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PD Compression: A Quasi-Static or Dynamic Process?

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ABSTRACT

Gas compression in a positive displacement (PD) type piston engine, or piston compressor, has long been modeled as a quasi-static process in which the piston is assumed to move infinitely slow inside a cylinder. The gas compression mechanism is therefore regarded as being governed by the basic thermodynamic principles. Energy transfer is accomplished quasi-statically with a piston reducing the volume a little at a time, small perturbation, while increasing gas pressure simultaneously. This is in direct contrast to a dynamic type gas compressor such as turbo compressor that achieves energy transfer by initially imparting kinetic energy to the gas and then converting that kinetic energy to gas pressure.

Is the quasi-static hypothesis still valid today with the quantum leaps made in piston speeds since the Watt’s days? Moreover, exactly how is the work done by the moving piston converted to gas pressure? This paper attempts to answer these questions by applying the classic shock tube theory to the transient dynamic process of high speed piston compression. The results reveal that the nature of the compression is in essence a dynamic process by actions of a pair of moving shock waves triggered by fast piston movement. A transient dynamic stage of a piston compression consists of a rotor with incident shock wave (ISW) compression and a stator with reflected shock wave (RSW) compression during a finite time interval; or a stage cycle of \( \Delta t \). The resulting energy transfer process is thus similar to that of a turbo compressor characterized by initially imparting kinetic energy to the gas through ISW action and then converting the kinetic energy to pressure by RSW reaction.

1. INTRODUCTION

1.1 Conventional PD Compression Cycle and Compressor Classification

Reciprocating compression, often called piston compression, is one of the earliest inventions by human in record that can be traced back to 1500 BC in ancient China and Egypt where man powered bellow style devices used to assist combustion in a furnace. The mechanical powered reciprocating compression was re-invented during the industrial revolution in England during the late 18th century and has since played a major role in the subsequent industrial revolution that has brought profound social and economic changes to the world.

Compression inside a piston compressor, during the compression stroke, where a piston moves in a cylinder via a connecting rod and crankshaft, decreases the trapped volume, so called internal compression, which results in an increase of the gas pressure as shown in Figure 1a. As an example; a typical compression cycle with corresponding basic thermodynamic processes is explained for a two stroke piston compressor. Compression occurs within the cylinder as part of a four-phase cycle that takes place with each advance and retreat of the piston, two strokes per cycle. The four phases of the cycle are intake (4→1), compression (1→2), discharge (2→3) and expansion (3→4), corresponding to the thermodynamic states on the P-V diagram as shown in Figure 1b.
Since a positive displacement always accompanies piston compression, or with similar processes such as rotary motions found in screw or scroll compression, it is often termed positive displacement (PD) compression in contrast to dynamic types identified as axial or centrifugal compression, as shown in a typical compressor classification in Figure 2. For simplicity, the piston compression will be used in this paper to illustrate the general principle of PD.

![Figure 2: Conventional compressor classification](image)

1.2 Quasi-Static Postulate for Conventional PD Compression Theory and its Limitation

Though Watt’s machines of the 18th century bear little resemblance to the piston type of today, the basic principle has surprisingly remained unchanged. The PD compression has been and still is regarded essentially as a thermodynamic process, as pointed out by Baehr (2005), Kestin and Glass (1950), Prasad (1999), that uses thermodynamic methods and principles to define the change in state of the gas and the work transfer, in contrast to dynamic types that are governed by aero-dynamic energy transfer process by initially imparting kinetic energy to gas and then converting that kinetic energy to pressure following Bernoulli principle. As a thermodynamic device, the compression inside a piston engine or compressor has been modeled as a quasi-static process that assumes the piston moves infinitely slow and results in a system in thermodynamic equilibrium, pressure and temperature are always uniform throughout the system, and can be represented as a continuous curve on P-V diagram. There is no aerodynamics involved and the kinetic energy of the piston is directly transferred to the internal gas energy in the form of work input as represented by the area under the compression curve (1→2) on the P-V diagram in Figure 1b.

