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Experimental and Numerical Optimization of a Variable-Geometry Ejector in a Transcritical CO₂ Refrigeration Cycle

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ABSTRACT

Implementation of an ejector for expansion work recovery in transcritical carbon dioxide (CO₂) cycles provides an opportunity to improve the efficiency of these environmentally-friendly refrigeration systems. However, literature outlining an approach to ejector design for a given application is lacking. This paper presents a tool to design a complete ejector applied in a vapor compression cycle. In this work, the developed design tool was validated using experimentally-derived polynomials for air-conditioning conditions. Then, constant values for nozzle and mixing section efficiencies were used as inputs into design tool to broaden the analysis outside of the application boundaries of the experimentally-derived polynomials to study a transcritical CO₂ system with an ejector operating in the evaporating temperature and gas cooler pressure in the range of -15 °C to 20 °C and 80 bar to 110 bar, respectively. The design tool allows for the calculation of the motive and suction nozzle throat diameters, the mixing section diameter, and the diffuser outlet diameter, as well as the lengths of each section, to output a full internal geometry of the ejector based on performance requirements. Individual component sub-models are presented within the proposed model structure. The model which forms the basis of the design tool was experimentally validated with a mean absolute error (MAE) between 3% to 4%. Additionally, the sensitivity of the ejector geometry and performance to component efficiencies, operating conditions, and component versus system optimization was investigated. The optimization and parametric studies provided novel insights into the impact of desired efficiency and operating conditions on ejector geometry, thus allowing a designer to make decisions based on the tradeoff between ejector size and performance. For example, as the diffuser length increased by 5.1 mm to obtain an efficiency increase, to obtain a further efficiency increase of the same amount would require a 17.1 mm length increase in diffuser length. Potential model improvements and other future work are also discussed.

1. INTRODUCTION

In an effort to develop environmentally-friendly refrigeration solutions, natural refrigerants such as carbon dioxide (CO₂) have gained significant attention in the past 25 years. However, the combination of a high critical pressure and a low critical temperature of CO₂ often results in the need for transcritical operation, which requires more compressor input power than a subcritical vapor compression cycle due to cycle thermodynamics. This then results in a lower coefficient of performance (COP) relative to cycles which utilize synthetic refrigerants. Using an ejector to recover a portion of the expansion work available is one example of a modification that has been proposed and validated to increase the efficiency of transcritical CO₂ refrigeration cycles. Since the primary function of an expansion device in a vapor compression cycle must be cycle control, variable geometry and multi-ejector concepts have received the most focus in recent years due to their ability to function over a range of operating conditions effectively (Elbel & Hrnjak, 2008; Hafner et al., 2014; Haida et al., 2016).

With respect to analysis of ejectors, there are a number of ways to conduct design and performance characterization. In order to balance numerical complexity with accuracy, many numerical 0-dimensional and 1-dimensional analysis approaches have been conducted and experimentally validated. Lucas and Koehler (2012) conducted experimental analyses on the performance improvement of a transcritical CO₂ refrigeration cycle with an ejector relative to a cycle using isenthalpic expansion, achieving an ejector efficiency of 22% and a COP increase of 17%. This earlier experimental work led to the insight that the motive nozzle throat became choked in transcritical operation, and also provided a simple but effective numerical modeling strategy. In a continuation of the CO₂ ejector work, Lucas et al. (2013) developed a simplified 0-dimensional numerical model that provided a correlation for overall ejector efficiency as a function of operating conditions, the motive throat diameter, and the mixing section diameter. Despite the lack of
component sub-models, the correlation was able to predict ejector efficiency within 10% of experimental data. Liu and Groll (2012) investigated the potential performance benefit of a variable geometry ejector in a transcritical CO₂ air conditioning cycle, achieving COP improvements of up to 36% at high ambient temperatures, and also provided experimental insights into how varying internal dimensions, such as the motive and suction nozzle effective diameters, would impact the overall ejector performance. Liu and Groll (2013) furthered this work through a 0-dimensional analysis conducted on experimental data to assess the impact of operating conditions and geometry on sub-component efficiencies. The numerical analysis of calculating these sub-component efficiencies facilitated the development of polynomials to predict the motive nozzle, suction nozzle, and mixing section efficiencies as a function of geometry and operating conditions. When these ejector component efficiency correlations were utilized in a full cycle model, COP and cooling capacity predictions were achieved within 8% and 12% of experimental data, respectively.

