Supersonic Axial Compressor Stage Simplified Analysis

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ABSTRACT

Supersonic axial stages with big pressure ratio are increasingly in demand. There is a problem to elevate pressure ratio of a stage up to 3 and more. Efficiency of a stage can be limited by shock wave losses at high supersonic speeds. The numerical analysis of losses was made in a plane cascade. Calculated losses in shock waves depend on a velocity coefficient and an angle of a shock wave. Pressure loss calculation in subsonic parts of a stage was made by a loss coefficient. Its value was chosen by an expert assessment. It is shown shock that wave losses are not an obstacle for an acceptable level of stage efficiency up to a velocity coefficient value 1.5.

Keywords: normal shock wave, oblique shock wave, plane cascade, loss coefficient, pressure ratio, efficiency.

1. INTRODUCTION

Application of supersonic axial compressor stages is an effective way to decrease mass and size of gas turbines. It is reported that stages with pressure ratio up to 2,8 and blade velocity about 450 m/s can operate quite satisfactory – Fig. 1.

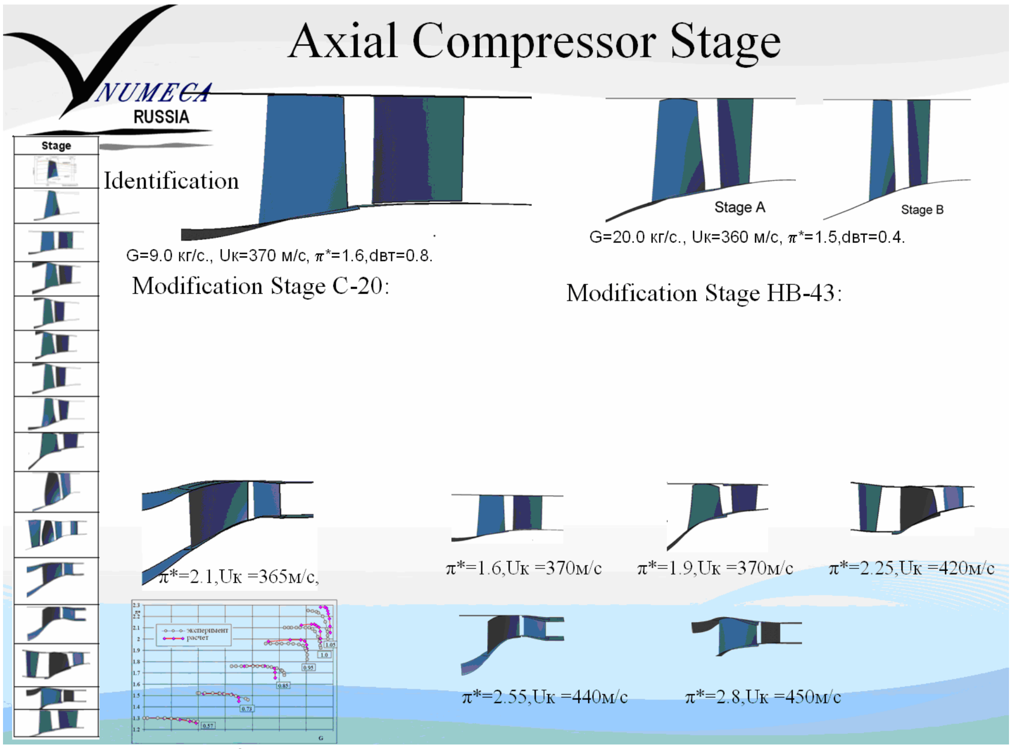

Fig. 1. Information on modern supersonic axial compressor stages [9]

Euler coefficients of stages presented in [9] were calculated by the Authors at the next suppositions: $\eta_{t\,ad}$ = 0,87, γ =1,4, c_p =1005 J/kg, T_{0t} = 288 K and presented in the Table 1. Equations used:

- Euler coefficient at an impeller outer diameter:

$$\psi_T = \frac{H}{U^2}, \tag{1}$$

- total enthalpy:

$$H = \frac{\gamma}{\gamma - 1} RT_{1t} \left(\left(\pi_t \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{t\,ad}}, \tag{2}$$

Table 1. Estimation of Euler coefficients of high pressure ratio axial stages presented in [9]

π_t	U , m/s	H , J/kg	ψ_T
1,6	370	47810	0,349
1,82	455	62080	0,300
1,9	370	66960	0,489
2,05	455	75850	0,366
2,25	420	86740	0,492
2,55	440	102000	0,527
2,8	450	113780	0,562

General rule is that the more is pressure ratio the more is Euler coefficient. The value ψ_T = 0,562 seems to be not low for an industrial centrifugal stage. It is three times more than of an usual value for a subsonic axial stage. It creates problems for stator part of a supersonic stage but the problem will not be touched here. The discussed problematic is efficiency of shock waves as diffusers.

