Lubrication Model

The whole path between the groove and the tooth flank can be divided into two parts by the meshing point, convergent region at the high pressure side, and divergent region at the low pressure side. For the divergent region, the pressure applied on the tooth flank can be seen as the suction pressure.

For the convergent region, the lubrication model introduced by Launder and Leschziner (1978) can be simplified as:

\[ \frac{dP(x)}{dx} = \frac{6\mu \rho}{h(x)^2} \frac{12 \mu M}{\rho h(x)^4} + \frac{1.2 M^2}{h(x)^2} \frac{dh(x)}{dx} - \frac{0.133 \rho \mu L^2}{h(x)^2} \frac{dh(x)}{dx} \] (15)

Where \( \rho \) is the density of the lubrication oil, \( h(x) \) is the geometric model of the clearance.

The boundary condition is

\[ \begin{cases} 
x = x_0, P(x) = P_g \\
x = x_0, P(x) = P_s \end{cases} \] (16)

Where \( P_g \) is the gas pressure in the compressing chamber, \( P_s \) is the suction pressure. The pressure distribution along the tooth surface in the convergent region is
\[ P(x) = P_0 + \int_{h(x)}^{h(x_0)} \frac{6 \mu U}{h(x)} \, dx - \frac{12 \mu M}{\rho} \int_{h(x)}^{h(x_0)} \frac{1}{h(x)} \, dx \]
\[ - \frac{1.2 M^2}{2 \rho} \left( \frac{1}{h(x)^2} - \frac{1}{h(x_0)^2} \right) - 0.133 \rho U^2 [\ln h(x) - \ln h(x_0)] \]

Where
\[ M = \frac{\frac{12 \mu}{\rho} \int_{h(x)}^{h(x_0)} \frac{1}{h(x)} \, dt}{(\frac{12 \mu}{\rho} \int_{h(x)}^{h(x_0)} \frac{1}{h(x)} \, dt)^2 + \frac{2.12}{\rho} \left( \frac{1}{h(x)^2} - \frac{1}{h(x_0)^2} \right) N} \]
\[ N = \int_{h(x)}^{h(x_0)} \frac{6 \mu U}{h(x)} \, dx - 0.133 \rho U^2 [\ln h(x_0) - \ln h(x)] + P_r - P_0 \]

\[ U \] is the relative velocity between the meshing pair
\[ U = \sqrt{(\omega r_2)^2 + (\omega r_{U(j)}))^2} \]

Then, the component forces caused by the oil film and the gas force, applied on the engaged element \( dl \), which are parallel to the upper surface of the star-wheel are
\[ \begin{aligned}
F_{\alpha j} &= \cos \alpha \int_{x_0}^{x_1} P(x) \, dx \\
F_{\alpha j} &= \cos \alpha \int_{x_0}^{x_1} P \, dx
\end{aligned} \]

Where \( \alpha \) is the groove inclined angle of the related meshing point.

The gas force, applied on the unengaged element \( dl \), which is parallel to the upper surface of the star-wheel is
\[ F_{\alpha} = u \cdot P_r \]

Therefore, the torques applied on each side of the tooth are
\[ \begin{aligned}
T_{\alpha j} &= \int_{l_j}^{l_{j+1}} F_{\alpha j} \cdot r_\alpha(l) \, dl + \int_{l_j}^{l_{j+1}} F_{\alpha j} \cdot r_\beta(l) \, dl + \int_{l_j}^{l_{j+1}} F_{\alpha j} \cdot r_\gamma(l) \, dl \\
T_{\alpha h} &= \int_{l_h}^{l_{h+1}} F_{\alpha h} \cdot r_\alpha(l) \, dl + \int_{l_h}^{l_{h+1}} F_{\alpha h} \cdot r_\beta(l) \, dl + \int_{l_h}^{l_{h+1}} F_{\alpha h} \cdot r_\gamma(l) \, dl
\end{aligned} \]

Where \( l_j, l_h \) and \( r_{\gamma}(l) \) can be got through equation (2~4).