Despite the amount of quality design, validation, and performance assessment work that has been conducted in the existing literature on ejectors applied in refrigeration cycles, discussion about the specific designs utilized is often secondary to the analysis conducted. Furthermore, there is limited work in the literature regarding the explicit design approach of an ejector using first-principle analytical models. The effects of component efficiencies and operating conditions on geometry is also lacking in the literature. Therefore, to facilitate informed design decisions to broaden the accessibility of ejector development without requiring significant computational power, there is a need to develop a high fidelity, 0-dimensional model to study the internal ejector geometries required to achieve a desired performance for a broad array of applications. This paper presents an ejector design tool that has been experimentally validated for a transcritical CO₂ vapor compression cycle but can be readily extended to other working fluids. The design tool was integrated into a full system model to assess the relative effects of individual component efficiencies as well as operating conditions on overall ejector efficiency and geometry. Differences in ejector geometry and efficiency when optimizing the ejector versus the entire cycle are presented. Further details on this model can be found in Barta et al. (2021). Lastly, potential model improvements and perspectives on future work are discussed.

## 2. APPROACH

This section describes the sub-models, governing equations, and overall system design used throughout this paper. For visualization, the internal processes of an ejector are shown in Figure 1 and in a pressure-specific enthalpy (P-h) diagram for a transcritical CO₂ cycle in Figure 2. Ejector operation begins at the motive nozzle inlet, $mi$, where the high-pressure flow from the gas cooler outlet is accelerated to the motive nozzle throat, $mb$. This high-velocity motive flow then entrains low-pressure vapor from the evaporator outlet into the suction nozzle inlet, $si$. The suction nozzle slightly accelerates this vapor into the throat of the suction nozzle, $sb$. Here, the two flows enter the suction chamber and begin to mix at the entrance to the circular, constant area mixing section, where they increase in pressure due to the mixing process as they reach the end of the mixing section, $mix$. Finally, the flow enters the diffuser where the flow pressure is further increased until exiting the ejector at a two-phase state, $d$.

![Figure 1: Ejector components and internal states.](image1.png)

![Figure 2: Ejector process in a P-h diagram.](image2.png)

### 2.1 Sub-Component Modeling Strategy

Four primary sub-component models are utilized in the overall ejector model, which are the motive nozzle, suction nozzle, mixing section, and diffuser. The fundamental equations are a combination of those utilized by Li and Groll (2005) and Liu and Groll (2013). As the motive nozzle was modeled as a converging nozzle, the maximum velocity...
at the outlet of the motive nozzle is limited by choked flow, which is achieved when the average flow velocity reaches the speed of sound. The method for estimating the speed of sound applied in this paper is adapted from Attou and Seynhaeve (1999). All components are assumed to be adiabatic and operate at steady-state, steady flow, with the nozzle work being zero due to it being a passive device. Additionally, the inlet velocity to both nozzles and the outlet velocity of the diffuser are assumed negligible, given the small impact those respective velocities would have on the first law. The equations applied for the isentropic efficiency of a nozzle, and the subsequent calculation of the nozzle outlet velocity, are shown in Equations 1 and 2, respectively. These equations are applied to both the suction and motive nozzles.

\[ \eta_{\text{is nozzle}} = \frac{h_{\text{in}} - h_{\text{out}}}{h_{\text{in}} - h_{\text{out,ts}}} \]  

(1)

\[ V_{\text{out}} = \sqrt{2(h_{\text{in}} - h_{\text{out}})} \]  

(2)

The convergence criterion for the choked flow motive nozzle model is to match the velocity resulting from the energy balance to the speed of sound. For the suction nozzle, similar isentropic efficiency and velocity equations are applied, but in this instance conservation of mass is used to estimate the outlet velocity. This is performed using Equation 3, and the area is calculated using a fixed ratio of throat diameter relative to the motive nozzle. This is achieved through the introduction of a pseudo-mass flow rate that is used to calculate ratios throughout. For example, assuming a motive mass flow rate of 1 kg s\(^{-1}\), the entrainment ratio is then used to solve for the suction mass flow rate, shown in Equation 4. Then, using Equation 5, a diameter for the motive nozzle is applied to determine the proposed suction nozzle diameter. With both an area and a mass flow rate known, Equation 3 is used to compare the outlet velocities from the mass flow rate and the energy balance to form the convergence criterion for the suction nozzle.

