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Equivalent Linkages of Compressor Mechanisms

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ABSTRACT

Frequently, the dynamics of a compressor's mechanism can be simplified and better understood by analyzing compressor's equivalent linkage. Although the equivalent linkage of a reciprocating piston compressor is well known, the equivalent linkages of other types of compressors are not. Presented are equivalent linkages of reciprocating-piston compressor, rolling-piston compressor, swing-piston compressor, rotary-vane compressor, and the scroll compressor. Because of the limited space detailed the analysis of a reciprocating piston compressor is presented only.

1. INTRODUCTION

The aim of dynamic analysis of an equivalent linkage is to find linear and angular velocities, and linear and angular accelerations. The most frequently used method is to find corresponding velocities and accelerations by taking derivatives of position coordinates of key points of an equivalent linkage. This leads to too long, and difficult to manage expressions. An analytical version of graphical method that was used by engineers in pre-computer era offers simpler approach that avoids derivatives, and that gives the same results. The method uses plans of velocities and accelerations.

2. RECIPROCATING PISTON COMPRESSOR

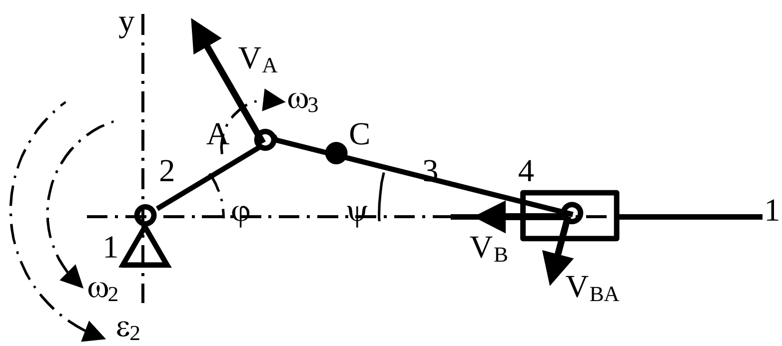

Figure 1: Slider-crank mechanism

It is customary to number individual links. In the Figure 1 link number one is the fixed frame of reference. Link number 2 is the crank, ling number 3 is connecting rod, and link number 4 is piston. The instantaneous position of the slider-crank mechanism is uniquely given by the angle φ that is measured from the positive x-axis in the counterclockwise direction. Angle ψ indicates position of the connecting rod. The relation between these two angles is

$$\sin\psi = \frac{R}{L}\sin\varphi \tag{1}$$

$$\cos\psi = \sqrt{1-\left(\frac{R}{L}\sin\varphi\right)^2} \tag{2}$$

Where
φ is angle indicating position of crankshaft [rad]
ψ is angle between axis of connecting rod and the axis of the cylinder [rad]
R is the radius of the crank [m]
L is length of connecting rod (length A-B in Figure 1) [m]

1.1 Velocity

In order to be able to take into consideration variability of moment of inertia of the driving shaft, we need to know velocity of the center if gravity C of connecting rod (link No. 3), and its angular velocity ω_3. We will use plan of velocities to do so.

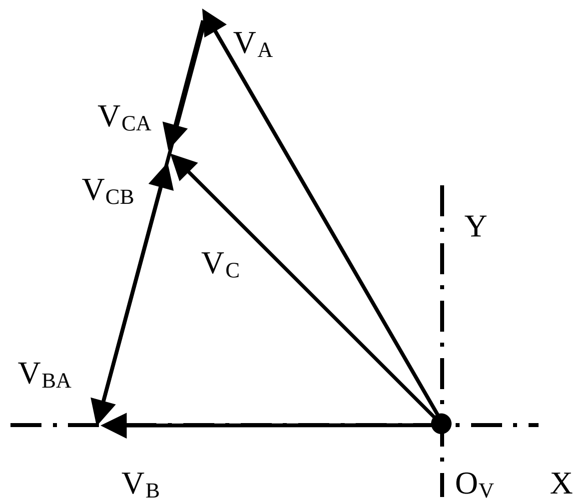

Figure 2: Plan of velocities

The idea is resolution of motion of connecting rod into two simultaneous motions. We assume all points of connecting rod move in the direction of velocity V_A, and having the same magnitude V_A, and rotation of the connecting rod about point A. The vector summation of these two motions results in rectilinear motion of point B. Expressed in vector form we have

