2014

Global Sensitivity Analysis of a Multi-Cylinder Automotive Reciprocating Compressor

Nasir Bilal
Purdue University, United States of America, nasirbilal@gmail.com

Global Sensitivity Analysis of a Multi-Cylinder Automotive Reciprocating Compressor

Nasir BILAL[1],
[1]Purdue University, School of Mechanical Engineering,
West Lafayette, IN, USA
nasirbilal@gmail.com

ABSTRACT

The objective of this paper is to perform sensitivity analysis of a compressor simulation model using a global sensitivity analysis method. In this paper, Sobol's method of global sensitivity analysis, which is based on decomposition of variance, is applied to a compressor simulation model. A previously developed and tested compressor simulation model is used to perform sensitivity analysis of gas pulsations in the suction manifold of a multicylinder reciprocating compressor. The focus of the research is to determine the sensitivity of gas pulsations in the suction manifold of the compressor to three design parameters, namely, radius, width and depth of the suction manifold. Sobol's method of global sensitivity analysis was used to calculate the first order effect and total effect of the suction manifold radius, width and depth on the manifold pressure response. It was also showed that suction manifold pressure response was most sensitive to changes in manifold radius, followed by manifold width and depth. This method of sensitivity analysis can be readily extended to any compressor simulation model.

1. INTRODUCTION

Gas pulsation in reciprocating compressor is a common problem and generally requires detailed mathematical modeling for noise reduction and design optimization (Park, et al., 2007) and (Park & et al., 2008). Mathematical models for compressors are generally complex, nonlinear and have a large number of operational and design parameters. There are inherent uncertainties associated with the input parameter and generally the model output is quite sensitive to certain input parameters (Bilal, 2011). However, fewer studies have focused on understanding how uncertainties and sensitivities in model parameters affect the validity of compressor simulations models when the model parameters are perturbed about the baseline set of parameter that are used to calibrate the model.

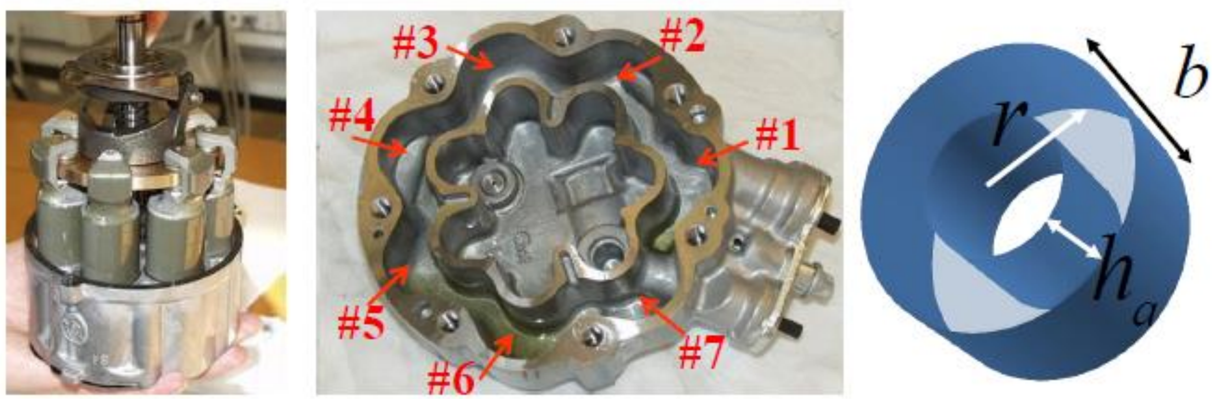
Generally two distinct approaches of uncertainty and sensitivity analysis are used, namely local sensitivity analysis and global sensitivity analysis. Local sensitivity analysis methods work well for models that are linear but fail to detect any discontinuities or nonlinear relationship between models and are also computationally expensive (Iman, 1986). The most commonly used local sensitivity analysis method is one-at-a-time (OAT) which is performed by varying one factor at a time while keeping the other factors constant about their nominal values. This method has very limited application, is confined to linear models and suitable for models that are computational inexpensive.

On the other hand, global sensitivity analysis methods explore the entire parameter space of each variable and generally all parameters are varied simultaneously (Saltelli et al., 2000). This method does not require the model to be additive or linear, and is independent of the model. The basic approach behind Global sensitivity analysis is to vary all variables simultaneously and compute the variance of the output. The relative sensitivity of an input variable is determined by fixing a particular variable, and determining the drop in the output variance. Usually, the larger the drop in output, the more sensitive the output is to the input variable. Some of the properties of global sensitivity analysis methods are that it should be able to take into account the effects of different input distributions and parameters changes, be model independent, be able to address the interaction effects, and be able to treat grouped factors as a single variable.