\[ \dot{m} = \rho Av \]  

(3)

\[ W = \frac{\dot{m}_{\text{suction}}}{\dot{m}_{\text{motive}}} \]  

(4)

\[ \frac{d_{\text{mb}}}{d_{\text{sb}}} = \text{Constant} \]  

(5)

Mixing section losses are primarily due to fluid flow friction in the mixing section and have the effect of decreasing the outlet velocity and the potential pressure lift across the ejector. The mixing section efficiency, \( \eta_{\text{mix}} \), accounts for the momentum losses in the mixing section. The mixing section efficiency is essentially applied as a scalar to reduce the kinetic energy contributions of the motive and suction nozzle flows to the mixing section outlet velocity in the conservation of momentum. Since both conservation of mass and energy are satisfied with the motive and suction nozzle models, the difference in velocity outputs of momentum and energy balances for the mixing section is used as the convergence criterion. For the former, conservation of momentum is manipulated to produce an expression for outlet velocity, shown in Equation 6. The energy balance is conducted by defining both nozzles and the mixing section as the control volume and utilizing both nozzle inlet states as the inputs, shown in Equation 7. This minimizes the propagation of error through the nozzle outlet velocity calculations in the mixing section velocity calculation.

\[ V_{\text{mix}} = \sqrt{2\left(\frac{h_{\text{mi}} + \eta_{\text{si}}}{1 + w} - h_{\text{mix}}\right)} \]  

(6)

\[ V_{\text{mix}} = \sqrt{\frac{1}{\rho_{\text{mix}}}} \left[ \alpha_{\text{mb}}(P_{\text{mb}} + \eta_{\text{mb}} \rho_{\text{mb}} V_{\text{mb}}^2) + \alpha_{\text{mix}}(1 + w) - \alpha_{\text{mb}}(P_{\text{mb}} + \eta_{\text{mix}} \rho_{\text{sb}} V_{\text{sb}}^2) - P_{\text{mix}} \right] \]  

(7)

where \( \alpha \) represents the inverse of mass flux, calculated as the ratio of specific volume to velocity. This value provides the bridge from intensive thermodynamic properties to physical dimensions. Because the units are area over mass flow rate, multiplying this value by a given mass flow rate returns the area necessary for the flow to occur.

The diffuser performance is calculated as presented by Liu and Groll (2013), with the only difference being the area ratio is calculated by a series of constant diameter ratios and angles. The diffuser lift coefficient, \( C_L \), is calculated through Equation 8, with the corresponding pressure lift calculated by Equation 9. The overall ejector efficiency is
calculated using Equation 10 from Koehler et al. (2007) and represents the ratio of recovered specific work to the recoverable specific work across the motive nozzle and is scaled by the entrainment ratio. It should be noted that the two ratios involving motive and suction nozzle diameters were varied between tests used for validation. The geometric values applied in this analysis correspond to the geometries that yielded the highest ejector efficiencies in the Liu and Groll (2008) investigation. The use of these ratios, angles, and the inverse mass flux is the key to transitioning these governing equations from an analysis to a design tool, as all that is necessary to go from an intensive property to an area is multiplication of $a$ at each key point by the mass flow rate through that portion of the ejector. Then, application of the ratios and angles are applied to find the remaining lengths.

$$C_t = 0.85 \frac{\rho_{mix}}{\rho_{mix}} \left[ 1 - \left( \frac{A_{mix}}{A_d} \right)^2 \right] \left[ \frac{x_{mix}^2 + \left( 1 - x_{mix} \right)^2}{\rho_{g,mix}} \right]$$  \hspace{1cm} (8)

$$C_t = \frac{\rho_{mix} - \rho_{mix}}{2\rho_{mix}^2}$$  \hspace{1cm} (9)

$$\eta_{ejector} = \frac{W}{h_{in} - h_{out}} = \frac{f(P_{d,m}^2) - f(P_{d,m}^2)}{f(P_{m}^2) - f(P_{d,m}^2)}$$  \hspace{1cm} (10)

