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ABSTRACT

The revolving vane (RV) compressor is a relatively new rotary compressor design and many of its performance characteristics would have to be evaluated. Vibration of compressors is one of these aspects and this paper presents the theoretical analysis for the vibration characteristics of the RV compressor. The analysis is done by dividing the compressor into two components – the rotational vibration of the cylinder-rotor assembly, and the resulting torsional twist of the stationary shell housing. The unique approach of using Lagrange's equations to formulate the equation of motion for the complex cylinder-rotor assembly is presented here.

1. INTRODUCTION

Global concern over the usage and depletion of fossil fuels has led to the motivation to develop more energy efficient systems to reduce power consumption. The revolving vane (RV) compressor was thus developed (Teh *et al*., 2009; Teh & Ooi, 2009a, 2009b) as an alternative to a more energy efficient rolling piston compressor. As the RV compressor is a relatively new design, some of its operational aspects are still relatively unknown. Vibration is a key concern since it affects the fatigue and wear of the components. This paper investigates the vibrational properties of the RV compressor.

Since the design of the RV compressor is derived from the rolling piston compressor, the basic theory for the vibrational analysis of the rolling piston compressor (Yanagisawa *et al.*, 1984) is referenced. In the analysis of the rolling piston compressor, it was conducted by separating into two components, namely the rotating part of the compressor and the stationary part of the compressor. Similarly, the analysis of the RV compressor is also divided into two separate components; namely the cylinder-rotor assembly and the compressor shell housing.

2. CYLINDER-ROTOR ASSEMBLY THEORETICAL MODEL

Figure 1 shows a cross-section of the cylinder-rotor assembly. The bushing in the rotor serves to accommodate the vane movements during operation.

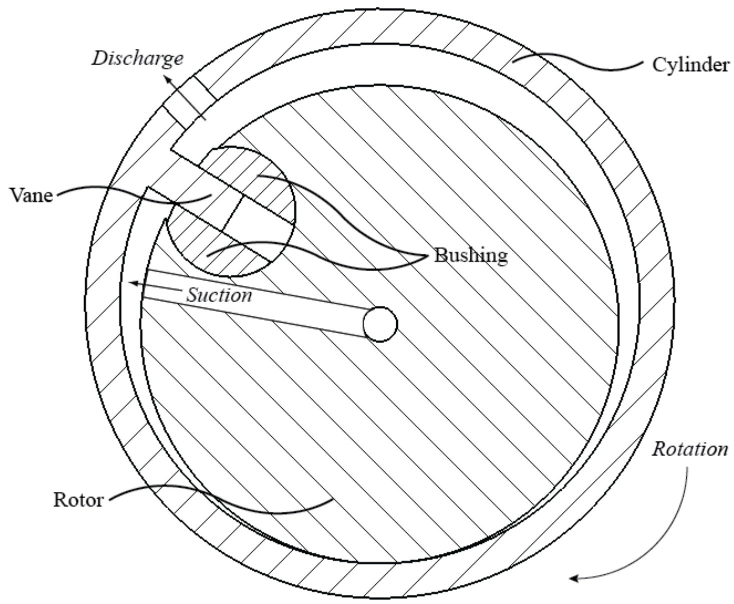

Figure 1: Cylinder-rotor assembly cross-section

The following assumptions are made in the analysis:

- The vibration of the rotor-cylinder assembly is purely in the rotational direction; i.e. there is no other translational motion of the assembly in its journal bearings.
- The dimensions of the components are perfect with no manufacturing tolerances; i.e. the geometric relations between the components hold true during all aspects of the compressor operation

Due to the multiple rotating reference frames of the components, Lagrangian mechanics would be employed to formulate the equation of motion for the entire system instead of traditional Newtonian mechanics. Holonomic constraints derived from geometric relations are used to constrain these Lagrange equations.