The goal of this paper is to perform global sensitivity analysis on a compressor simulation model. An already development and tested model for calculating the gas pulsations in the suction manifold of an automotive reciprocating compressor (Park, 2004), (Park et al., 2007) and (Park et al., 2008) is used to perform sensitivity analysis on the model parameter. Three design parameters of the suction manifold of the reciprocating compressor will be used to analyze the sensitivity of gas pulsation to these input parameters. Sobol's method of global sensitivity analysis (Sobol, 1993), which is a variance based method, is used to perform sensitivity analysis of this model.

2. BACKGROUND INFORMATION ABOUT SIMULATION MODEL

This research is performed using a detailed multi-cylinder automotive compressor model developed by (Park, 2004), which focused on modeling and simulating pressure pulsations in the suction manifold of a reciprocating compressor. The actual compressor model and the suction manifold, and simplified geometric model are shown below in Figure 1.



(a) (b) (c)

Figure 1: (a) A multi-cylinder reciprocating compressor and; (b) the suction manifold of the compressor with suction valves marked; (c) simplified geometric representation of the suction manifold, where r, b, and h_a respectively represent radius, depth and width of the suction manifold.

This model calculated the cylinder pressure and temperatures during a cycle, predicted the valve dynamic responses, and calculated mass flow rate through each of the suction ports. Mass flow rate was used as an exciting function in the acoustic model to calculate the gas pulsation in the suction manifold. The schematic of the basic simulation model is shown in Figure 2 and details could be found in (Park, 2004).

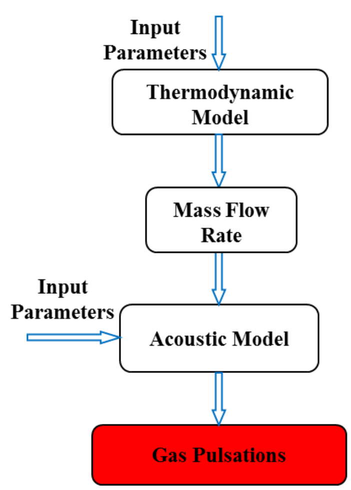


Figure 2: The basic compressor simulation model

A typical frequency pressure response at a particular cylinder location is shown in Figure 3. It shows the amplitude of pressure pulsation at each frequency in the 0-1200 Hz range. Since it is a multicylinder compressor, pressure varies for each cylinder location and is calculated and plotted separately.

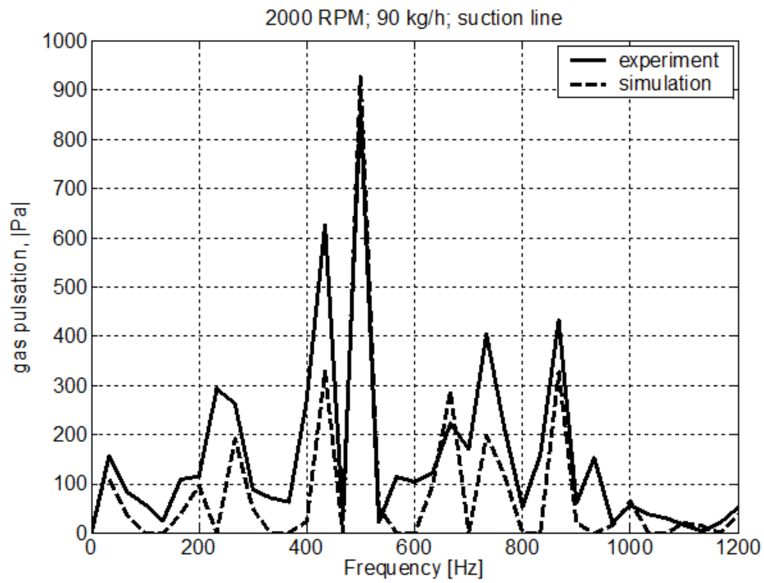


Figure 3: Comparison of pressure response in the suction manifold compared with the experimental data at 90 Kg/h and 2000 rpm.

3. SENSITIVITY ANALYSIS

Sensitivity analysis has been defined by (Saltelli et al., 2000) as “the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variations, and of how the given model depends upon the information fed into it.”

Global sensitivity analysis methods are variance-based methods and rely heavily on sampling methods and input parameter distribution. The concept of using variance as an indicator of the importance of an input parameter is the basis for many variance-based sensitivity analysis methods. The importance of the given input factor can be measured by a term defined as the sensitivity index, which is the fractional contribution to the output variance due to uncertainties in the inputs. While there are many methods available for analyzing the decomposition of variance as a sensitivity measure, the Method of Sobol (Sobol, 1993) is used to show the step involved in implement the equations. This method is one of the most established and widely used methods and is capable of computing the ‘Total Sensitivity Indicies’ (TSI), which measures the main effects of a given parameter and all the interaction (of any order) involving that parameter (Sobol, 1993). For example, if there are only three input parameters, A, B, and C, then the total effect of parameter A, for instance, on the output is,

$$TS(A) = S(A) + S(AB) + S(AC) + S(ABC) \tag{1}$$

where $TS(i)$ is the total sensitivity index of parameter *i*, and $S(A)$ denotes the first order sensitivity index for parameter A, $S(A_j)$ denote the second-order sensitivity index for the parameter A and j $(for\ j(\neq A))$, i.e. the interaction between parameters A and $j(\neq A)$, and so on.