2. OBJECT

The simplified calculation model of a stage is an object of an analysis below. A stage is presented as an elementary supersonic blade cascade of an impeller – Fig. 2. Shock wave losses are calculated by known equations [1]. (Losses in a subsonic part of an impeller and in a stator or in an exit guide vanes are calculated without description of details.) Losses in a subsonic part of a stage are estimated by applying an expertly appointed loss coefficient. The choice of its value is based on results of numerical investigations of subsonic stages that are published in [2, 3, 4, 5, 6, 7, 8].

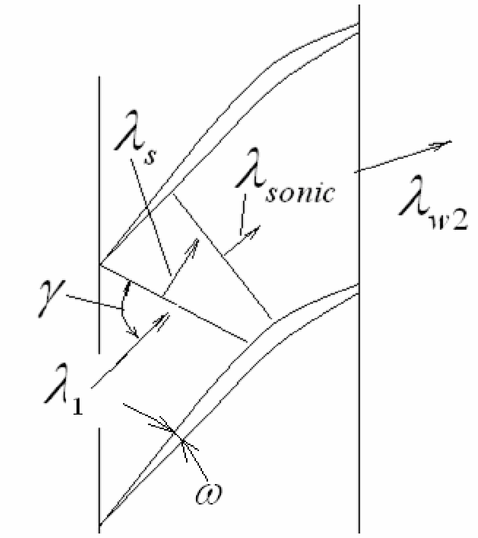

Fig. 2. Elementary supersonic blade cascade and oblique – direct shock wave scheme

It is assumed that supersonic flow in elementary blade cascade with sharp leading edges of blades produces oblique shock wave. The flow can become subsonic or can remain supersonic after an oblique shock. It depends on an inlet velocity coefficient and an angle γ_{os} between shock wave front and flow direction. The normal shock wave occurs if a flow is still supersonic after an oblique shock. The result of one of the Authors' CFD – calculation demonstrates validity of the "oblique – direct shock" scheme - Fig. 3.

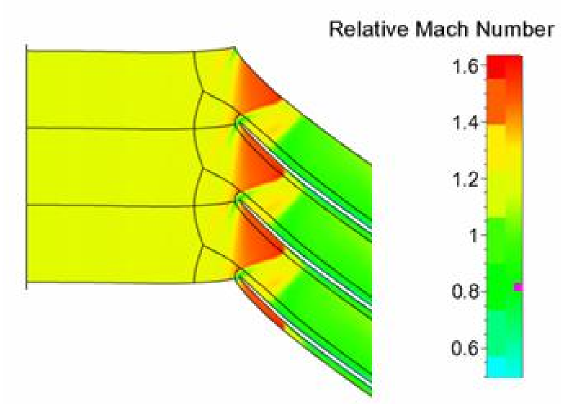

Fig. 3. Typical Mach number field at an impeller inlet (NUMECA Fine/AutoGrid [9])

3. SCHEME AND EQUATIONS

Oblique shock angle depends on a leading edge angle ω and Mach number:

$$tg\left(\gamma_{os}-0{,}5\omega\right)=\frac{\gamma-1}{\gamma+1}\left(1+\frac{2}{\gamma+1}\frac{1}{M_1^2\,sin^2\gamma_{os}}\right)tg\gamma_{os}\cdot \tag{3}$$

Minimal value of an oblique shock wave is function of a Mach number:

$$sin\gamma_{os0}=\frac{1}{M_1}\cdot \tag{4}$$

There is no shock but a sound wave only if an angle value is γ_{os0}.

The shock parameters are easier analyzed by velocity coefficient $\lambda = w/\sqrt{\frac{2\gamma}{\gamma+1}RT_t}$ as it directly proportional to flow velocity. Mach number and velocity coefficient are connected by Eq. (5):

$$M_1=\lambda_1\sqrt{\frac{\frac{2}{\gamma+1}}{1-\frac{\gamma-1}{\gamma+1}\lambda_1^2}}\,. \tag{5}$$

The next equation defines a velocity coefficient after an oblique shock:

$$\lambda_{1s}=\sqrt{\lambda_1^2\cos^2\gamma_{os}+\frac{\left(1-\frac{\gamma-1}{\gamma+1}\lambda_1^2\cos^2\gamma_{os}\right)^2}{\lambda_1^2\left(1-\cos^2\gamma_{os}\right)}}\,. \tag{6}$$