### 2.2 Cycle Description

A schematic of the cycle used in the system analysis is shown in Figure 3. The two primary differences between this cycle and a four-component vapor compression cycle are the utilization of ejector and the phase separator. As the diffuser outlet state will be two-phase, the phase separator is necessary to ensure that the compressor suction receives saturated vapor. Furthermore, in reality, many transcritical CO$_2$ cycles utilize a semi-hermetic reciprocating compressor, where the suction flow enters the compressor and flows over the motor before entering the compression chamber. This cools the motor and superheats the vapor before it enters the suction of the compression chamber, which makes this architecture more robust in actuality than it may appear. Additionally, the bypass from state 1 to state 6 is there to provide additional control should instabilities arise during ejector operation.

![Figure 3: Schematic of a cycle for optimization.](image)

### 3. NUMERICAL DESCRIPTION AND SOLUTION SCHEMES

In this section, numerical strategies for solving the ejector model as well as a complete cycle with an ejector are discussed. First, two general numerical strategies are discussed for the standalone ejector model. The first strategy is used in the validation of the ejector numerical model with experimental data and physical geometry, while the second is for applying the same fundamental model as an ejector design tool. Then, a numerical strategy for a complete cycle analysis with an integrated ejector along with an approach for optimization is presented. The overall solution scheme for the ejector model receives inputs of the two inlet states, characterized by temperature and pressure for these single-phase states, and the outlet state, which is defined by pressure and an entrainment ratio. The entrainment ratio is calculated based on the ratio of suction to motive mass flow rates, which then facilitates the calculation of different ejector geometric parameters.
In this solution scheme, first, the suction and motive nozzle throat pressures are iterated to satisfy the convergence criterion defined based on throat velocities as discussed in Section 2.1. Following the convergence of both nozzles, the specific enthalpy and pressure at the outlet of the mixing section are iterated to achieve conservation of momentum, and conservation of energy, respectively. The flowchart utilized for solving the ejector model for validation as well as design is shown in Figure 4. Both solution schemes solve the motive nozzle, suction nozzle, and mixing section outlet states sequentially, in that order. The two main differences between the two solution schemes are the convergence criteria for mixing pressure ($P_{mix}$) iterations and the component efficiencies utilized.

3.1 Numerical Strategy for Validation
The validation solution entails employing governing equations, experimentally-derived efficiency polynomials, and experimental data from Liu and Groll (2013) to calculate ejector geometries used to achieve a given efficiency for given operating conditions in experimental data. The experimental data set used in this validation was the same as was utilized for the development of ejector component efficiency polynomials by Liu and Groll (2008). The parameters for the validation of the ejector modeling and solution approach are the diameters of the motive nozzle throat, suction nozzle throat, mixing section outlet, and diffuser outlet. Using the solution scheme shown in Figure 4, the ejector inlet and outlet states are matched as closely as possible to the experimental data. For model validation, the convergence criterion for mixing pressure ($P_{mix}$) iterations is agreement between calculated and experimental diffuser outlet pressures, as shown in Equation 11. The mass flow rates from the experimental data are utilized in the model to calculate the four primary diameters within the ejector.

$$|P_{d,\text{calc}} - P_{d,\text{data}}| < \text{tol?}$$

(11)

3.2 Solution Scheme for Design
The overall numerical structure of the design scheme shares the solution scheme shown in Figure 4. However, for the design solution scheme, the convergence criterion for $P_{mix}$ iterations is defined based on the relationship between the diffuser outlet quality and the ejector entrainment ratio, given in Equation 12. The design scheme allows for the calculation of ejector geometric parameters and outlet states for given inlet states to the ejector, motive mass flow rate, and ejector component efficiencies. With respect to ejector component efficiencies, it has been shown that the polynomials used in the validation scheme are more accurate than constant efficiency assumptions, given the logical conclusion that a nozzle, mixing section, and diffuser outlet pressures will vary for different operating conditions and entrainment ratios. However, those polynomials were derived from air conditioning application testing data, limiting them to somewhat high evaporation temperatures.

$$|x_d(1 + w) - 1| < \text{tol?}$$

(12)

This ejector model utilizes the same governing equations regardless of whether constant or variable ejector component efficiencies are applied. This broadens the applicability of this model to most vapor compression cycles. Furthermore, the relationship between the ejector outlet quality and entrainment ratio can be modified in the case of more advanced cycles. For example, if the phase separation process at the diffuser outlet has more than the standard two outlets more complex expressions can replace the simple convergence criterion at the diffuser outlet, and as long as the fundamental definitions of quality and entrainment ratios are satisfied, the model will still be able to estimate the mixing section pressure.