The composite torque applied on one tooth is as follows:
\[ T_i = T_{h_i} + T_{f_i} \]

During operation, there are usually three or four star-wheel teeth participating in the engaging at the same time. Thus, the total torque applied on the star-wheel is
\[ T = \sum_i T_i \]
3. RESULTS

In this paper, two different designs of the two profiles for the same 2 m³/min oil-flooded single screw air compressor are taken as calculation examples. The Dimensions of the single screw compressor is shown in Tab. 1.

**Table 1:** Dimensions of the single screw compressor

<table>
<thead>
<tr>
<th>Main Parameter</th>
<th>Value</th>
<th>Main Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screw rotor radius $R_1$/mm</td>
<td>65</td>
<td>Tooth length $L$/mm</td>
<td>30</td>
</tr>
<tr>
<td>Star-wheel radius $R_2$/mm</td>
<td>61</td>
<td>Center distance $a$/mm</td>
<td>96</td>
</tr>
<tr>
<td>Tooth width $b$/mm</td>
<td>18</td>
<td>Envelope column radius $r$/mm</td>
<td>6</td>
</tr>
<tr>
<td>Rotating speed / $r\cdot min^{-1}$</td>
<td>2970</td>
<td>Discharge pressure $P_{out}/10^5 Pa$</td>
<td>10</td>
</tr>
<tr>
<td>Design clearance $\xi$/mm</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2:** Design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$n$/mm</th>
<th>$t$/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth top of Column O₁</td>
<td>-4.754</td>
<td>7.292</td>
</tr>
<tr>
<td>Column O₂</td>
<td>-2.674</td>
<td>4.542</td>
</tr>
<tr>
<td>Column O₃</td>
<td>2.674</td>
<td>0.958</td>
</tr>
<tr>
<td>Tooth top of Column O₄</td>
<td>4.739</td>
<td>-1.792</td>
</tr>
<tr>
<td>Column O₅</td>
<td>-2.674</td>
<td>4.542</td>
</tr>
<tr>
<td>Column O₆</td>
<td>2.674</td>
<td>0.958</td>
</tr>
</tbody>
</table>

Tab. 2 is the design parameters related to the two different profiles. For the double-column, $\beta_f=2.327^\circ$, $\beta_b=-2.298^\circ$.

Fig. 4 is the pressure distributions of the two types meshing pair under the discharge starting rotation angle ($\phi_2=35^\circ$) at the tooth top.
In both types of the meshing pair, the pressures increase in the convergent region, and will reach the peak value at the meshing point. For the double-column meshing pair, the peak pressure is $1.773 \times 10^6$ Pa. For single-column meshing pair, the peak pressure is $1.89 \times 10^6$ Pa.

The total torques of the two types meshing pair through the rotation angle $\phi_2$ are shown in Fig.5. Obviously, the torques for both profiles have periodical changes during the operation. For the double-column, the total torque changes from 1.3 N·m to 3.7 N·m. For the single-column, the total torque changes from 2.92 N·m to 4.5 N·m. At any same rotation angle, the total torque applied on the double-column star-wheel is always smaller than that of the single-column. According to the study of Post and Zwaans (1986), there will be clearance changes between the meshing pair and the single-column meshing pair will have a larger clearance change because of the larger unbalanced torques. From the researches of Jisheng et al. (1997), Huang et al. (2007), Kim et al. (2014) and Larsson (2009), if the change reaches the critical point, the flow regime will change form hydrodynamic lubrication to boundary lubrication, which will cause a metal contact between the meshing pair and lead to the wear out of the tooth flank. That means the double-column have a better lubrication situation.
4. CONCLUSIONS

In this paper, a simulation model is used to analyses the oil lubrication in two types of meshing pair. Real design parameters of these two types meshing pair for the same single screw compressor are given as calculation examples. Several conclusions can be made according the calculating results:

- The pressure increases to a peak value from the start of the convergent region to the meshing point.
- The total torques caused by the oil film forces are unbalanced, which would lead to a deflection of the tooth in order to regain balance. Furthering result in changes in the clearances between the meshing pair and may cause metal contact of the meshing pair reducing the operation life.
- The total torque generated by the double-column meshing pair profile is smaller than that of the single-column at any same rotation angle, which indicates the double-column meshing pair profile has a better hydrodynamic lubrication state and a longer operation life.
- The analysis can also be used to optimize the meshing pair profile design for single screw compressors.

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