2.1 Geometric Relations

Figure 2 illustrates the geometrical relations between the cylinder, rotor and bushing components. R_b denotes the radial distance of the bushing center from the rotor center. The centers of the cylinder, rotor and bushing are denoted by O_c, O_r, and O_b respectively as seen in Figure 2. Figure 3 shows these relations without the compressor cross-section. Note that certain dimensions have been exaggerated for clarity in Figure 3. ε denotes the eccentricity of the compressor and is the difference between the cylinder and rotor radius ($\varepsilon = R_c - R_r$). R_2 denotes the radial distance of the bushing centre from the cylinder centre.

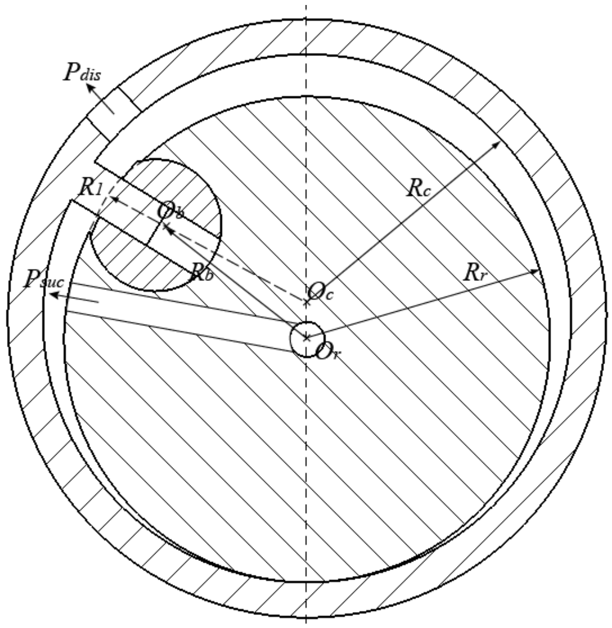

Figure 2: Cylinder-rotor assembly geometric relations

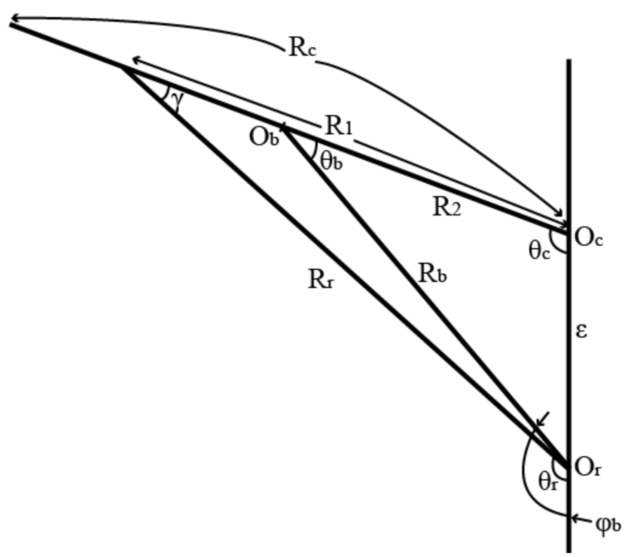

Figure 3: Cylinder-rotor assembly geometric relations

$$\left.\begin{aligned} R_1 &= \varepsilon \cos\theta_c + \sqrt{R_r^2 - \varepsilon^2 \sin^2\theta_c} \\ \cos\gamma &= \frac{R_r^2 + R_1^2 - \varepsilon^2}{2R_r R_1} \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} R_2 &= \varepsilon \cos\theta_c + \sqrt{R_b^2 - \varepsilon^2 \sin^2\theta_c} \\ \cos\theta_b &= \frac{R_b^2 + R_2^2 - \varepsilon^2}{2R_b R_2} \end{aligned}\right\} \tag{2}$$

The geometric relations in equations (1) and (2) are crucial in formulating the equation of motion for the cylinder-rotor assembly and will hold true for all instances during compressor operation.