3.1 VARIANCE BASED SENSITIVITY ANALYSIS

The basis of most of the variance-based methods depend on the estimation of the following quantity

$$\frac{Var_X[E(Y|X)]}{Var(Y)}$$

Where Y denotes the output variable, X denotes an input variable, $E(Y|X)$ denotes the expectation of Y conditional on a fixed value of X, and the variance is taken over all possible values of X, and $V(Y)$ is total output variance of the output Y.

4. SOBOL'S METHODS OF GLOBAL SENSITIVITY ANALYSIS

Sobol's method is used to calculate global sensitivity indices of the input parameters for nonlinear models that are defined either by analytical methods or by a simulation model. The basis of Sobol's method is the decomposition of variance of the model output function into a sum of variances in combinations of input parameters in increasing dimensionality (Zheng & Rundell, 2006). In the method of Sobol, each effect (main or otherwise) is computed by evaluating a multidimensional integral via the Monte Carlo integration (Saltelli, et al., 2000). The first order effect, or the main effect, of a single input parameter is the effect of a single parameter on the model output variance. The total effect of a parameter is the sum of the parameter's main effects and all the interactions (of any order) involving that parameter (Saltelli, et al., 2000). The appropriate equations to implement the method of decomposition of variance can be found in (Zheng & Rundell, 2006).

4.1 Implementation of the Sobol's Method to Compressor Simulation Model

The method of Sobol was implemented by following a series of steps as outlined below:

- The parameters and their ranges of variation were defined as shown in Table 1.
- A probability distribution to sample the variables was defined. In this case, a uniform distribution to generate the input vector.
- Simulations were run for 295 iterations. The implementation of Sobol's method required two sets of parameter spaces to be sampled. The first one is called the primary parameter space and the second one is called the complementary parameter space. For details on implementation see Bilal (2011) and Zheng & Rundell (2006).
- The simulation code for this method was developed in MATLAB® and was applied to the acoustic model and the manifold radius *r*, the manifold depth *d*, and the manifold width *ha* were used. A complete description of the parameters and their ranges is shown in Table 1.

Table 1: Numerical values of the variables used in simulation

Variables	Mean (mm)	Minimum Value (mm)	Maximum Value (mm)
Manifold radius, ***r***	50	30	70
Manifold depth, ***b***	27	20	40
Manifold width, ***ha***	14	8	20

These parameter values were chosen so that some of the geometric constraints of the manifold were still satisfied, and at the same time, the system was sufficiently perturbed in order to observe the effects of sensitivity analysis. The operating conditions for the thermodynamic model were chosen to be a speed of 2000 rpm and a mass flow capacity of 90 kg/hr. The thermodynamic model was run at the nominal values and no variables were changed in that model to calculate the mass flow rate which is an excitation function for the acoustic model. The output of this model was used in conjunction with acoustic model to calculate the pressure pulsation in the suction manifold.

5. RESULTS AND DISCUSSION

5.1 First Order Effects

The first order effects of the response are shown in Figures 4-6. In comparison to Figure 3, which shows the manifold pressure response versus frequency, Figures 4-6 shows the value of sensitivity indices for each frequency in the range 0-1200 Hz. It may be useful to recall that the greater the first order effects, the more sensitive the model output is to the given input parameters. Figure 4 shows the effects changes in of manifold radius at all cylinder locations in the suction manifold. If the sensitivity index at particular frequency is low, it shows that at that particular frequency the pressure response is less affected by varying the parameter of interest. For example, in Figure 4 it is seen that the manifold pressure response is less sensitive to variation in radius at lower frequencies, i.e. up to 300 Hz, because the value of the sensitivity index is low. However, beyond this range, the variations in manifold radius have a much more prominent effect on gas pulsations, as evidenced by the higher sensitivity values. One possible explanation for the higher sensitivity of the gas pulsations to the changes in the manifold radius at higher frequencies is that the acoustic resonances of the compressor are in the same frequency range.