If velocity coefficient $\lambda_{1s}>1$ a normal shock follows with velocity coefficient $\lambda_{sonic}<1$ (subsonic velocity) after it;

$$\lambda_{sonic}=\frac{1}{\lambda_s}\cdot \tag{7}$$

Isentropic equations connect total and static pressures at a cascade inlet, after oblique and normal shocks. They are:

$$\frac{p_{1t}}{p_1}=\left(\frac{1}{1-\frac{\gamma-1}{\gamma+1}\lambda_1^2}\right)^{\frac{\gamma}{\gamma-1}},\ \frac{p_{st}}{p_s}=\left(\frac{1}{1-\frac{\gamma-1}{\gamma+1}\lambda_s^2}\right)^{\frac{\gamma}{\gamma-1}},\ \frac{p_{sonict}}{p_{sonic}}=\left(\frac{1}{1-\frac{\gamma-1}{\gamma+1}\lambda_{sonic}^2}\right)^{\frac{\gamma}{\gamma-1}}. \quad (8\text{-}10)$$

Static pressure ratios in oblique and normal shocks are given by equations:

$$\frac{p_s}{p_1}=\frac{\lambda_1^2\left[1-\frac{4k}{(\gamma+1)^2}\cos^2\gamma_{os}\right]-\frac{\gamma-1}{\gamma+1}}{1-\frac{\gamma-1}{\gamma+1}\lambda_1^2}. \quad (11)$$

$$\frac{p_{sonic}}{p_s}=\frac{\lambda_s^2-\frac{\gamma-1}{\gamma+1}}{1-\frac{\gamma-1}{\gamma+1}\lambda_s^2}. \quad (12)$$

The equations (8 - 12) define total pressure loss in shock waves:

$$\frac{p_{sonict}}{p_{1t}}=\left(\frac{p_s}{p_1}\cdot\frac{p_{sonic}}{p_s}\cdot\frac{p_{sonict}}{p_{sonic}}\right)/\frac{p_{1t}}{p_1}. \quad (13)$$

Losses in a subsonic part are calculated by loss coefficient ζ_{ad} :

$$H_{wad}=\zeta_{ad}\frac{w_{sonic}^2}{2}. \quad (14)$$

This coefficient is connected with velocities in a subsonic part:

$$\frac{w_{sonic}^2-w_2^2}{2}=H_{ad}+H_{wad}. \quad (15)$$

Pressure ratio in a subsonic part could be derived from Eq. (14, 15):

$$\frac{p_2}{p_{sonic}}=\left(1+\frac{\gamma-1}{\gamma+1}\frac{\lambda_{sonic}^2\left[1-(\lambda_2/\lambda_{sonic})^2\right]-\zeta_{ad}}{1-\frac{\gamma-1}{\gamma+1}\lambda_{sonic}^2}\right)^{\frac{\gamma}{\gamma-1}}. \quad (16)$$

Isentropic equation connects total and static pressures at the exit of the cascade:

$$\frac{p_2^*}{p_2}=\left(\frac{1}{1-\frac{\gamma-1}{k+1}\lambda_2^2}\right)^{\frac{\gamma}{\gamma-1}}. \quad (17)$$

The ratio $\lambda_{w2}/\lambda_{sonic}$ in Eq. (16) is a parameter for calculations. A stator part of the stage model is taken into consideration indirectly by the value of the coefficient ζ_{ad}. The calculations were made for a gas with γ = 1,4 in a range of velocity coefficient λ_1 =1,1 – 1,8. Shock wave angle was varied in a range γ = 90^0 – γ_0.

4. CALCULATED PARAMETERS

The following parameters are presented as result of calculations:

- velocity coefficients after oblique and normal shock waves $\lambda_s=f(\lambda_1,\gamma_{os})$, $\lambda_{sonic}=f(\lambda_1,\gamma_{os})$,

- static pressure ratios after oblique and normal shock waves and in (model as a whole of) a stage, subsonic part included: $\pi_s = \dfrac{p_s}{p_1} = f(\lambda_1, \gamma_{os})$, $\pi_{sonic} = \dfrac{p_{sonic}}{p_s} = f(\lambda_1, \gamma_{os})$, $\pi = \dfrac{p_{ex}}{p_1} = f(\lambda_1, \gamma_{os})$.
- polytrophic efficiency and loss coefficient of shock waves or in a stage (in a whole if subsonic part is taken into account):

$$\eta = \frac{\lg\left(\dfrac{p_2}{p_1}\right)}{\dfrac{\gamma}{\gamma-1}\lg\left(\dfrac{1-\dfrac{\gamma-1}{\gamma+1}\lambda_2^2}{1-\dfrac{\gamma-1}{\gamma+1}\lambda_1^2}\right)}, \tag{18}$$

$$\zeta = (1-\eta)\left(1-\frac{\lambda_2^2}{\lambda_1^2}\right). \tag{19}$$

5. EFFICIENCY AND SHOCK WAVES PARAMERETS

Graphics in the Fig. 4 show two zones of a cascade possible operation:
- subsonic flow after an oblique shock wave with angles bigger than 72^0-62^0 (the bigger value corresponds to a smaller velocity coefficient);
- supersonic flow after an oblique shock wave with angles smaller than 72^0-62^0 and a normal shock wave after.

The next Fig. 5 demonstrates velocity coefficients after normal shock if a flow is supersonic after an oblique shock.