$$\mathbf{V_B} = \mathbf{V_A} + \mathbf{V_{BA}} \tag{3}$$

Where
$\mathbf{V_B}$ is velocity of the piston ([link No. 4) [m.s^{-1}]
$\mathbf{V_A}$ is known velocity of point A (the crankpin) [m.s^{-1}]
$\mathbf{V_{BA}}$ is instantaneous rotation of point B (wristpin) around point A as a pivot [m.s^{-1}]

When we project vectors in Eq. 3 into x and y axis, we get

$$V_B \cdot \cos\pi = V_A \cdot \cos\left(\varphi + \frac{\pi}{2}\right) + V_{BA} \cdot \cos\left(\frac{3\pi}{2} - \psi\right) \tag{4a}$$

$$V_B \cdot \sin\pi = V_A \cdot \sin\left(\varphi + \frac{\pi}{2}\right) + V_{BA} \cdot \sin\left(\frac{3\pi}{2} - \psi\right) \tag{4b}$$

Because the magnitude of velocity of point A, V_A is $V_A = R \cdot \omega_2$, we can now solve Eqs. 4a and 4b for two unknown magnitudes of V_B and V_{BA}. The result is

$$V_{BA} = R \cdot \omega_2 \cdot \frac{\cos\varphi}{\sqrt{1-\left(\frac{R}{L}\cdot\sin\varphi\right)^2}} \tag{5}$$

$$V_B = R \cdot \omega_2 \cdot \sin\varphi \cdot \left(1+\frac{R}{L}\cdot\frac{\cos\varphi\cdot}{\sqrt{1-\left(\frac{R}{L}\cdot\sin\varphi\right)^2}}\right) \tag{6}$$

From Eq. 5 we get instantaneous angular velocity of connecting rod

$$\omega_3 = \frac{V_{BA}}{L} = \frac{R}{L}\cdot\omega_2\cdot\frac{\cos\varphi}{\sqrt{1-\left(\frac{R}{L}\cdot\sin\varphi\right)^2}} \tag{7}$$

In the Figure 2 we can se that we can find velocity of the center of gravity, point C on connecting rod (link 3), by the use of one of the following vector equations.

$$\mathbf{V_C} = \mathbf{V_A} + \mathbf{V_{CA}} \tag{8}$$

$$\mathbf{V_C} = \mathbf{V_B} + \mathbf{V_{CB}} \tag{9}$$

Where

$$V_{CA} = \omega_3 \cdot a \tag{10}$$

$$V_{CB} = \omega_3 \cdot (L-a) \tag{11}$$

Where

a is the distance of CG from the crankpin [m]

1.2 Acceleration

Figure 3 shows directions of accelerations of point A and B.

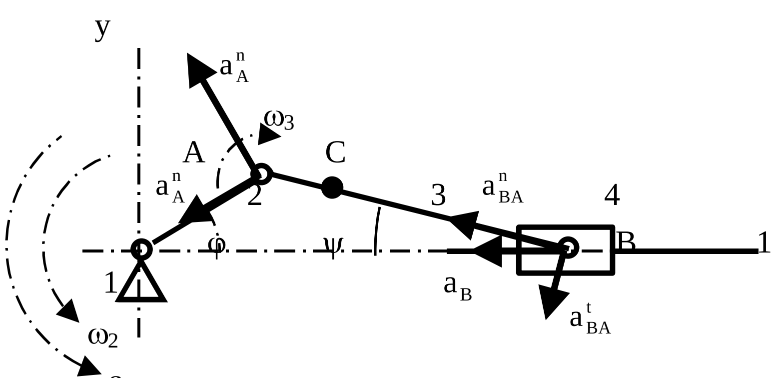

Figure 3: Acceleration

In the Figure 3 we have

$\mathbf{a}_A^n$ vector of normal acceleration of pint A [m.s^{-2}]