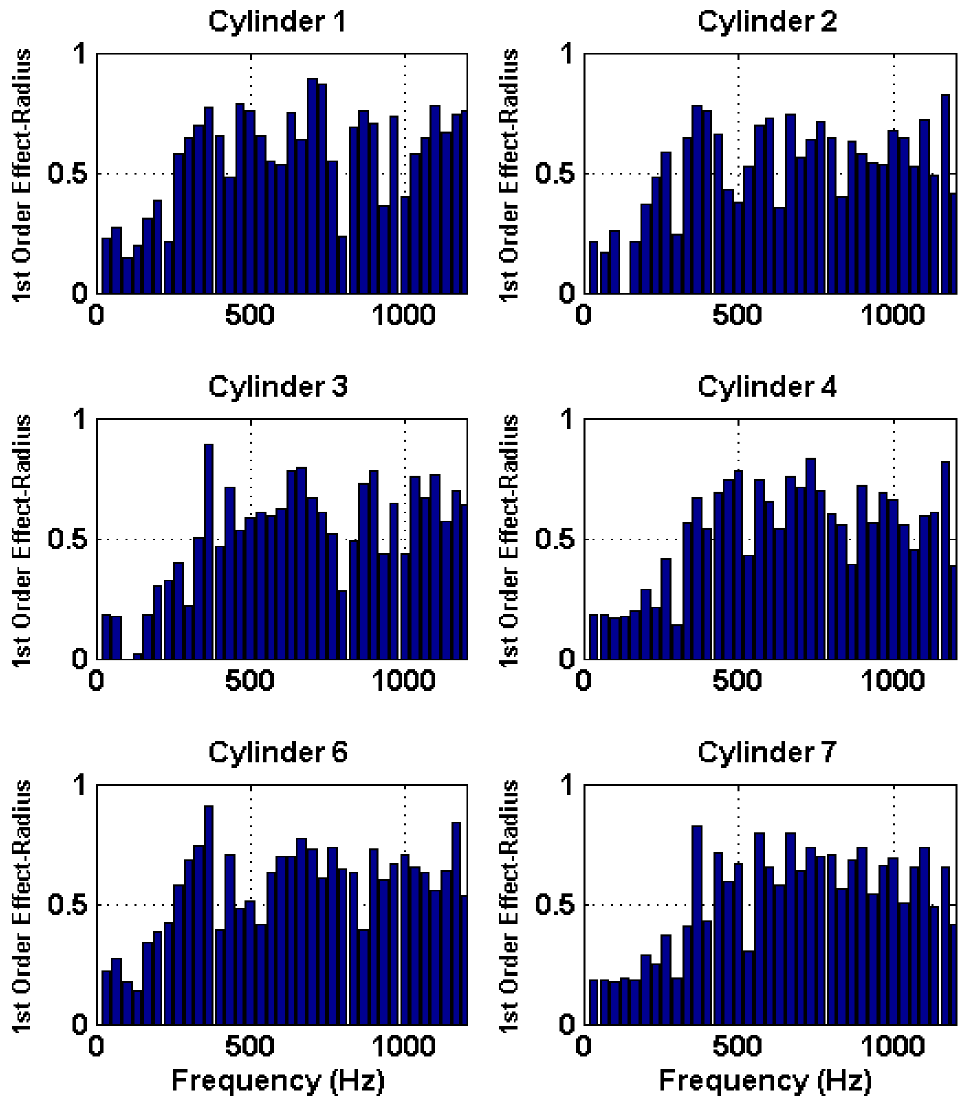


Figure 4: First order indices of manifold radius using Sobol's method

From Figure 5, it is seen that manifold depth does not have much effect on the gas pulsations. The response is somewhat sensitive at lower frequencies, but close to acoustic resonances, the manifold does not seem to have much effect.