However, it is important to take notice that no real compression process is quasi-static, especially for modern day high speed engines or compressors that have witnessed quantum leaps on the construction material and precision manufacturing since the late 18th century. A criterion has been suggested by Baehr (2005) that a process can be treated as quasi-static as long as the piston velocity is less than 2-10 m/s or the relative Mach number is below 2-3%. However, peak piston velocities between 20-60 m/sec are common nowadays and could be over 100 m/s within the next 50-100 years if the current trend continues. Under that circumstance, the thermodynamic approach may not be sufficient to model the high speed compression process as criticized by Kestin and Glass (1950), Prasad (1999). Particularly in question is the quasi-static postulate that assumes small perturbation and ignores perhaps the most important transient dynamic aspect of the process played out between the high velocity piston and cylinder. So fundamental questions need to be raised, such as; what is the physical nature of a high speed PD compression; is it quasi-static or dynamic? How energy is transferred from the moving piston to the gas inside the cylinder? More importantly, how to calculate the transient compression process for high speed compression? Searching for answers,
we came across a powerful analytical tool, the Shock Tube, which is simple to use yet well established for studying the transient behavior under a dynamic situation.

1.3 A Transient Dynamic Approach: The Shock Tube Analogy
This paper attempts to address these questions by focusing on the essential physics during the transient portion of the PD compression in order to build simple hypothesis and theory that give understanding and suggestive insight rather than exact calculation. Because of the difficulty in directly solving non-linear PDEs for high velocity compressible flow, a different approach, using an analogy with the shock tube, is employed in this paper. Analogy is a diversionary cognitive process by correlating and transferring information from a known source, shock tube process in this paper, to the unknown target, transient PD compression process. Historically, especially before the emergence of the modern digital computer, it played a significant role in problem solving in science and technology as described by Holyoak (1996), such as by Newton in comparing a planet’s motion to a stone thrown with greater and greater force on the earth and by Franklin in linking the lightening with the Leyden jar. The effectiveness of this method depends largely on how strong the source causality is and how closely the premises of an argument’s claim are correlated between the source and target as formulated being a material analogy by Hesse (1966).

The shock tube analogy was first proposed by Huang (2012a and 2012b) in analyzing the transient behaviors during an under compression or gas pulsation phenomena. The selection of the shock tube as the source is primarily because its physics is clear and easy to visualize yet well-established over the years from analytical, experimental, and numerical studies. Secondly, the existence of high speed piston or rotary screw compressor suggests that the dynamic large perturbation shock tube is a closer approximation than the quasi-static and small perturbation approximation.

2. TRANSIENT DYNAMIC PD COMPRESSION: A SHOCK TUBE MECHANISM

2.1 Introduction to Shock Tube Theory
The shock tube is a device used to study transient aerodynamic phenomena under a wide range of temperatures and pressures for a variety of gases (Liepmann and Roshko, 1957; Rudinger, 1969 and Anderson, 1982). It has been studied extensively by researchers from the start of the last century and has effectively served the developments of the supersonic and hypersonic flights, a range of transient devices such as pulse-jet engines and wave rotors for supercharging internal combustion engine called Comprex or for topping gas turbine (Mueller, 2006; Weber, 1995). However, the physical phenomena observed in a shock tube and the well-established shock tube theory have thus far not served for examining, hence determining the transient dynamics of PD compression process in general.

An ideal piston-triggered shock tube as shown in Figures 3a-3b is a half-closed gas tube with a closed end on one side and moving piston on the other side. In between is filled with gas with initial gas pressure p1. Then the piston is suddenly accelerated towards the closed end from zero to some finite velocity U producing a series of compression waves...
waves, each increasing the speed of sound behind them so that they quickly coalesce into an incident shockwave (ISW) propagating through the stationary gas at velocity $W_i$ increasing gas pressure adiabatically from $p_1$ to $p_2$. At the instant the incident shock impinges on the stationary end wall, a reflected shock wave (RSW) travelling in opposite direction is instantly generated in such a way that the originally induced gas motion with velocity $U$ is stopped dead in its track. The RSW further raises gas pressure from $p_2$ to $p_5$ when sweeping through at velocity $W_r$ as shown in Figures 3c-3e.

The analytical solutions for shock wave and shock tube were available more than a century ago, thanks to independent efforts by Rankin and Hugoniot. The key results are summarized by (Anderson, 1982) as below:

$$E_2 - E_1 = (p_2 + p_1) \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) / 2$$  \hspace{1cm} (1)

$$p_2 - p_1 = \rho_1 W_i U$$  \hspace{1cm} (2)

$$p_5 - p_2 = \rho_2 W_r U$$  \hspace{1cm} (3)

Equation (1), known as the Hugoniot equation, relates changes of thermodynamic variables across a normal shock wave, while equations (2-3) relate the abruptly arisen pressure ($\Delta p$) to shockwave speed of $W$, initial gas density $\rho$ and piston velocity of $U$. Moreover, if there is no heat exchange with surroundings and the piston velocity $U < 150$ m/s, the pressure ratio of the incident shock waves (ISW) and reflective shockwave (RSW) can be approximated, within an engineering tolerance of 5%, by:

$$p_2/p_1 = p_5/p_2 = (p_5/p_1)^{1/2}$$  \hspace{1cm} (4)

Take note that the only condition that is sufficient to trigger a shock tube type transient behavior from static to dynamic is an impulsive piston velocity $U$, or the instant piston acceleration from zero to some finite velocity $U$.