3.3 Cycle Analysis and Optimization
To study the effects of operating conditions, individual component efficiencies, and other design parameters on the overall system and ejector performance, the ejector solver model is integrated into an overall system model. Furthermore, this model formulation can be used to perform optimization on design parameters for a target system performance as per user application. Figure 5 outlines a flowchart to solve the system with an integrated ejector as shown in Figure 3 for an array of operating conditions and design parameters. The idea behind the numerical solution scheme is to solve the different components either sequentially or simultaneously with additional constraints to ensure model convergence at different component interfaces. An example interface would be the outlet of the evaporator to the suction nozzle inlet. Here, in addition to ejector internal states shown in Figure 4, diffuser outlet pressure ($P_d$) and entrainment ratio ($w$) are iterated simultaneously to find a solution that satisfies the convergence criteria for the ejector as well as different component and cycle models.
In this study, a 10-coefficient compressor map was developed using data from the manufacturer and utilized to model a fixed speed compressor with a volumetric displacement rate of 1.75 m$^3$ hr$^{-1}$ at 60Hz. In this work, the primary focus was to study the system performance with an ejector, as well as how different operating conditions and ejector geometric design parameters affect the overall system and ejector performance. Therefore, a simple heat exchanger model was considered for the evaporator as well as for the gas cooler. Here it was assumed that the heat exchangers are sized properly to have constant pinch point, subcooling and superheat values with no pressure drop over a range of operating conditions. However, the user can implement more detailed models based on their application and design purpose.

### 4. RESULTS AND DISCUSSION

This section provides a summary and discussion of the primary findings from the presented analysis strategies. Validation is covered first, followed by a component sensitivity analysis. A parametric study showing component geometry variation with ambient and low-side conditions is then presented. As opposed to inputting a fixed geometry and operating conditions and outputting component efficiencies, as is done in many previous analyses on two-phase flow ejectors, the analyses conducted herein aim to input a fixed efficiency and an operating condition, then to output...
the associated geometry. Therefore, geometric variation with operating conditions can be isolated and assessed. Furthermore, keeping the sub-component efficiencies constant is no longer a simplification. This is due to the fact that the model utilized herein is of the same complexity as a model that could receive geometry and operating conditions and output variable efficiencies.

4.1 Experimental Validation
Validation of the model against experimental data was taken in two steps. First, individual component sub-model results were compared against experimental data, then the entire model was integrated within the numerical solution scheme. For the sub-model validations, experimental data for the motive and suction nozzles, including throat diameters, was used for validation. Then, the experimental data from nozzle outlets was applied as an input to the mixing section and diffuser models for validation. This allowed assessment of the geometry calculations for both nozzles as well as the mixing section and diffuser and facilitated isolation of individual sub-models for identification of any possible sources of error. The sub-model validation results are shown in Figure 6. Once confidence grew in the sub-models, the entire ejector model was run numerically. The model received only temperatures and pressures at the two nozzle inlets, pressure at the diffuser outlet, and the suction and motive mass flow rates from experimental data. With this data, the model calculated four primary component diameters, being the motive nozzle throat, suction nozzle throat, mixing section, and diffuser outlet, which are shown relative to the physical dimensions in Figure 7. The mean absolute error (MAE) for each simulated parameter relative to experimental data fell at or under 4% for all numerical assessments.

Figure 6: Individual components sub-model validations.
Figure 7: Validation of integrated model in estimating component diameters.