$\mathbf{a}_\mathbf{A}^\mathbf{t}$ vector of tangential acceleration of pint A [m.s^{-2}]

$\mathbf{a}_\mathbf{BA}^\mathbf{n}$ vector of normal acceleration of rotation of point B about point A [m.s^{-2}]

$\mathbf{a}_\mathbf{BA}^\mathbf{t}$ vector of tangential component of acceleration of rotation of point B about pint A [m.s^{-2}]

$\mathbf{a}_\mathbf{B}$ vector of rectilinear acceleration of piston (point B) [m.s^{-2}]

Equations (12) and (13) describe plan of accelerations that is shown in Figure 4.

$$\mathbf{a}_\mathbf{B} = \mathbf{a}_A + \mathbf{a}_{BA} \tag{12}$$

$$\mathbf{a}_\mathbf{B} = \mathbf{a}_\mathbf{A}^\mathbf{n} + \mathbf{a}_\mathbf{A}^\mathbf{t} + \mathbf{a}_\mathbf{BA}^n + \mathbf{a}_\mathbf{BA}^\mathbf{t} \tag{13}$$

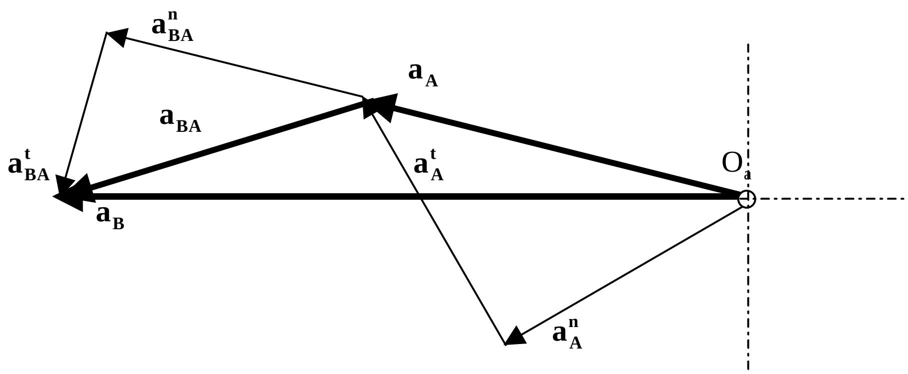

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Figure 4: Plan of accelerations

The projection of Eq. (13) into horizontal and vertical directions yields

$$a_B \cdot \cos\pi = a_A^n \cdot \cos(\varphi+\pi) + a_A^t \cdot \cos\left(\varphi+\frac{\pi}{2}\right) + a_{BA}^n \cdot \cos(\pi-\psi) + a_{BA}^t \cdot \cos\left(\frac{3\pi}{2}-\psi\right) \tag{14}$$

$$a_B \cdot \sin\pi = a_A^n \cdot \sin(\varphi+\pi) + a_A^t \cdot \sin\left(\varphi+\frac{\pi}{2}\right) + a_{BA}^n \cdot \sin(\pi-\psi) + a_{BA}^t \cdot \sin\left(\frac{3\pi}{2}-\psi\right) \tag{15}$$

There are only two unknowns in Eq.s. (14) and (15), a_B and a_{BA}^t. When we simplify trigonometric expressions in the above equations, we will get

$$a_{BA}^t = \frac{a_A^n \cdot \sin\varphi + a_A^t \cdot \cos\varphi + a_{BA}^n \sin\psi}{\cos\psi} \tag{16}$$