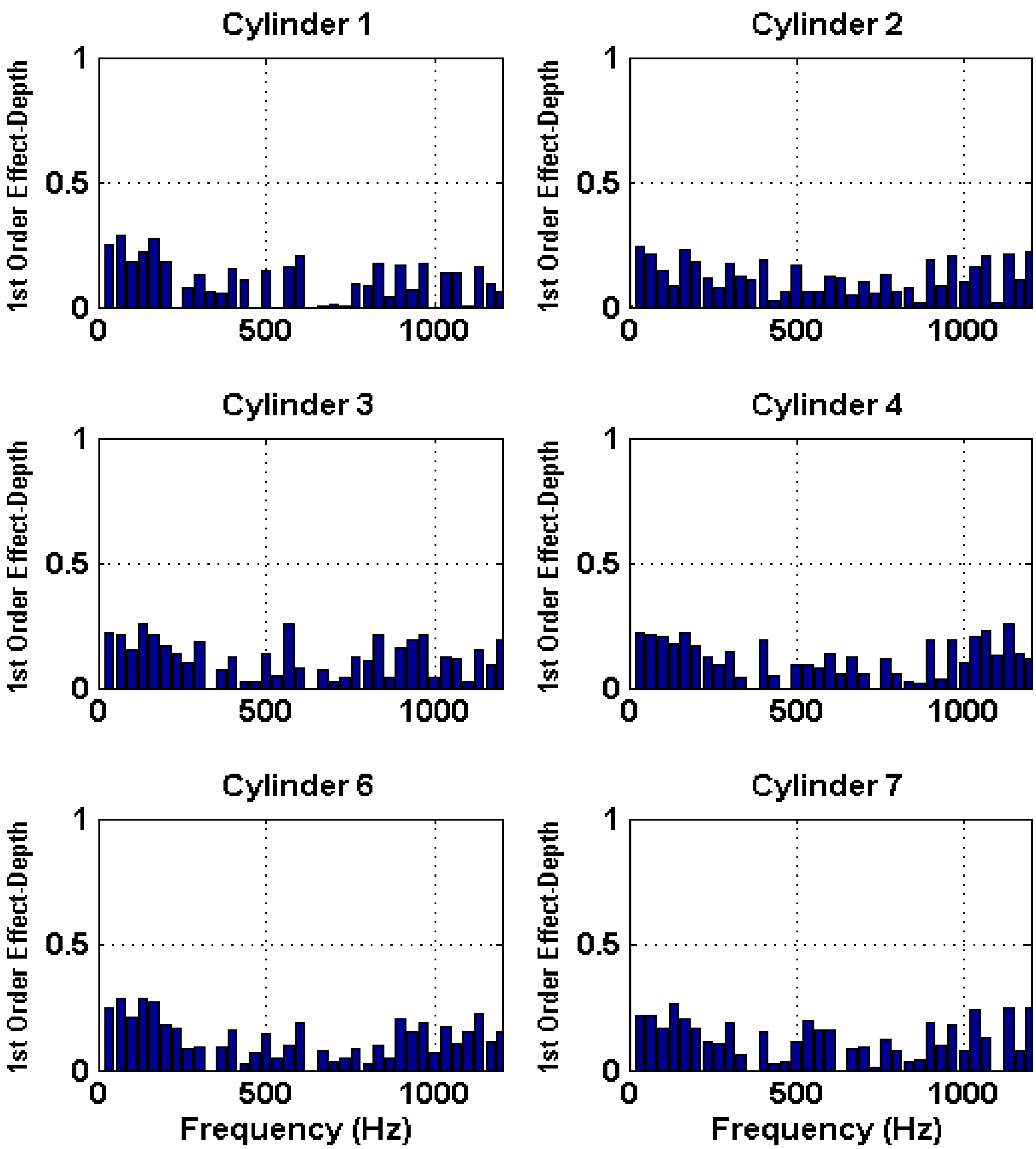


Figure 5: First order indices of manifold depth using Sobol's method.

The manifold width is more influential than the depth of the manifold, but less influential compared to the radius as shown in Figure 6.