2.2 Constrained Lagrange's Equation

The general form of Lagrange's equation (Thornton & Marion, 2004) is shown in equation (3) below:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_k^n \lambda_k(t)\frac{\partial f_k}{\partial q_j} \quad \begin{cases} j = 1,2,\dots,m \\ k = 1,2,\dots,n \end{cases} \tag{3}$$

In the equation, L is known as the Lagrangian of the system and f represents the different holonomic constraints. Subscripts j and k denote the individual generalized coordinates and the individual holonomic constraints respectively. Q represents forces due to the non-conservative work done by the system. For the case of the RV compressor, the non-conservative work includes the gas compression work and work done against friction.

With respect to the geometric relations established in Figure 3, θ_c, θ_r, θ_b, φ_b have been established as the generalized coordinates for the system.

2.3 Equation of Motion Formulation

Based on the generalized coordinates, the kinetic energy of the system is simply the sum of the rotational kinetic energies of the cylinder, rotor and bushing about their respective centres of rotation, and the translational kinetic energy of the bushing about the rotor centre. The expression for the total kinetic energy of the system can be written as shown in equation (4):

$$K.E. = \frac{1}{2}\left(I_c\dot{\theta}_c^2 + I_r\dot{\theta}_r^2 + I_b\dot{\theta}_b^2 + m_b R_b \dot{\varphi}_b^2\right) \tag{4}$$

The total potential energy for the cylinder-rotor assembly is zero since it is only in pure rotational motion. Hence, the Lagrangian of the system is thus given by equation (5):

$$\begin{gathered} L = K.E. - P.E. \\ \rightarrow L = \frac{1}{2}\left(I_c\dot{\theta}_c^2 + I_r\dot{\theta}_r^2 + I_b\dot{\theta}_b^2 + m_b R_b \dot{\varphi}_b^2\right) \end{gathered} \tag{5}$$

In addition, additional torques also affect the rotational motion of the cylinder-rotor assembly and these include the motor torque used to drive the assembly (T_m), the gas compression torque required to do work (T_g) and torques resulting from frictional losses (T_f). These torques constitute the non-conservative work done by the cylinder-rotor assembly and each components have their own affecting torques. The external torques affecting each generalized coordinate of the components are summarized in equation (6). Each of the frictional torques (T_f) has an extra subscript to denote the component that they affect.

$$\begin{aligned} \text{Cylinder: } Q_{\theta_c} &= T_m - T_g - T_{f,c} \\ \text{Rotor: } Q_{\theta_r} &= -T_{f,r} \\ \text{Bushing: } Q_{\theta_b} &= -T_{f,b} \\ Q_{\varphi_b} &= -T_{f,\varphi} \end{aligned} \tag{6}$$

The holonomic constraints of the system are derived from the geometric relations using the sine rule in equation (7).

$$\left.\begin{aligned} \frac{\sin\theta_r}{R_1} = \frac{\sin\theta_c}{R_r}, &\quad \rightarrow f_1 = \frac{\sin\theta_r}{R_1} - \frac{\sin\theta_c}{R_r} = 0 \\ \frac{\sin\varphi_b}{R_2} = \frac{\sin\theta_c}{R_b}, &\quad \rightarrow f_2 = \frac{\sin\varphi_b}{R_2} - \frac{\sin\theta_c}{R_b} = 0 \\ \frac{\sin\theta_b}{\varepsilon} = \frac{\sin\theta_c}{R_b}, &\quad \rightarrow f_3 = \frac{\sin\theta_b}{\varepsilon} - \frac{\sin\theta_c}{R_b} = 0 \end{aligned}\right\} \tag{7}$$