In the Eq. (16) there is $a_A^n = R \cdot \omega_2^2$, $a_A^t = R \cdot \varepsilon_2$, and $a_{BA}^n = R \cdot \omega_3^2$.

$$a_B = a_A^n \cdot \cos\varphi + a_A^t \cdot \sin\varphi + a_{BA}^n \cdot \cos\psi + a_{BA}^t \sin\psi \tag{17}$$

The last thing we need to know is angular acceleration of connecting rod. This acceleration is

$$\varepsilon_3 = \frac{a_{BA}^t}{L} \tag{18}$$

The acceleration of the CG of connecting rod can be found from

$$\mathbf{a}_\mathbf{C} = \mathbf{a}_\mathbf{A} + \mathbf{a}_\mathbf{CA}^\mathbf{n} + \mathbf{a}_\mathbf{CA}^\mathbf{t} = \mathbf{a}_\mathbf{B} + \mathbf{a}_\mathbf{CB}^\mathbf{n} + \mathbf{a}_\mathbf{CB}^\mathbf{t} \tag{19}$$

Because the magnitudes of $a_{CA}^n, a_{CA}^t, a_{CB}^n, a_{CB}^t$ are proportional to the distance of CG from point A and B, we can find acceleration of CG the similar way we found velocity of CG.

2. THE ROLLING PISTON COMPRESSOR

To make sure the stationary vane of a rolling-piston compressor is always in contact with the rolling piston we need to find velocity and acceleration of the vane. The tip of the vane that is in contact with the piston may of three types. Vane may have convex tip, concave tip, and the flat tip. Figure 5 shows equivalent linkage of rolling piston compressor with the vane that has convex tip. Figure 6 shows case with concave tip, and Figure 7 shows compressor with vane tat has flat tip of the vane.

In the Figure 5 the contact between two curved surfaces is modeled by a link that is actually connecting rod of a slider crank mechanism. Connecting centers of curvature of the two contacting surfaces creates this link. The crank radius is equal to the eccentricity e, and the length of connecting rod is $L = R_P + r$, which is the sum of radius of piston and the radius of the tip of the vane.

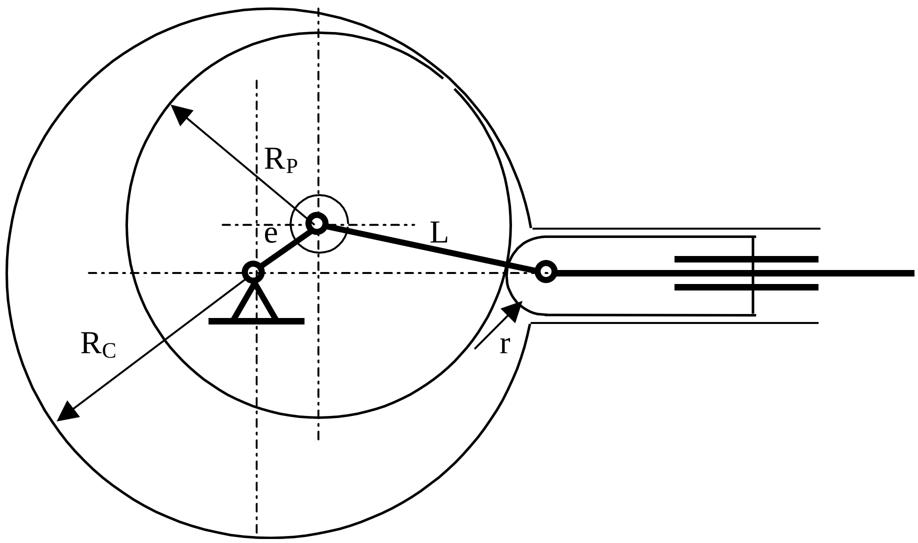

Figure 5: Equivalent linkage of a rolling-piston compressor with convex tip of vane