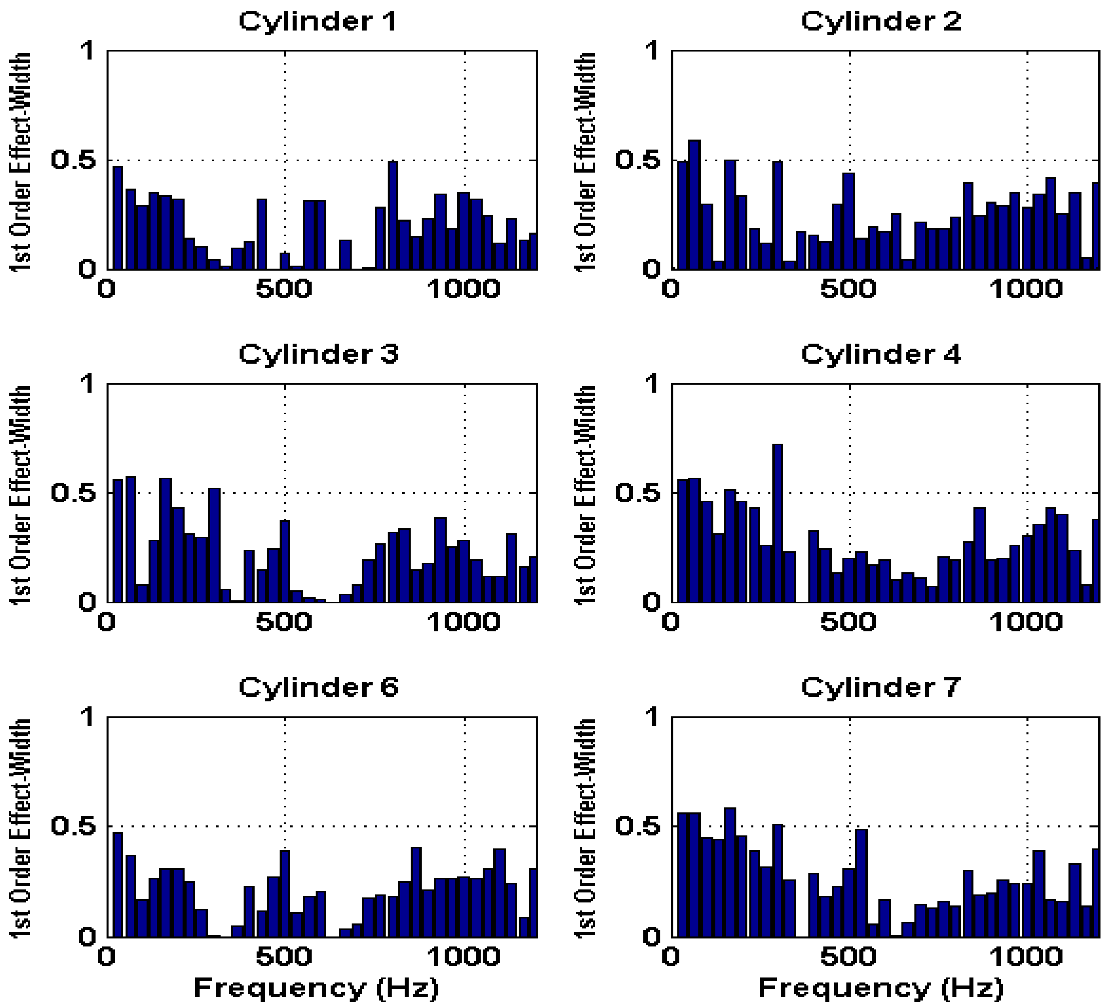


Figure 6: First order indices of manifold width using Sobol's method.

5.2 Total Effects

It may be useful to recall that the total effects include the first order effects and all the interactions involving that parameter. The total effects of the response are shown in Figure 7-9.

The total effect of manifold radius r describes the effects of the interaction of changes in radius with manifold depth and width as explained in Equation (1). For a linear system, the total effect should add up to unity. Figure 7 shows that parameter radius has much more impact across the entire frequency range. At close observation, the value of radius at certain frequencies is more than one, which indicates that there are nonlinear interactions taking place by changing the parameter radius. Also, the total sensitivity coefficients are much higher indicating that there is a significant nonlinear interaction taking place with variations in radius.

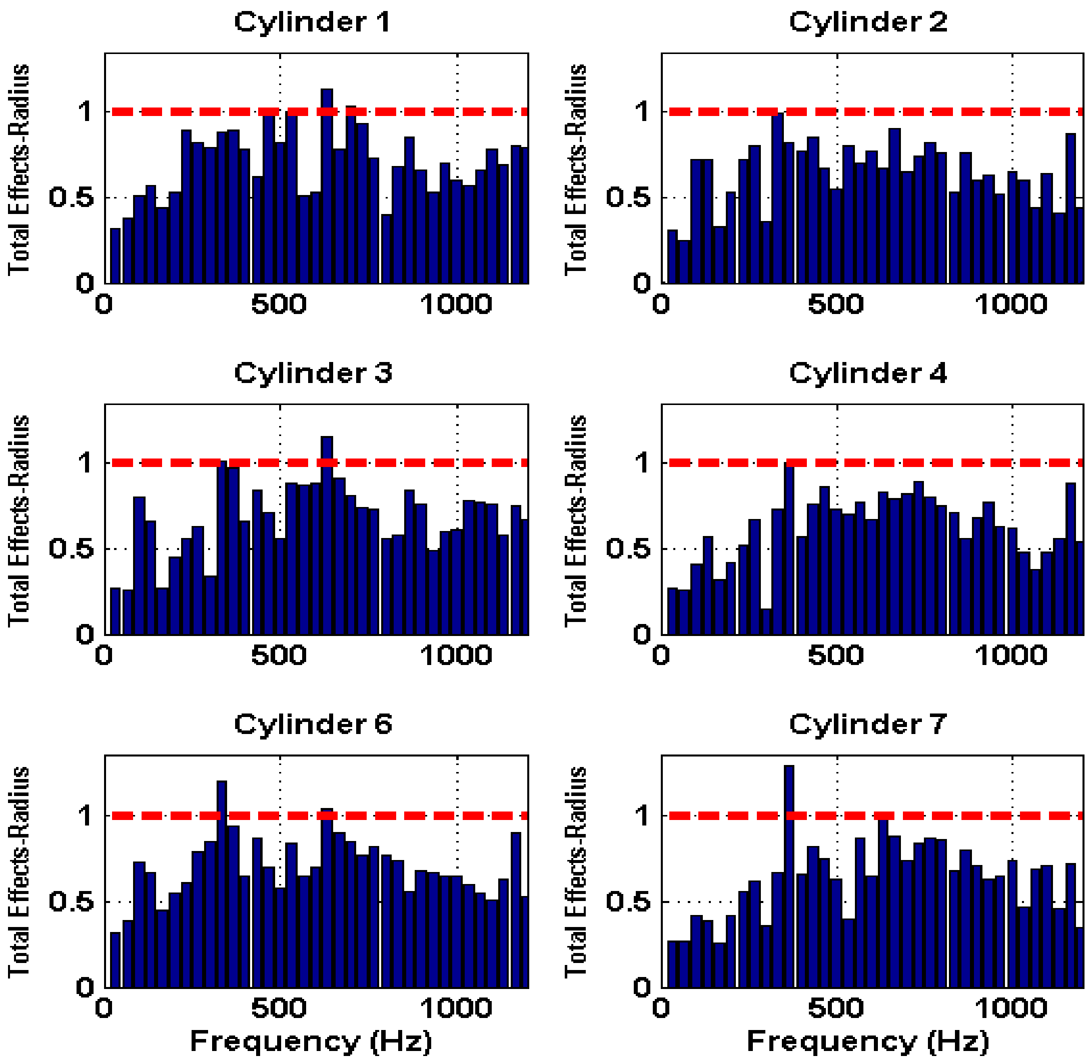


Figure 7: Total effects of manifold radius using Sobol's method.