### 2.2 Piston Velocity Distribution and Transient Dynamic Compression Postulate

In real world, the piston or lobe moves at high speed during compression phase. An example is illustrated in Figure 4a where the piston first accelerates from zero to the highest velocity near mid-stroke, and then decelerates to a dead stop. The compression ratio depends on the volume reduction ratio within the cylinder when the pressure within the cylinder reaches the pressure of the discharge header and discharges out before reaching the dead end of the cylinder. For modern day piston engines or compressors, the peak piston velocity can range up to 20 to 60 m/sec.

Suppose that we divide the continuous piston velocity distribution in Figure 4a to a series of velocity segments as an approximation as illustrated in Figure 4b, then each velocity impulse $U_j$ can be interpreted as an instant acceleration from zero to $U_j$, then after a short but finite time interval $\Delta t_j$, the piston decelerates instantly from $U_j$ to zero. The determination of $\Delta t_j$ will be discussed in next section 2.3. In other words, the piston movement segmented into a series of finite velocity impulses will interact with gases within the cylinder dynamically, which is in direct contrast to the quasi-static postulate that the piston is assumed to move infinitely slow inside the cylinder. This insight immediately suggests a possible shock tube mechanism for the transient dynamics of a piston compression.

Figure 4(a-b): Velocity distribution of a piston stroke inside a cylinder: continuous vs. segmented

2014 Purdue Compressor Engineering Conference, July 14-17, 2014
2.3 Transient Dynamic Stage of PD Compression: A Shock Tube Analogy

As analyzed above, the segmented velocity impulse concept is linking the phenomena of the transient dynamic piston compression process to the piston shock tube process from the moment of an impulsive acceleration to the moment of an impulsive deceleration, that is, from the instant of ISW generation by piston to the instant when RSW meets piston again and disappears. Hence, a transient dynamic piston compression theory can be proposed by the following postulate:

The continuous piston movement can be approximated by a series of velocity segments $U_j$ that each first accelerates instantly from zero to $U_j$, then after a finite time interval of $\Delta t_j$, decelerates instantly from $U_j$ to zero.

This postulate is fundamental and significant here not only because it establishes the link between a single piston velocity impulse and the piston shock tube, but also implies that what happens after the sudden piston movement can be applied to the transient piston compression by having fulfilled the sufficient condition in the material analogous reasoning, (Hesse, 1966), with the shock tube. Therefore, the well-established results of the shock tube accumulated over the past 110 years, though degraded somehow by the ideal gas and one dimensional assumption, can be readily applied to examine, hence reveal, the transient dynamic PD compression process without the need to solve any non-linear PDEs.

![Diagram](image)

**Figure 5(a-d):** Transient dynamic stage of PD compression

**Figure 6(a-c):** Dynamic stage of a multistage axial compression

Now let’s take a new look at the piston compression process by a single velocity impulse as shown in Figure 5d in light of the new theory. The instantly accelerated piston from zero velocity to $U$ at $t = t_1$ as shown in Figure 5 resembles the same moment of a piston shock tube as shown in Figure 3. An incident shock wave (ISW) is generated travelling right at velocity $W_i$, while inducing a gas flow $U$ behind as ISW sweeps through. The pressure of the gas between the ISW and piston is increased from $p_1$ to $p_2$, corresponding to the piston movement from I→II as shown in Figure 5a-5c. At $t = t_2$, the ISW impinges on the end wall, instantly generating a reflected shock wave (RSW) which further increases the gas pressure from $p_2$ to $p_3$ when sweeping through in opposite direction at velocity $W_r$, corresponding to the piston movement from II→III. At $t = t_3$, the RSW suddenly disappears as soon as
it meets the instantly stopped piston. The time it takes from I\(\rightarrow\)II\(\rightarrow\)III is:

\[ \Delta t_{I-II} = t_2 - t_1 = L / W_i \]  
\[ \Delta t_{II-III} = t_3 - t_2 = (L - X_{III}) / W_r \]  
\[ \Delta t_{I-III} = \Delta t_{I-II} + \Delta t_{II-III} = L / W_i + (L - X_{III}) / W_r \]