The smaller angle γ_{os} is the less intensive is an oblique shock wave and the higher is a subsonic velocity coefficient after a shock. (A velocity coefficient is equal to 1 after an oblique shock at a certain value of γ_{os}.) The flow is subsonic after an oblique shock in the region on the right of λ_{sonic} =1. Velocity coefficients on the left of λ_{sonic} =1 are result of a normal shock following an oblique one.

Efficiency and loss coefficients of shock waves are presented in the Fig. 6. The subsonic flow zone after an oblique shock is on the right of “d.l.” dividing line.

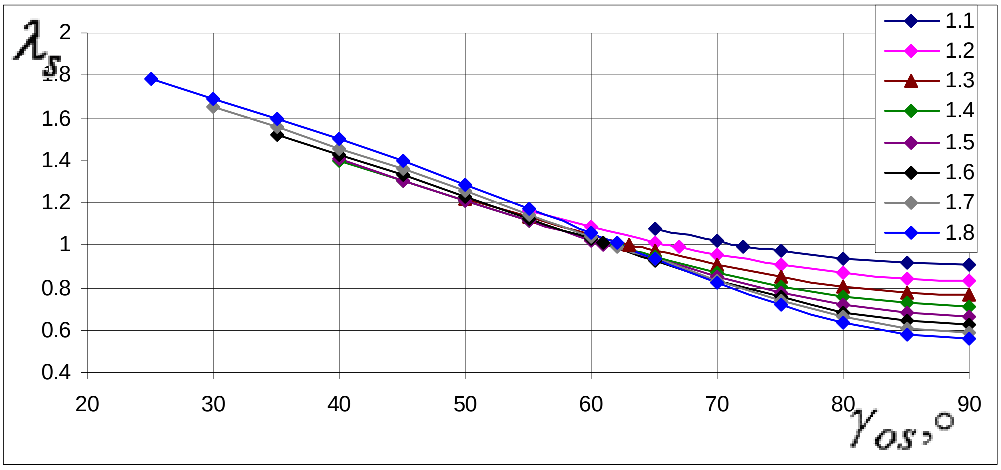

Fig. 4. Velocity coefficients after an oblique shock wave

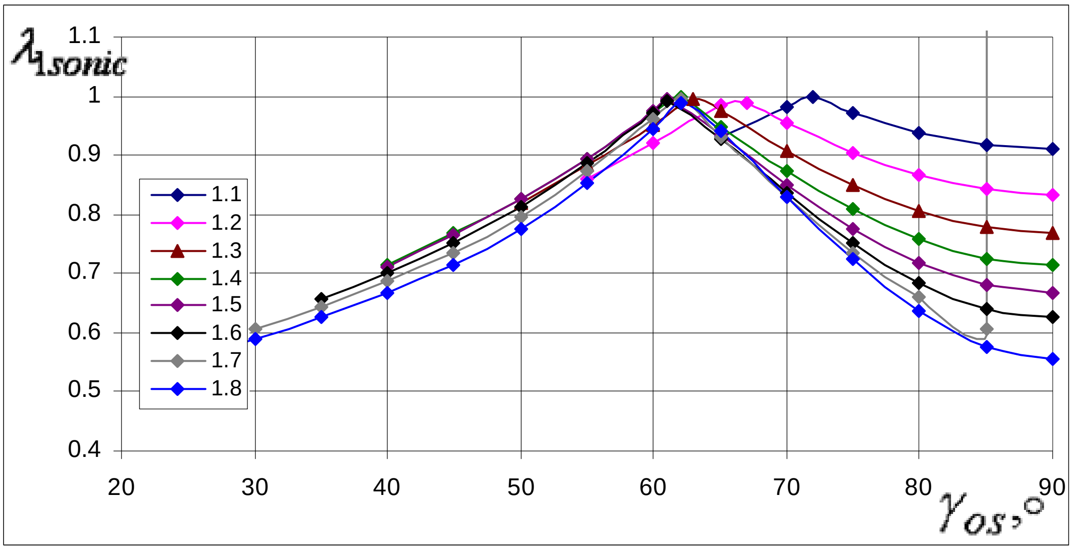