The sensitivity of manifold depth is quite low, and compared to other factors, the value of depth variation index never goes above 0.5 indicating that there are not many interactions and no nonlinear effects. Also, as seen in Figure 8, the response is more significant at lower frequencies.

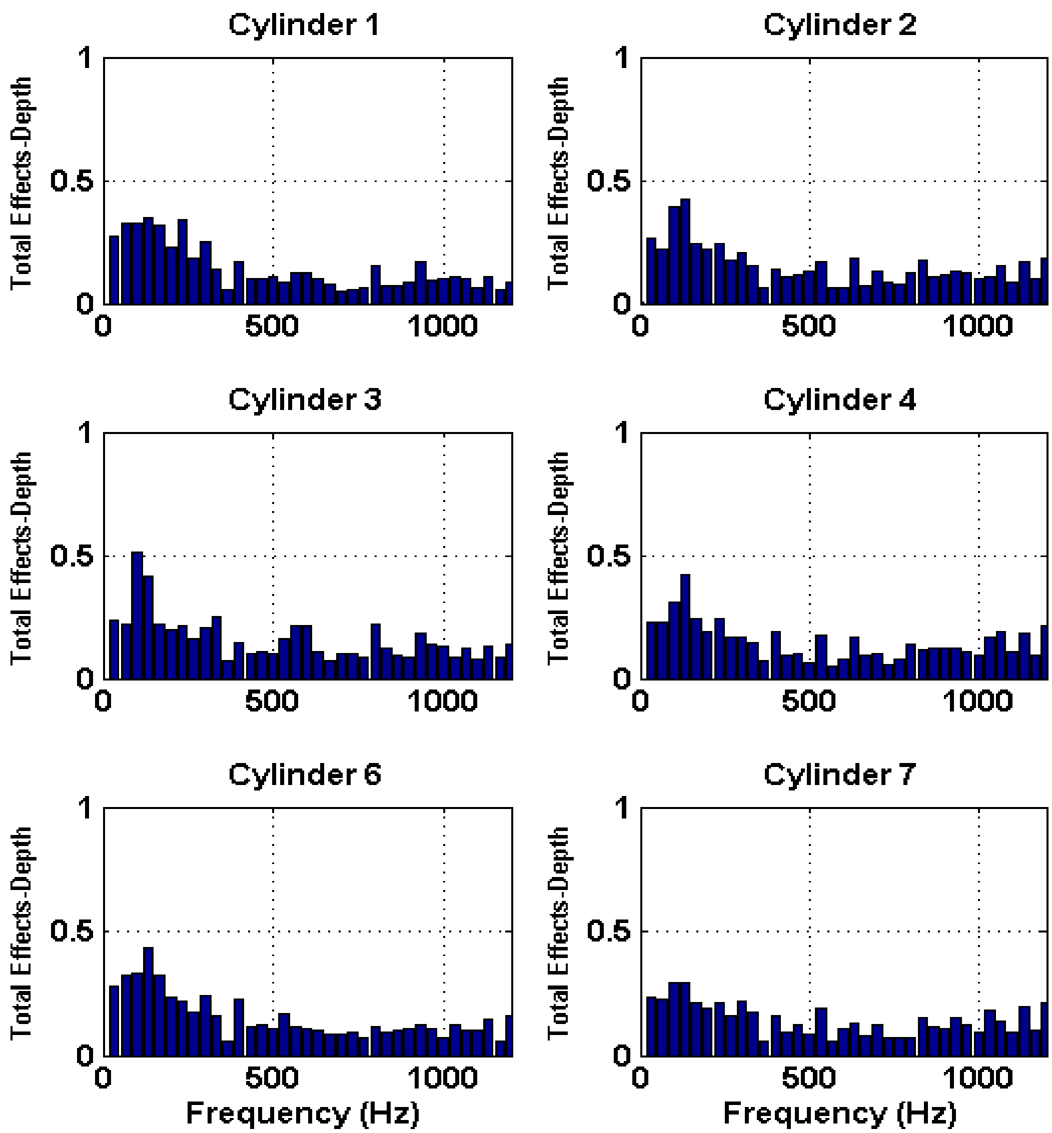


Figure 8: Total effects of manifold depth using Sobol's method.