Where L is the cylinder length and X_{III} can be calculated as follows:

\[ X_{III} / L = (1 / W_i + 1 / W_r) / (1 / U + 1 / W_r) \]  

2.4 Transient Dynamic Stage of PD Compression vs. Dynamic Stage of an Axial Compression

The energy transferring process from the moving piston to the gas during \(\Delta t_{I-III}\) is as follows: the instantly accelerated piston with a velocity U first imparts its kinetic energy to generate an ISW in the gas. Then the stored energy carried by the ISW is transferred to the gas in the form of a static pressure rise, \(p_1\rightarrow p_2\), and the kinetic energy of induced mass behind the ISW during I\(\rightarrow\)II. It takes another shock wave, the RSW, to convert the kinetic energy of the gas into further static pressure rise, \(p_2\rightarrow p_3\), during II\(\rightarrow\)III. The above process, I\(\rightarrow\)II\(\rightarrow\)III, is dynamic in nature, resembling the energy transfer process of a traditional dynamic type compressor. So a dynamic stage during \(\Delta t_{I-III}\) can be defined for the transient piston compression with the ISW functioning as a dynamic rotor during \(\Delta t_{I-II}\) and the RSW as a dynamic stator during \(\Delta t_{II-III}\). For comparison purpose, a dynamic stage of a traditional multistage axial compressor is illustrated in Figure 6, where the energy transfer is achieved first by imparting kinetic energy from rotor cascade to gas and raising its static pressure, then converting the gas kinetic energy to more static pressure in stator cascade. Pay attention to Equations (2) & (3) that state that pressure rise through a shock wave is proportional to \(W \times U\); the multiplication of shock wave velocity by fluid velocity. By comparison, the pressure rise through Bernoulli Principle is proportional to \(U^2/2\); the fluid velocity squared. This difference could hold the key to explain why PD types tend to have much higher pressure rise than a dynamic type for the same fluid and drive velocity.

It can also be seen that the invisible shock waves play the role of blades of a conventional dynamic compressor. In other words, it’s the moving shock wave that actually compresses the gas while the piston is used here only to facilitate the generation and degeneration of the shock wave. However, there are some differences with a turbo-compressor. Firstly, the speed of compression is at the speed of moving shock wave that is fast, no hardware inertia to overcome, yet has no material strength concerns because of the absence of material blades. Secondly, it takes less space since the invisible ISW and RFW are generated as needed and disappear after finishing the job. More importantly, this shock wave compression is much more efficient than a conventional hardware compression through a diffuser or impeller if the dynamic stage pressure ratio is not over 2, to be explained later in case studies. Theoretically, this shock wave compression process is governed by Rankine-Hugoniot equation (1) and is adiabatic in nature. There is an entropy increase associated with the shock wave compression, or dynamic loss, even though there is no frictional effect assumed for the gas.

2.5 Transient Dynamic Stage of PD Compression vs. Thermodynamic Quasi-static Compression

The classic thermodynamics is based on a quasi-static postulate that assumes the piston velocity is infinitely slow so that it results in a system in thermodynamic equilibrium, that is, the pressure and temperature are always uniform throughout the gas as the cylinder volume is reduced. Under this assumption, the state of the gas and the process of the compression can be represented on a P-V diagram as a point and a continuous curve mathematically. There is no macro gas movement and the kinetic energy of the piston is directly transferred to the internal gas energy in the form of work as represented by the area under the compression curve on a P-V diagram. The piston work for a quasi-static process can be calculated as:

\[ \text{Piston Work} = \int_{V_1}^{V_2} PdV \]  