Fig 5. Velocity coefficients in a subsonic part of a cascade

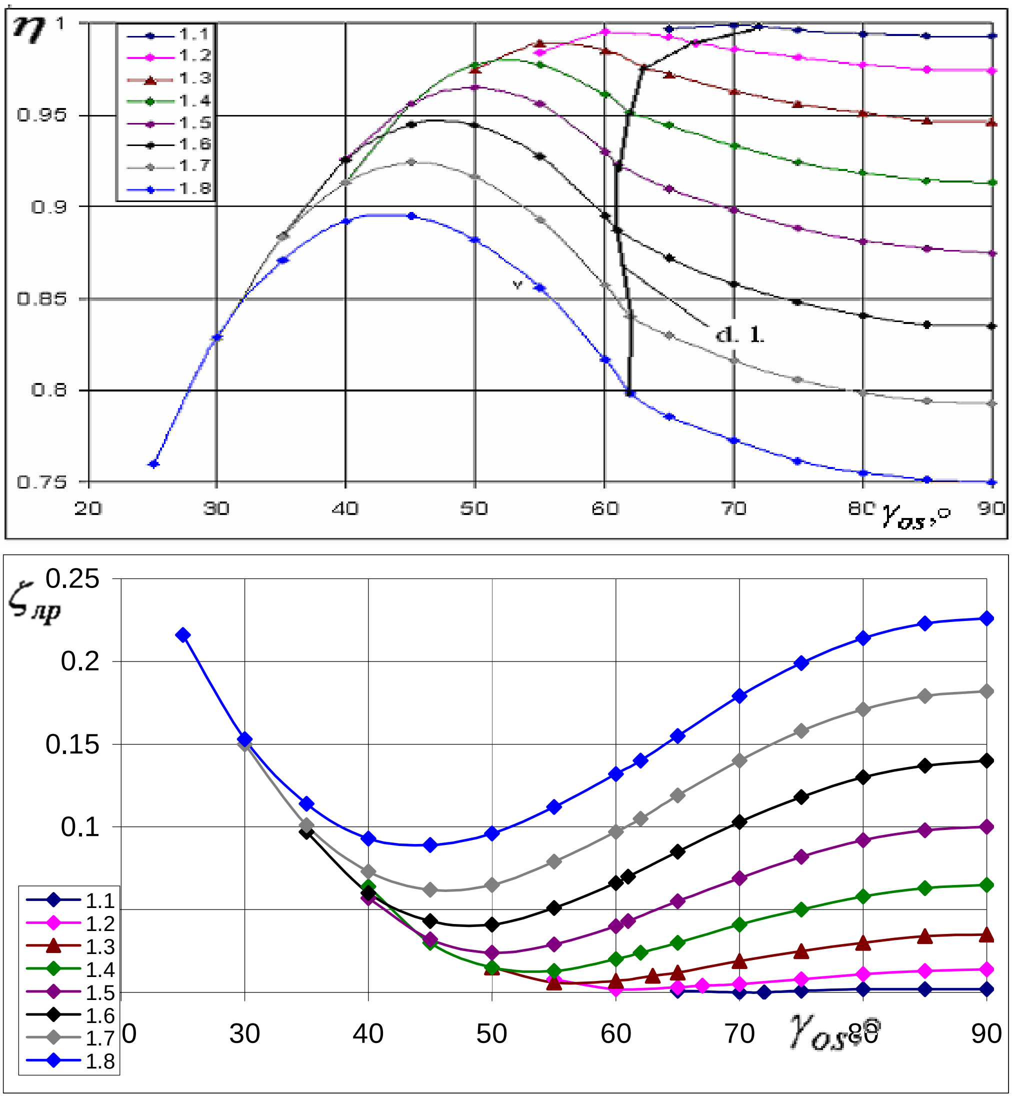

Fig. 6. Efficiency and loss coefficient of shock waves as diffusers versus γ_{os} and λ_1

The border between subsonic flow and supersonic flow after an oblique shock corresponds to (its front angle) $\gamma_{os} \approx 62^0$ for values of velocity coefficient λ_1 = 1,8-1,3. The left zone corresponds to the oblique – normal shock combination. The most effective flow diffusion with $\eta \approx$ 0,89-0,99 takes place in a combination of shocks and lies in a range 45-60^0 for values of λ_1 = 1,8-1,2.

Static pressure ratios in an oblique shock, in a following normal shock and in their combination are shown in the Fig. 7.