Figure 9 shows that manifold width has more impact at lower frequencies where it is quite influential. Also, at some of the lower frequencies, it is seen that the sensitivity index is close to 1, which means that some sort of nonlinear interaction cannot be discounted.

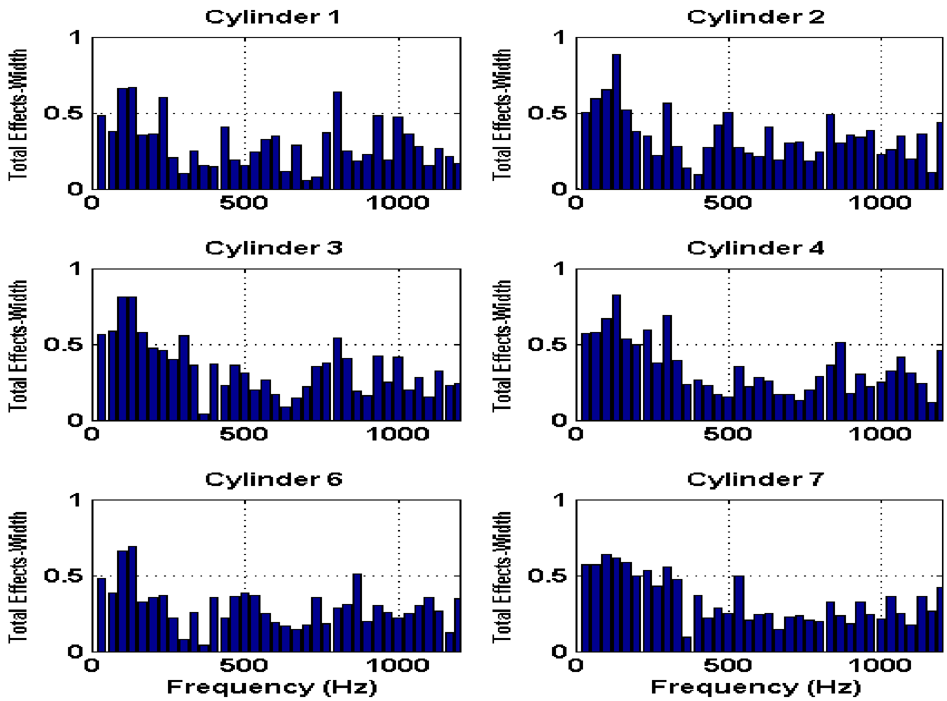


Figure 9: Total effects of manifold width using Sobol's method.

5. CONCLUSION

Sobol's method of global sensitivity analysis was used to calculate the first order effects and total effects of the gas pulsation due to three design parameter, namely, suction manifold radius, width and depth. This method was used to determine not only the uncertainty and sensitivity of individual parameters but also the interaction between these parameters. It was also showed that suction manifold pressure response was most sensitive to changes in manifold radius, followed by manifold width and depth. It was also seen that at certain frequencies the total effects were greater than one which indicates that there is interaction between parameters. In the next step, the original simulation model should be used to support the finding of this method. This method of global sensitivity analysis could be readily applied to any compressor simulation model.

REFERENCES

Bilal, N., 2011, Design Optimization of the Suction Manifold of a Reciprocating Compressor Using Uncertainty and Sensitivity Analysis, West Lafayette: Purdue University.

Bilal, N., Adams, D. E. & Park, J. I., 2006, Uncertainty and Sensitivity Analysis of Gas Pulsations in the Suction Manifold of a Multi-Cylinder Automotive Compressor, International Compressor Engineering Conference at Purdue University.

Iman, R. L., 1986, Modeling Input processes, *Winter Simulation Conference*, Washington, DC.

Park, J., Bilal, N. & Adams, D. E., 2007, Gas Pulsation Reductions in a Multi-Cylinder Compressor Suction Manifold using Valve-to-Valve Mass Flow Rate Phase Shifts. *Vibration and Acoustics*, Vol. 129, p. 406.

Park, J. & et al., 2008, Numerical and Experimental Studies of Gas Pulsations in the Suction Manifold of a Multi-Cylinder Automotive Compressor, *Vibration and Acoustics*, Volume 130, p. 0110114.

Park, J. I., 2004, Mathematical Modeling and Simulation of a Multi-cylinder Automotive Compressor, West Lafayette, Purdue University.

Saltelli, A., Chan, K. & Scott, E. M., 2000, Sensitivity Analysis, John Wiley & Sons Ltd.

Sobol, I. M., 1993. Sensitivity analysis for nonlinear mathematical models. *Mathematical Modeling & Computational Experiment*, vol. 1, p. 407-414.

Zheng, Y. & Rundell, A., 2006, Comparative Study of Paramete Sensitivity Analyses of the TCR-activated ERK-mapk Signalling Pathway, *Proceedings of Systems Biology*, vol. 153, no. 4, p. 201-21.