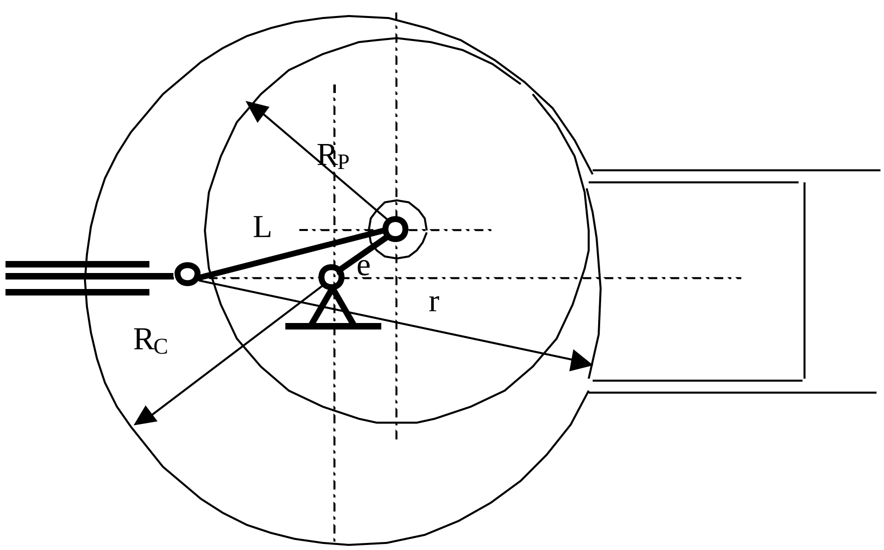

Figure 6: Equivalent linkage of a rolling-piston compressor with concave tip of vane

The equivalent linkage of the compressor with concave tip of the vane is also slider-crank mechanism. The length of connecting rod in Figure 6 is $L = r - R_P$. The radius r must be $r > R_P$. The thickness of the vane has to be $h > 2.e$.

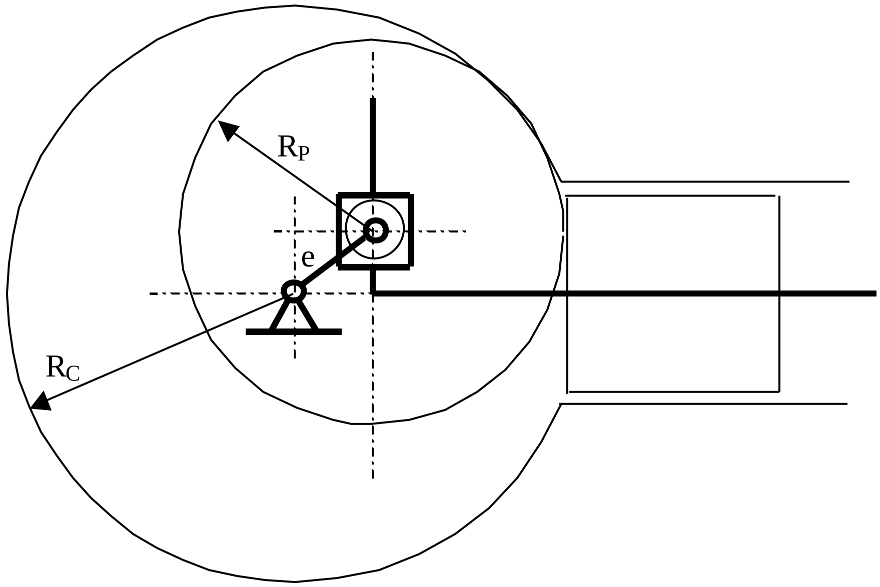

Figure 7: Equivalent linkage of a rolling-piston compressor with the flat tip of vane.

The equivalent linkage of the rolling-piston compressor with the flat tip of the vane is a Scotch-Yoke mechanism. The thickness of the sliding vane in Figure 7 has to be $h > 2.e$. Where e is the eccentricity.

3. THE SWING PISTON COMPRESSOR

The equivalent linkage of a swing-piston compressor is mechanism that was used by Stephenson on his famous steam locomotive (1802).