With the Lagrangian, holonomic constraints and external torques defined, the Lagrange's equations of the system are formulated as shown in equation (8(8):

$$\left.\begin{aligned} I_c\ddot{\theta}_c = T_m - T_g - T_{f,c} - \lambda_1\left(\frac{\sin\theta_r}{R_1^2}\cdot\frac{dR_1}{d\theta_c}+\frac{\cos\theta_c}{R_r}\right) - \lambda_2\left(\frac{\sin\varphi_b}{R_2^2}\cdot\frac{dR_2}{d\theta_c}+\frac{\cos\theta_c}{R_b}\right) - \lambda_3\frac{\cos\theta_c}{R_b} \\ I_r\ddot{\theta}_r = -T_{f,r} + \lambda_1\frac{\cos\theta_r}{R_1} \\ I_b\ddot{\theta}_b = -T_{f,b} + \lambda_3\frac{\cos\theta_b}{\varepsilon} \\ m_b R_b\ddot{\varphi}_b = -T_{f,\varphi} + \lambda_2\frac{\cos\varphi_b}{R_2} \end{aligned}\right\} \tag{8}$$

Equation (8) represents the system of Lagrange's equations for the cylinder-rotor assembly system in which λ_1, λ_2 and λ_3 represent the Lagrangian multipliers. This set of Lagrange's equations together with the holonomic constraints form a system of seven simultaneous equations which must be solved simultaneously to obtain expressions for the generalized coordinates and Lagrangian multipliers. Simplifying, eliminating the Lagrangian multipliers and consolidating the frictional loss terms from each generalized coordinate into a single term (T_F) results in equation (9) which describes the motion of the entire cylinder-rotor assembly.

$$I_c\ddot{\theta}_c = T_m - T_g - T_F - \frac{R_1}{R_r\cos\gamma}\left(I_r\ddot{\theta}_r\right) - \frac{R_2}{R_b\cos\theta_b}\left(m_b R_b\ddot{\varphi}_b\right) - \frac{\varepsilon\cos\theta_c}{R_b\cos\theta_b}\left(I_b\ddot{\theta}_b\right) \tag{9}$$

The main focus here is the generalized coordinate θ_c, since it is the only independent variable that is representative of the entire cylinder-rotor assembly system and all other generalized coordinates can be expressed in terms of just θ_c when solving equation (9) for the rotational vibration characteristics of the cylinder-rotor assembly.

3. COMPRESSOR SHELL HOUSING

Most of the external torques acting on the cylinder-rotor assembly are also experienced by the compressor shell housing except that these torques act in the opposite direction for the shell housing. These torques include the motor torque (T_m), frictional loss torques (T_{jbr}) from the journal bearings and refrigerant fluid shear (T_{shear}). There is also a damping (T_{damp}) and support (T_{base}) torque from the rubber mounts similar to the rolling piston compressor (Yanagisawa *et al.*, 1984). Figure 4 shows the components in the shell housing and figure 5 depicts the torques acting on the compressor shell housing.