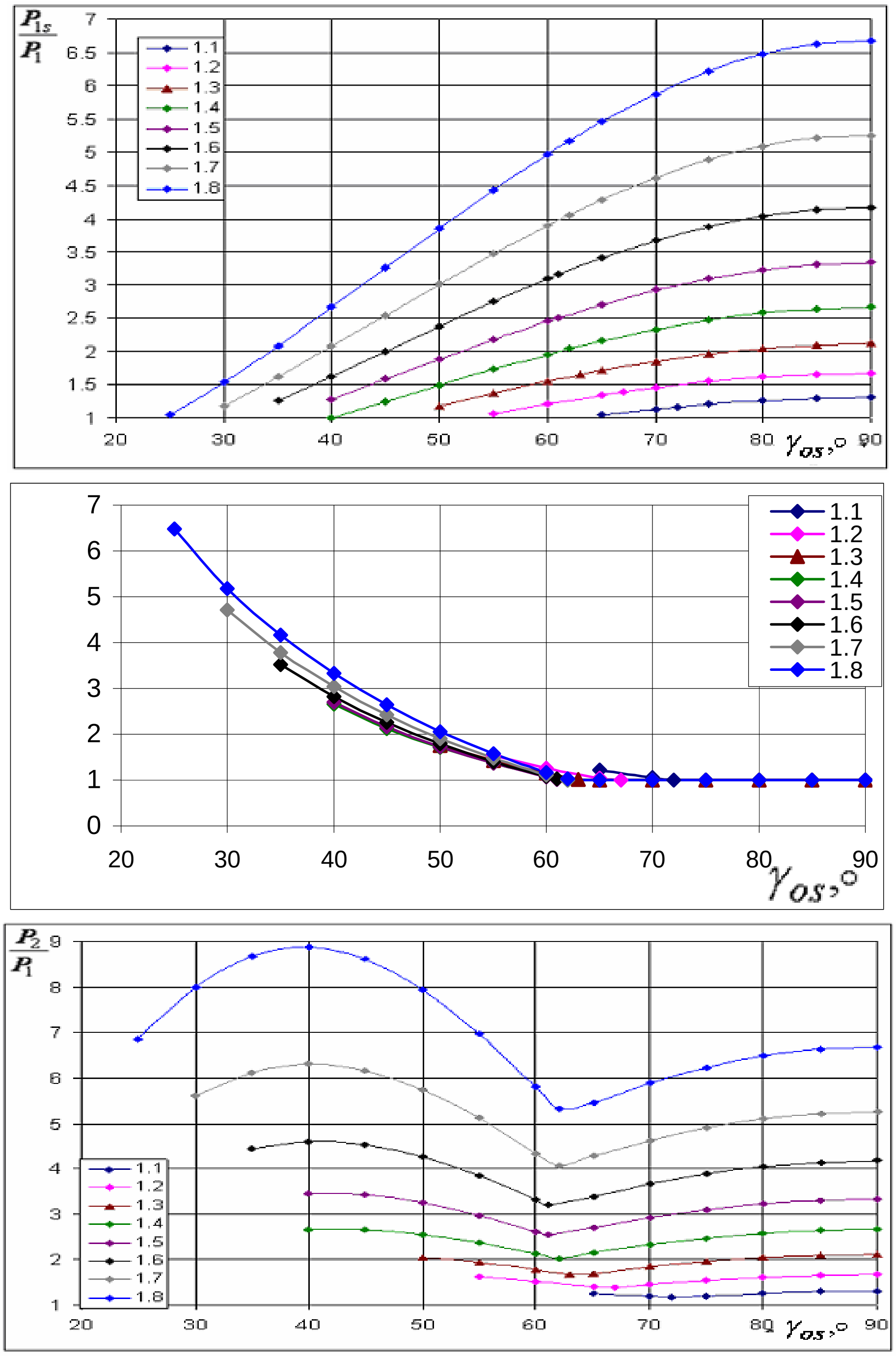

Fig. 7. Pressure ratios in an oblique shock, in a following normal shock and in their combination

The highest pressure ratio at given velocity coefficient take place when front angle is about 40^0. Extremely high pressure ratios at high velocity coefficients $\lambda_1 > 1{,}5$ are hardly realistic as values λ_1 = 1,6-1,8 correspond to a blade speed $U > 600$ m/s. The maximum pressure ratio 2,8-8,9 for λ_1 = 1,4-1,8 corresponds to an angle $\gamma_{os} \approx 40^0$. Corresponding shock efficiency values (Fig. 6) are 0,89 – 0,925.

6. STAGE EFFICIENCY SIMPLIFIED ANALYSIS

The simplified model represent (simulates) a stage as a combination (sum) of shock wave system and a subsonic part. A stage subsonic part pressure ratio is calculated by Eq. (14). Flow parameters in a shock system and in a subsonic part are enough to calculate static stage efficiency by Eq. (15). Calculations by Eq. (14) are made with expertly appointed values of loss coefficient ζ_{ad} = 0,085 and $\lambda_{w2} / \lambda_{sonic}$ = 0,60.