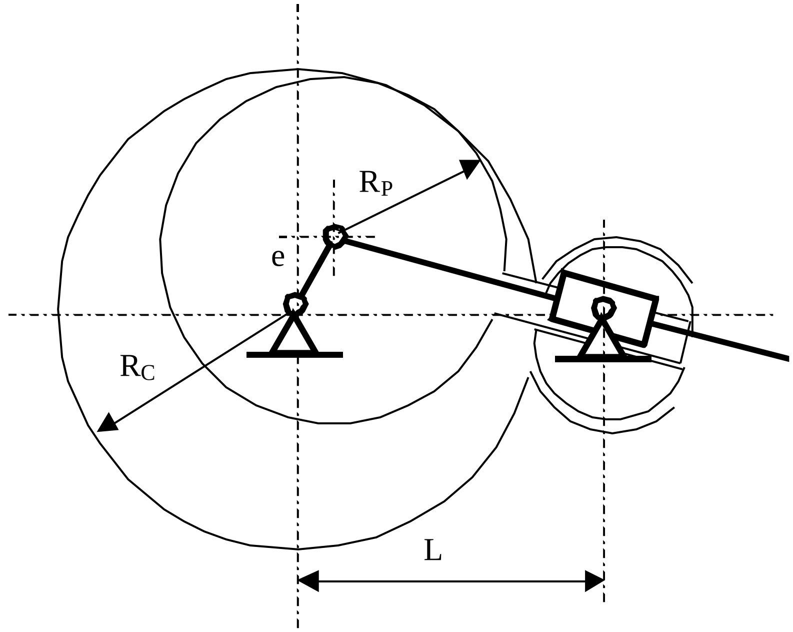

Figure 8: Equivalent linkage of a swing-piston compressor.

Although the analysis of velocities may be straightforward, one shall not forget Coriolis acceleration.

4. THE ROTARY VANE COMPRESSOR

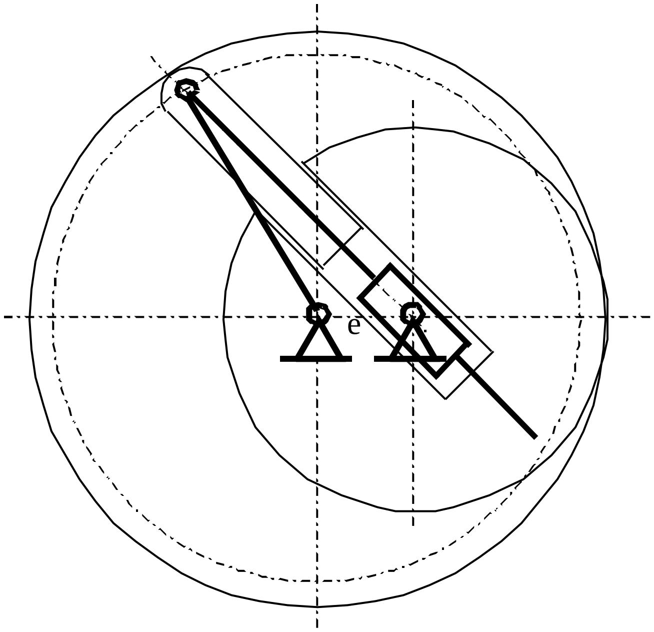

Figure 9a: Equivalent linkage of rotary-vane compressor with concentric radial vanes