By the shock tube theory, however, the piston velocity is large enough to cause a macro gas movement and the gas pressure and temperature are no longer uniform inside the cylinder. As shown in Figure 7 for one stage dynamic piston compression during I\(\rightarrow\)II\(\rightarrow\)III, the pressure on the piston face does not increase gradually as traditional thermodynamic model would predict. Instead, the pressure jumps first from \(p_1\rightarrow p_2\) as vertical line of I-A, then stays at a constant pressure \(P_{II}\) as volume is changed from \(V_I\) to \(V_{III}\) represented as the horizontal line of A-II-B, then another sudden jump from \(p_2\rightarrow p_5\) as the vertical line of B-III. Hence, the dynamic process represented by
I→A→II→B→III is a non-continuous process in contrast to the continuous curve under the quasi-static assumption. Furthermore, only at states I, II and III are the gas pressure and temperature inside the volume uniform and can be represented as an equilibrium point on the P-V diagram in Figure 7. The rest of the process is dominated by a discontinuity caused by the moving shock and can only be represented as dotted line on the P-V diagram. The piston work in this case is the area under the constant $p_2$ line and can be calculated as:

$$\text{Piston Work} = p_2 \cdot (V_1 - V_{III})$$  \hspace{1cm} (10)

To plot a non-quasi-static process on a P-V diagram, we could connect point’s I-II-III with an approximated quasi-static continuous curve in such a way that the area under this curve is the same as the area under the constant $P_{II}$ line, that is, $\text{Work}_{\text{quasi-static}} = \text{Work}_{\text{dynamic}}$. This equivalent thermodynamic curve of a transient dynamic piston compression is shown as the solid curve in Figure 7. The difference between this curve and the constant $p_2$ line, as shown as shaded areas KE_A and KE_B, are then equal to the kinetic energy of the compressed gas according to the law of energy conservation, note that gas movement is ignored under the quasi-static postulate.

For a piston velocity distribution that can be divided into multiple velocity impulses $U_j$, multi-stage dynamic compressions, according to the stage cycle $\Delta t_j$ calculated from equations (5-8), this results in a saw-shaped line on top of an approximated quasi-static curve on the P-V diagram as shown in Figure 8. The ripples on the curve are an indication of ongoing dynamic energy exchange between the piston and the gas carried out by moving shock waves. In the extreme case of infinitesimal $U_j$, the saw-shaped line merges into a continuous compression curve, a true quasi-static process.

Figure 7: Dynamic compression stage in comparison with quasi-static compression

Figure 8: Dynamic compression in comparison with quasi-static compression

Figure 9: Under compression: conventional backflow compression vs. shock wave compression

Figure 10: Comparison of compression processes: Hugoniot (shock) vs. isentropic
2.6 Transient Dynamic PD Compression vs. Under-Compression (UC)
For PD compressor, when the discharge pressure is below the system back pressure, an under-compression condition takes place that is traditionally represented as a sudden pressure rise on a P-V diagram as shown in Figure 9a, or an isochoric process. The corresponding theory uses backflow compression as a mechanism for UC as shown in Figure 9b. However, a different explanation based on an analogy with the diaphragm shock tube is proposed by Huang (2012a) arguing that it is actually a shock wave generated by the sudden opening of a pressure difference that compresses the gas during the transient under-compression process. It further demonstrates that the thermodynamic process of UC is an adiabatic process in nature inherent with shock compression. It is interesting to briefly compare the shock compression in UC with the shock compression by a high speed piston. In essence, both are achieved by the shock waves while differing only on the means the shocks are generated, with one by a sudden opening of a pressure difference and the other by an impulsive piston movement. Both methods generate moving shock waves that obey the same laws as can be found from Anderson (1982). However, a side-effect of the shock compression for UC is the associated gas pulsations that are, in essence the expansion waves and induced gas flow generated simultaneously as the shock compression wave. The interested readers can refer to Huang (2012b, 2014) for the gas pulsation mechanism and the control method by a pulsation trap.

2.7 Rules for Transient Dynamic Stage of PD Compression
For the convenience of effectively using the analytical results of the shock tube theory, the following Rules for transient dynamic stage of a piston compression are summarized. In principle, these rules are applicable to different gases and for different types of PD compressors:

1. Rule I: if a piston velocity impulse relative to the speed of sound inside a cylinder is small (U/a < 5%), then piston compression can be approximated as a quasi-static process as in thermodynamics;
2. Rule II: If the piston velocity impulse U/a > 5%, then aerodynamic effects become significant, resulting a transient dynamic stage of a piston compression consisting of a rotor with ISW compression and a stator with RSW compression during a stage cycle of \( \Delta t \) with their corresponding magnitudes as follows:
   
   \[
   \text{ISW} = p_2 - p_1 = \rho_1 W_{\text{i}} U \\
   \text{RSW} = p_5 - p_2 = \rho_2 W_{\text{r}} U \\
   \Delta t = \frac{L}{W_{\text{i}}} + \frac{(L - X_{\text{III}})}{W_{\text{r}}} 
   \]

3. Rule III: ISW is an action force from piston to gas while RSW is a reaction force from cylinder, end wall, to gas. They have the same magnitude in terms of pressure ratio but move in opposite direction.