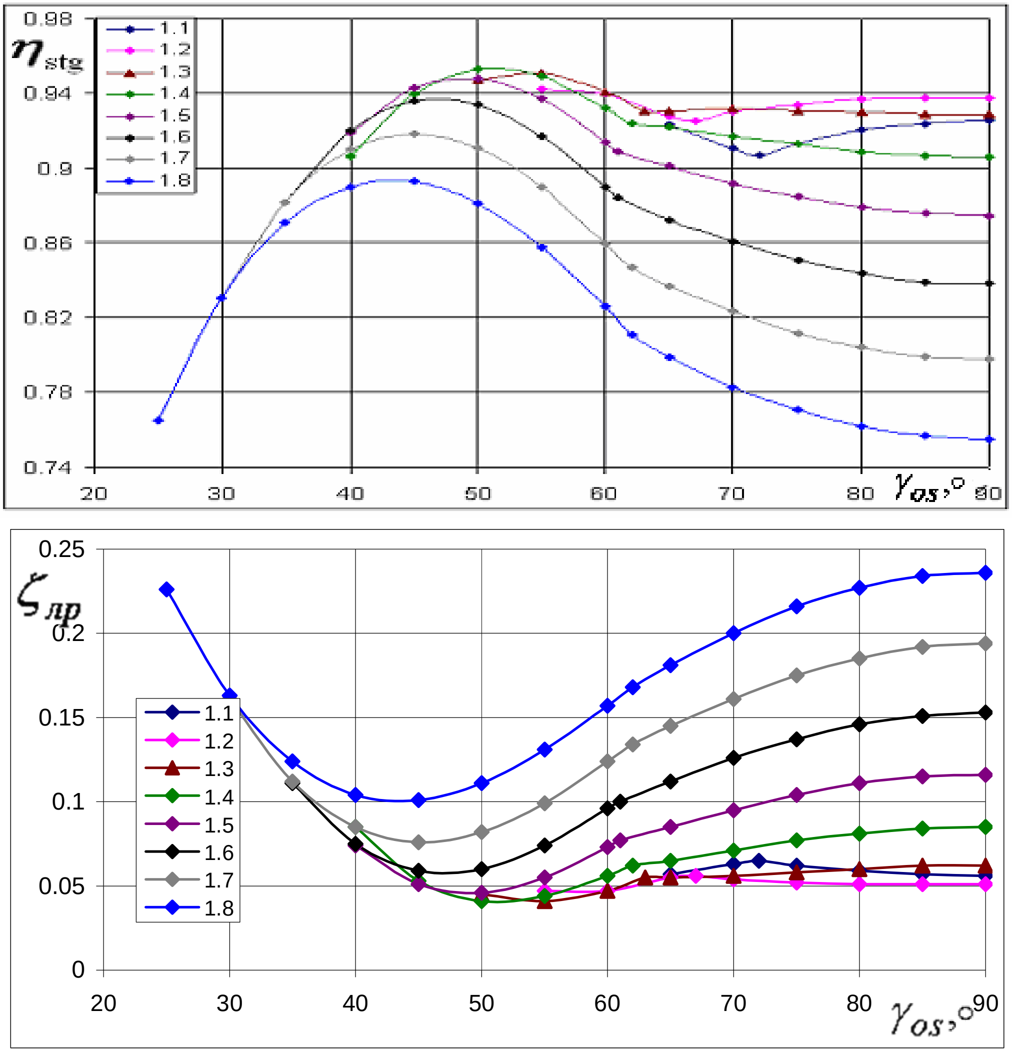

Fig. 8. Stage efficiency (static parameters) and loss coefficient versus γ_{os} and λ_1. (Constant values)

ζ_{ad} = 0,085, $\lambda_{w2} / \lambda_{sonic}$ = 0,60

Stage efficiency (static parameters) and loss coefficient versus γ_{os} and λ_1 are presented in the Fig. 8.

Maximum efficiency corresponds to oblique – normal shock system at (an oblique front angle about) $\gamma_{os} \approx 45^0$ for velocity coefficient values 1,5-1,8. (as for a system of shocks in the Fig. 6.) Unlike shock system the efficiency of a stage does not decrease (in a monotonous way) with diminishing of a velocity coefficient. The highest efficiency of about 95% corresponds to λ_1 =1,4 and γ_{os} = 50^0. Maximum efficiency of about 94% takes

place in a zone of a single shock for λ_1 =1,2 and γ_{os} = 90^0. A normal shock is more effective at λ_1 =1,1-1,3 as subsonic velocities after a normal shock are low. (It diminishes losses at a subsonic part of a stage.)
Static pressure ratio for calculation model of a stage is presented in the Fig. 9.