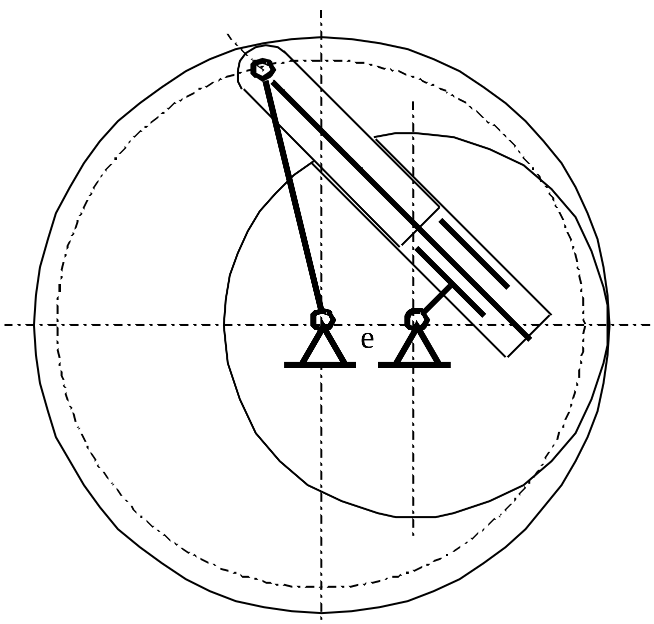

Figure 9b: Equivalent linkage of rotary-vane compressor with eccentric radial vanes

5. THE SCROLL COMPRESSOR

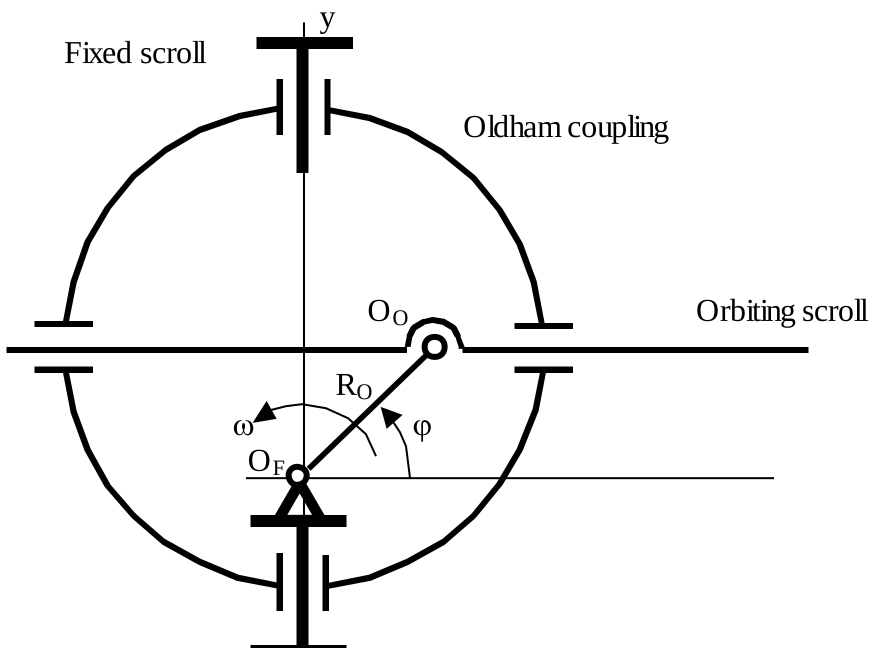

Figure 10: Equivalent linkage scroll compressor

The equivalent linkage of a scroll compressor is Scotch Yoke mechanism. Oldham coupling follows sinusoidal motion while the orbiting scroll moves on a circle.

6. CONCLUSIONS

The analysis of velocities and accelerations of equivalent linkages that uses vector diagrams of velocities and accelerations enables to avoid sometimes very complicated derivatives of position of points of interest. Due to limited space only slider-crank mechanism was analyzed in detail. The same approach can be used to analyze other types of mechanisms.

REFERENCES

Beer F.,Johnston Jr.,Cornwell P., *Vector Mechanics For Engineers: Dynamics*, 2012, McGraw-Hill, 832 p.

Kolovsky M., Efgrafov A., Semnov Yu., Sloush A., Lilov L., *Advanced Theory of Mechanisms and Machines, 2000*, Springer, Berlinn, 396 p.