Rule I is about the static condition when the conventional quasi-static postulate is true and thermodynamic laws govern. For air with initial condition of atmosphere, this velocity is about 17-20 m/s. Rule II is about the transient dynamics of PD compression; that is, the relationship between piston motion and forces generated from piston and cylinder on the gas. It also points out that the basic dynamic stage of a PD compression is defined by its dynamic energy transfer process; first imparting kinetic energy to the gas and increasing its pressure and then converting that gas kinetic energy to further increasing its pressure. Moreover, it indicates the means through which the energy is transferred are the moving shock wave. Rule III shows further the moving shock waves, or the moving forces, are in pairs so that for every ISW, there is always an equal but opposite RSW in terms of pressure ratio. Moreover, ISW is functioning like a rotor that imparts kinetic energy into the gas and raises its static pressure by \( p_2 - p_1 \), while RSW is functioning as a stator that further converts the gas kinetic energy into more static pressure by \( p_5 - p_2 \). The details and implications of these Rules are to be explored in their application to case studies next.

3. CASE STUDY - APPLICATION OF PD DYNAMIC COMPRESSION RULES

3.1 Case Study –Single Stage Transient Dynamic PD Compression
The application of the above Rules for the transient dynamic stage is explored through case studies. The purpose is to illustrate the underlining physics and provide quantitative guidelines by using a convenient on-line Shock Tube Calculator from WiSTL Gas Dynamics Lab. Table1 calculates, for a single stage, for known piston velocity impulse and initial condition, the gas pressure and temperature rises in the rotor and stator. In addition, transient dynamic variables such as gas and shock wave velocities are also calculated. The dynamic rotor & stator time and stage cycle
are obtained by Equations (5-8). The rotor adiabatic efficiency, or Hugoniot compression efficiency, is also calculated with respect to an ideal isentropic process by:

$$\eta_{Hugoniot} = 100 \times \frac{T_1}{\eta_{isentropic}} \left[ \frac{(p_2/p_1)^{(k-1)/k}}{(T_2/T_1)} \right]$$  \hspace{1cm} (14)

Figure 10 shows the Hugoniot compression efficiency at different pressure ratios and Hugoniot compression curve, P-V relationship, that is the locus of all possible pressure-volume conditions behind a single normal shock of various strengths from one specific initial condition. The bottom four rows show the calculated rotor and stator pressure rises and pressure ratios of ISW and RSW compression in order to partially validate the Rule III.

<table>
<thead>
<tr>
<th>Table 1: Calculation for single stage transient dynamic PD compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(For air at initial gas condition: (T_1=300) K, (p_1 = 1.0) bar and cylinder stroke length (L=0.2) m)</td>
</tr>
<tr>
<td>(U, \text{ m/sec, Impulsive piston velocity or induced gas velocity})</td>
</tr>
<tr>
<td>(W_i, \text{ISW, Mach# / m/s})</td>
</tr>
<tr>
<td>(p_{r2}, \text{bar, rotor outlet pressure})</td>
</tr>
<tr>
<td>(T_{r2}, \text{K, rotor outlet temperature})</td>
</tr>
<tr>
<td>(\Delta t_{rotor}, \text{mSec, rotor time})</td>
</tr>
<tr>
<td>(\eta_{Hugoniot}, \text{%, rotor efficiency (relative to isentropic process)})</td>
</tr>
<tr>
<td>(W_r, \text{RSW, Mach# / m/s})</td>
</tr>
<tr>
<td>(p_{s2}, \text{bar, stator outlet pressure})</td>
</tr>
<tr>
<td>(T_{s2}, \text{K, stator outlet temperature})</td>
</tr>
<tr>
<td>(\Delta t_{stator}, \text{mSec, stator time})</td>
</tr>
<tr>
<td>(\eta_{stage}, \text{%, stage efficiency (relative to isentropic process)})</td>
</tr>
<tr>
<td>(\Delta t_{stage}, \text{mSec, stage cycle})</td>
</tr>
<tr>
<td>(p_{s}/p_{r}, \text{stage pressure ratio})</td>
</tr>
<tr>
<td>((p_{r2}-p_{r1})/(p_{r2}-p_{r1}), %, stage reaction)</td>
</tr>
<tr>
<td>(p_{r2}-p_{r1}, \text{rotor pressure difference})</td>
</tr>
<tr>
<td>(p_{s2}-p_{r2}, \text{stator pressure difference})</td>
</tr>
<tr>
<td>(p_{s2}/p_{r2}, \text{rotor pressure ratio})</td>
</tr>
<tr>
<td>(p_{s2}/p_{s2}, \text{stator pressure ratio})</td>
</tr>
</tbody>
</table>