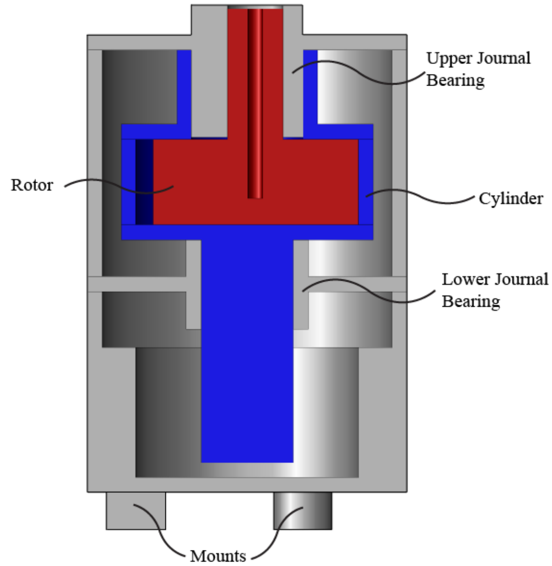

Figure 4: Compressor Shell Housing Cross-section

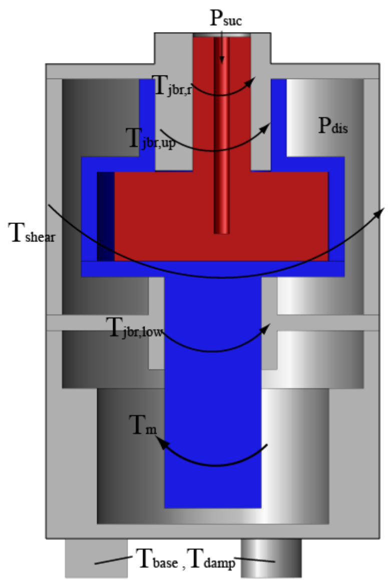

Figure 5: Compressor Shell Housing Counter-torques

From the counter torques depicted in Figure 5, the equation of motion for the compressor shell housing can be then formulated using Newtonian mechanics ($F = ma$) as shown in equation (10).

$$I\alpha = \Sigma T$$
$$\rightarrow I_h\ddot{\theta}_h = T_{jbr,up} + T_{jbr,low} + T_{jbr,r} + T_{shear} - T_m - T_{base} - T_{damp} \quad (10)$$

4. PRELIMINARY RESULTS

The dimensions of the RV compressor used for the analysis can be found in Table 1. R134a is employed as the working fluid.

Table 1: RV Compressor Dimensions

Component	Dimension (mm)
Compressor Shell Radius (inner)	39.00
Compressor Shell Length	65.00
Compressor Shell Thickness	4.00
Cylinder Radius (inner) – R_c	30.00
Cylinder Thickness	4.00
Rotor Radius - R_r	27.60
Cylinder Length	23.03
Lower Cylinder Shaft Diameter	15.00
Lower Cylinder Shaft Length	20.00
Cylinder Journal Bearing Length	15.00
Rotor Shaft Diameter	21.20
Rotor Shaft Length	20.00
Rotor Journal Bearing Length	15.00
End-Face Clearance	0.01
Journal Bearing Clearance	0.01

The motor torque profile and empirical constants for the coefficient of friction (μ_f), damping (C_h) and support (k_h) torques affecting the compressor shell housing are obtained from the rolling piston analysis (Yanagisawa *et al.*, 1984) since the above dimensions in are almost similar to the rolling piston in the literature. These values are listed in Table 2.

Table 2: Empirical Constants

Constant	Value
μ_f	0.15
C_h	0.69 N m s
k_h	127 Nm/rad

Equations (9) and (10) are solved to obtain the vibration characteristics of the RV compressor. Evaluation of the total frictional loss (T_F) torque for the cylinder-rotor assembly during each phase of operation is a complex task. As a preliminary study, it is assumed to be 10% of the average gas compression torque. In addition, the fluid shear torque (T_{shear}) was assumed to be negligible and was omitted.

The variation in the rotational speed of the cylinder-rotor assembly is shown in Figure 6 and the variation in the torsional twist of the compressor shell housing is shown in Figure 7. Despite the shape of the graph in Figure 6, the speed fluctuation is 6.60% of its mean value at 3470 RPM.