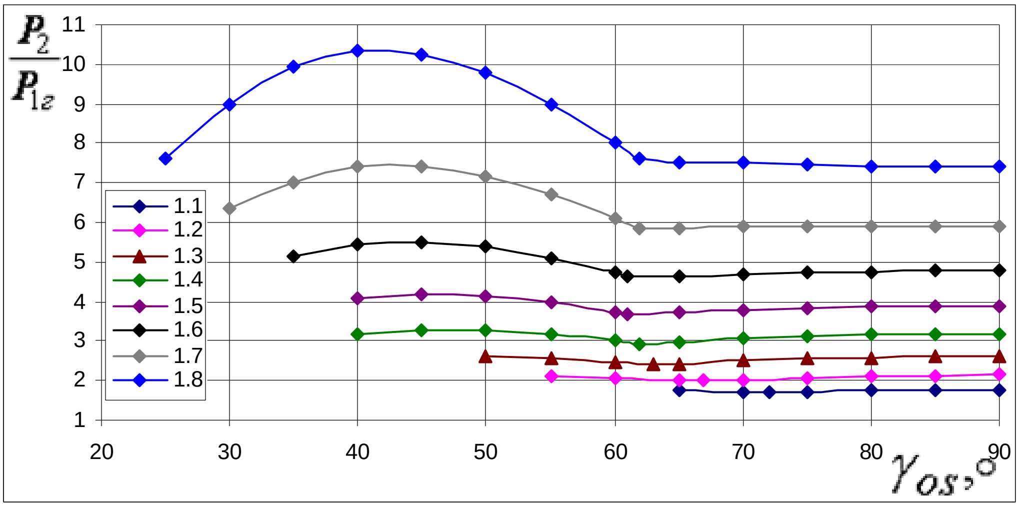

Fig. 9. Pressure ratios in a model of a stage coefficient versus γ_{os} and λ_1

A pressure ratio does not depend of a shock angle practically when flow is subsonic after the first shock. (It rises up to range of 1,8-7,4 for a velocity coefficient range 1,1-1,8.) For advanced (sophisticated) stages with total pressure ratio about 3,0 velocity coefficient could be 1,4-1,5. The efficiency could be about 0,86-0,91 if a normal shock is at a cascade inlet. The system “normal – oblique shock” increases calculated efficiency up to 95% if a front angle is 50^0.

7. CONCLUSION

The above calculations have demonstrated that despite a head losses in shock waves the efficiency of a plane cascade can be (reach) more than 90% for (sophisticated) stages with $\pi_t \approx$ 3,0. There are several serious problems to be solved though to reach the goal for a real stage.
Eq. (1) shows that (an oblique shock with any necessary) angle γ_{os} can be done by proper choice of leading edge angle ω (of a profile). It is not clear if an optimal angle γ_{os} can be always made at a real cascade entrance though. A profiles interaction and blade load influence on a flow structure.
Shock wave on a surface provokes flow separation. It can reduce efficiency of a subsonic part of a cascade more seriously than it was predicted by loss coefficient choice at presented calculations.
The problem of effective 3-D design of impellers and stators for high supersonic stages is still unsolved too. Effective elementary cascade is the first necessary step for final decision of the problem but not the last one.

8. NOMENCLATURE

c_p	specific heat;
F_d	diffusion factor;
H_{ad}	adiabatic head;
h_m	theoretical head;
i	enthalpy;
k	isentropic coefficient;
M	Mach number;
$\overline{m}$	mass flow rate;
p	pressure, Pa;
R	gas constant;
T	temperature;
U	impeller speed;

w	flow relative velocity;
π	pressure ratio;
ζ	loss coefficient;
η	efficiency;
ψ_T	Euler coefficient;
γ	isentropic coefficient,
γ_{OS}	oblique shock wave angle;
λ	velocity coefficient.

Subscripts

0	impeller eye condition;
1	impeller blade inlet condition;
2	impeller tip condition;
3	vaned diffuser inlet condition;
ad	adiabatic;
ex	exit;
max	maximum;
p	polytropic process;
sonic	subsonic parameter after a shock wave;
s	supersonic parameter after an oblique shock wave,
t	total parameter.

REFERENCE

1. Abramovich G.N. Applied gas dynamics. The edition 4 corrected and added. Main edition of physical and mathematical literature of publishing house «Science». - 1976. – 888 (in Russian).
2. Galerkin Y. B., Popov Y. A. The analysis of efficiency of spatial scapular lattices of axial compressors according to purges of flat lattices // Compressors & Pneumatics. – Moscow. - 2005. – No. 3. – P. 33-38 (in Russian).
3. Galerkin Y. B., Popov Y. A. Settlement analysis of characteristics of axial compressor stages. // Compressors & Pneumatics. – Moscow. - 2005. – No.5. – P.26-33.
4. Galerkin Y. B., Popov Y. A. The analysis of a spatial stream in "not homogeneous" axial compressor stages. // Compressors & Pneumatics. – Moscow. - 2006. – No.4. – P.11-19 (in Russian).
5. Galerkin Y. B., Popov Y. A. Optimization of flow part of axial compressors at a stage of alternative calculation. Part 1 // Compressors & Pneumatics. – Moscow. - 2009. – No.5. – P.2-9 (in Russian).
6. Galerkin Y. B., Popov Y. A. Optimization of flow part of axial compressors at a stage of alternative calculation. Part 1// Compressors & Pneumatics. – Moscow. - 2009. – No.6. – P.1-19 (in Russian).
7. Galerkin Y. B., Popov Y. A. Optimization of flow part of axial compressors at a stage of alternative calculation. Part 1 // // Compressors & Pneumatics. – Moscow. - 2009. – No.7. – P. 6-12 (in Russian).
8. Galerkin Y. Turbo compressors. // LTD information and publishing center. - Moscow. – 2010 (in Russian).
9. Transactions of the 2nd Open Conference of Users of NUMECA in Russia. - 2013.