From Table 1, it can be seen that the higher the piston velocity, the more dynamic the compression will be, characterized by more kinetic energy and shorter cycle time. Up to \(U=66\) m/s, the shock wave compression has an efficiency of 99%, almost same as an ideal isentropic compression based on quasi-static process. Even at \(U=150\) m/s, the efficiency is close to 96%, corresponding to a rotor pressure ratio of 1.784 and stage pressure ratio of 3.042. As piston velocity further increases, the shock loss increases faster, say at \(U=360\) m/s, the efficiency drops to 83%. Figure 10 shows the efficiency and volume curve, also called Hugoniot curve, of a single stage shock wave compression with different pressure ratios with respect to an isentropic compression. Hence, for the same total pressure ratio, a multi-stage shock compression is always more efficient than a single stage. For example, one-stage dynamic PD compression with \(U=360\) m/s has a stage efficiency of 83% while a three-stage compression with \(U=111\) m/s has a stage efficiency around 98%. It is also interesting to compare shock compression efficiency with a traditional dynamic stage of an axial compressor. For example, an one-stage dynamic PD compression with \(U=111\) m/s and stage pressure ratio of 2.32 is considerably more efficient (98.2%) than a corresponding stage of a high speed axial compressor that is typically around 90%. In addition, no blades are needed and the piston velocity is also well below the tip velocity of modern impeller that is around 300-400 m/s. At the same time, the dynamic effects from the shock wave make the process more and more non-linear demonstrated by the bottom 4 rows that show a more equal pressure ratio rotor/stator behavior instead of an equal pressure difference. In other words, the stage reaction is decreasing from ideal 50% for small disturbance or a linear weak shock wave compression. The equal pressure ratio rotor/stator behavior works well, less than 5% deviation, up to \(U=150\) m/s that is well above the peak.
piston velocity of today. This also partially validates Rule III.

3.2 Case Study – Multi Stage Transient Dynamic PD Compression

Table 2 is another example illustrating the calculation procedure for a multi-stage dynamic PD compression. The piston velocity distribution is assumed known and segmented into 5 velocity impulses or stages as exemplified in Figure 4b. The calculation starts from the 1st stage with known gas type and initial pressure temperature. It continues to the 2nd stage, taking the 2nd stage inlet pressure and temperature from the 1st stage outlet pressure and temperature. This process continues until a desired final pressure is reached. The purpose of this case study is to demonstrate the sequence of calculation using the same tools as the above single stage. From Table 2, it can be seen that even for piston velocity ranging 17-65 m/s; only 5 stages are required to reach a pressure ratio of 5.5 while the dynamic compression remains very efficient above 98%. In other words, the shock losses are low as long as stage pressure ratio is below 2.

Table 2: Calculation for multi stage transient dynamic PD compression (For air with MW=28.97, k=1.4)