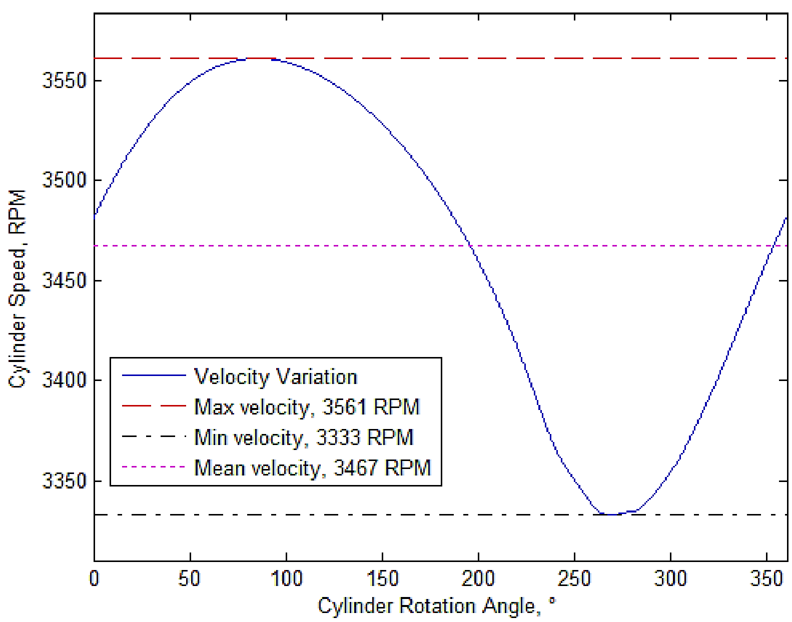

Figure 6: Cylinder-rotor Assembly Velocity Variation

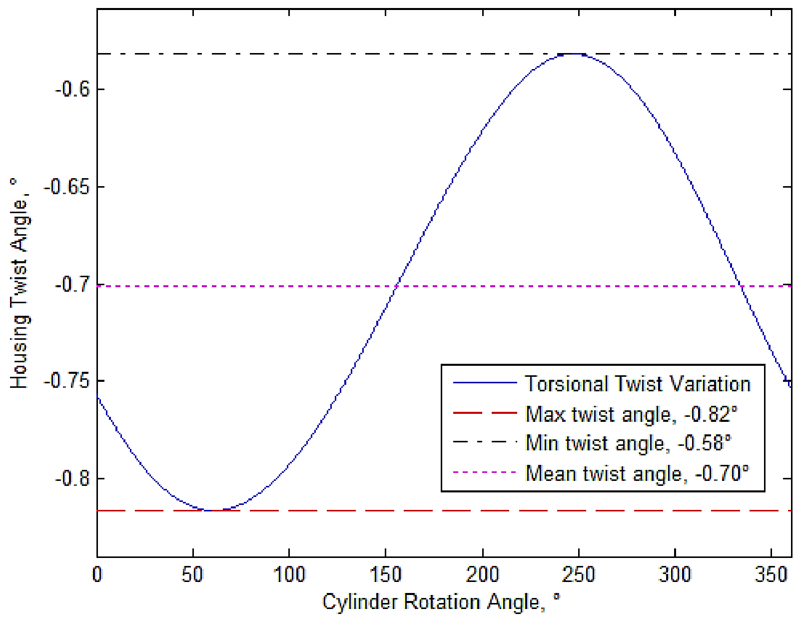

Figure 7: Compressor Shell Housing Torsional Twist

5. CONCLUDING REMARKS

This paper has presented a Lagrangian approach towards formulating the equation of motion for the complex cylinder-rotor assembly of the RV compressor and the preliminary theoretical results for vibration analysis. A summary of the results is as follows:

- The total variation of the rotational velocity of the cylinder-rotor assembly is at 6.60% with an average operating velocity of 3470 RPM.
- The torsional twist in the compressor shell housing is always opposite to the cylinder-rotor assembly rotation direction.
- The variation of the torsional twist experienced by the compressor shell housing is at 0.12° around the average value of -0.70°.

NOMENCLATURE

C	Damping constant	(Nms)
f	holonomic constrain	(m^{-1})
k	Spring constant	(Nm/rad)
L	System Lagrangian	(J)
Q	Non-conservative force	(Nm)
q	Generalized coordinate	(rad)
R	Radius/Radial length	(m)
T	Torque	(Nm)
γ	Angle between vane and rotor center	(rad)
ε	Eccentricity	(m)
θ	Rotation angle	(rad)
λ	Lagrange multiplier	(–)

μ	Coefficient of friction	(–)
φ	Angle of bushing center from rotor center	(rad)

Subscript

b	bushing
base	housing base
c	cylinder
damp	damping
f	friction
g	fluid/gas
h	housing shell
j	generalized coordinate number
jbr, low	lower journal bearing
jbr, up	upper journal bearing
k	holonomic constrain number
m	motor
r	rotor
shear	fluid/gas shear

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