<table>
<thead>
<tr>
<th>“Stage” #</th>
<th>1st stage</th>
<th>2nd stage</th>
<th>3rd stage</th>
<th>4th stage</th>
<th>5th stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1, bar, rotor inlet pressure</td>
<td>1.0</td>
<td>1.146</td>
<td>1.634</td>
<td>2.629</td>
<td>3.842</td>
</tr>
<tr>
<td>T1, K, rotor inlet temperature</td>
<td>300</td>
<td>312</td>
<td>346</td>
<td>397</td>
<td>443</td>
</tr>
<tr>
<td>U, m/sec, Impulsive piston velocity</td>
<td>17.1</td>
<td>45.1</td>
<td>65.5</td>
<td>55.6</td>
<td>54.2</td>
</tr>
<tr>
<td>W1, ISW, Mach# / m/s</td>
<td>1.03/358</td>
<td>1.081/376</td>
<td>1.111/414</td>
<td>1.087/435</td>
<td>1.08/456</td>
</tr>
<tr>
<td>p2, bar, rotor outlet pressure</td>
<td>1.071</td>
<td>1.372</td>
<td>2.081</td>
<td>3.186</td>
<td>4.588</td>
</tr>
<tr>
<td>T2, K, rotor outlet temperature</td>
<td>306</td>
<td>329</td>
<td>371</td>
<td>420</td>
<td>466</td>
</tr>
<tr>
<td>W2, RSW, Mach# / m/s</td>
<td>1.03/344</td>
<td>1.079/346</td>
<td>1.107/362</td>
<td>1.085/390</td>
<td>1.078/456</td>
</tr>
<tr>
<td>p3, bar, stator outlet pressure</td>
<td>1.146</td>
<td>1.634</td>
<td>2.629</td>
<td>3.842</td>
<td>5.455</td>
</tr>
<tr>
<td>T3, K, stator outlet temperature</td>
<td>312</td>
<td>346</td>
<td>397</td>
<td>443</td>
<td>490</td>
</tr>
<tr>
<td>ηHugo, %, compressor efficiency</td>
<td>99.3</td>
<td>98.2</td>
<td>98.4</td>
<td>98.4</td>
<td>98.5</td>
</tr>
<tr>
<td>p5/p1, compressor pressure ratio</td>
<td>1.146</td>
<td>1.634</td>
<td>2.629</td>
<td>3.842</td>
<td>4.588</td>
</tr>
</tbody>
</table>

The sequence of the calculation also resembles the multi-stage dynamic type compressors such as centrifugal or axial by starting from the first stage and ending at the last stage. However, there is no stage performance curve needed. In its place are the invisible moving shock waves whose characteristics are determined by Hugoniot equation (1) that is in turn related to piston velocity. In other words, the intangible shock waves in place of tangible blades of conventional dynamic compressors are adaptable to different piston speeds without hardware restriction from geometry, surface friction and material strength.

4. CONCLUSIONS

High speed PD compression is difficult to model dynamically yet deserves the attention due to its importance to the widely used modern internal combustion engines and PD compressors. So an indirect approach is attempted in this paper by analogizing the shock tube theory to the transient process of a dynamic piston compression. By this idealized yet revealing method, some insights are gained into the transient dynamics of a piston compression:

1. High speed piston compression may not be an exact quasi-static process as theorized today, but more likely a transient dynamic process by actions of a pair of moving shock waves, ISW and RSW, obeying Hugoniot compression law.
2. The work done by the high speed piston may not be directly converted into gas static pressure and internal energy. Instead, the piston triggered moving shock waves are the intermediary driving forces that first induces gas movement, a kinetic energy, and pressure rise with ISW and then converts the gas kinetic energy into more pressure rise by RSW.
3. Dynamic piston compression process and dynamic variables such as gas kinetic energy and strength of the shock waves can be calculated analytically or numerically by the shock tube theory from known piston velocity distribution and initial gas parameters.
For the convenience of practical applications, transient dynamic Rules of PD compression are summarized and demonstrated through case studies. It has not escaped our notice that three Rules somehow resemble the three Newton’s Laws of Motion. This could be interpreted as one form of the manifestation of Newton’s Laws of Motion in its application to the shock tube and its subsequent application to the transient PD compression phenomena.

Finally, we hope that following this simple analogous analysis, more research can be directed in this direction so that more realistic and accurate CFD models can be established and the full potential of shock wave compression can be explored for its high efficiency and absence of hardware blades. More importantly, experiments with transient measuring capabilities should be carried out to validate the proposed theory. With a better understanding of the transient dynamic PD compression, it is anticipated that future generations of internal combustion engines and PD compressors can be designed to be even more reliable and predictable than today.

**NOMENCLATURE**

- **a** speed of sound
- **E** gas internal energy
- **ISW** incident shock wave
- **k** ratio of specific heat
- **KE** gas kinetic energy
- **L** cylinder stroke length
- **M** Mach number
- **p** absolute gas pressure
- **RSW** reflected shock wave
- **t** time
- **T** absolute temperature
- **U** piston or induced gas velocity
- **V** cylinder volume
- **W** shockwave velocity
- **X** piston position
- **ρ** gas density
- **Subscripts**
  - **i** incident shock wave
  - **r** reflective shock wave
  - **1** initial gas state in shock tube
  - **2** gas state after incident shockwave
  - **5** gas state after reflective shock wave

**REFERENCES**


**ACKNOWLEDGEMENT**

The author would like to thank Sean Yonkers for proof reading and for providing some of the figures and piston velocity data. A special thank-you note is in debt to WiSTL Gas Dynamics Lab for its convenient on-line Shock Tube Calculator.