4.2 Trends of Component Efficiency and Geometry
It is vital to understand the relative impact of individual component efficiencies on the overall ejector efficiency and geometry when designing an ejector. Figure 8 shows the effects of varying three primary component efficiencies, being motive nozzle, suction nozzle, and mixing section efficiencies, on the overall ejector efficiency for transcritical CO₂ system operation. Here, only one of the component efficiencies was varied at a time while the other two were held constant at 0.75. Furthermore, operating conditions were kept constant with a gas cooler outlet pressure of 90 bar and outlet temperature at 30 °C. The evaporation temperature was held constant at a temperature of -5 °C with 5 °C pinch point and 5 °C superheat. The compressor was modeled to run at a fixed speed of 1750 revolutions min⁻¹. For ejector geometric parameters, the ratios of motive nozzle throat diameter to suction nozzle throat diameter and mixing section area to diffuser outlet area were also kept constant. The range of efficiencies was motivated by the literature (Fang Liu, 2014) as well as experimental data utilized in this investigation (F. Liu & Groll, 2008). COP is defined as the ratio of cooling capacity to compressor input power, as fan power was neglected. It can be seen that the mixing section losses have the most significant impact on the overall ejector efficiency with an almost-linear direct trend. On the other hand, changes in motive and suction efficiencies at lower values have a substantial effect on overall ejector efficiency. However, as these component efficiencies increase, the added benefit to overall ejector efficiency
decreases, which provides a designer an opportunity to prioritize maximizing the efficiency of the components with the most significant effect on overall ejector efficiency.

Figure 8: Ejector efficiency variation with component efficiency.

Figure 9: Ejector mixing section diameter variation with component efficiency.

Figure 9 shows the variation of mixing section diameter with component efficiencies for the same parametric study used in Figure 8. With respect to the mixing section diameter, the effects of the two nozzle efficiencies and mixing section efficiency can be related to the diameter through consideration of two-phase density. The actual mix state is the outlet of the mixing section, which is the portion of the section that is at the highest pressure because overall ejector pressure rise occurs primarily in the mixing and diffuser sections. Therefore, for a given mass flow rate, the outlet of the mixing section has the lowest density and thus, represents the smallest area which would satisfy conservation of mass for a given flow rate. With the relationship between density and area in mind, Figure 9 shows that the mixing section efficiency has an inverse relationship with mixing section diameter. This inverse relationship is logical, given that the mixing section efficiency primarily represents the adverse effects of friction in the mixing section, as shown in Equation 7. Therefore, the mixing section outlet velocity, pressure, and density are directly proportional to the mixing section efficiency and inversely proportional to the mixing section outlet diameter.

The motive nozzle efficiency shares a similar relationship to mixing section diameter because of its direct relationship to motive nozzle outlet velocity. The higher the motive nozzle outlet velocity is, the more effective the entrainment process and the more kinetic energy can be converted to pressure across the ejector. Additionally, a higher isentropic efficiency of a motive nozzle expanding from supercritical flow to subcritical flow would result in a higher outlet density, allowing more mass flow rate through the ejector for a given area, or a smaller area for a given mass flow rate. On the other hand, mixing section diameter varies directly with the suction nozzle efficiency. Because the suction nozzle area is solved as a ratio to the motive nozzle area in this model, that ratio is held constant when the suction nozzle efficiency is varied. Therefore, the outlet pressure must be varied to reach an outlet state that satisfies both conservation of mass and conservation of energy, shown in Equations 3 and 2, respectively, which results in an increase in suction nozzle outlet pressure with suction nozzle efficiency. An increased pressure at the outlet of the suction nozzle results in a higher density fluid, which allows more mass flow to be entrained when all other parameters are held equal and thus, increases the entrainment ratio shown in Equation 4.

A similar study was conducted for the motive nozzle throat diameter sensitivity to component efficiencies. It was concluded that the motive nozzle throat diameter is most sensitive to motive nozzle efficiency with an exponentially decreasing effect. The suction nozzle and mixing section efficiencies were found to have negligible effects on the motive nozzle throat diameter, which is primarily due to the choked condition at the throat of the nozzle.

4.3 Effects of Operating Condition on Performance and Geometry
A system usually operates over a wide range of operating conditions. Therefore, it is important to understand the effects of operating conditions on the overall system and ejector performance, as well as on ejector geometric parameters. A parametric study was performed for different operating conditions of a transcritical CO2 system based
on the numerical scheme outlined in Section 3.3. The gas cooler pressure was varied from 80 bar to 110 bar and the evaporating temperature was varied from -15 °C to 20 °C to simulate both refrigeration and air-conditioning system operation. The evaporator superheat and pinch point were kept constant at 5 °C. The gas cooler outlet temperature was fixed at 30 °C, with the compressor running at a fixed speed, similar to the study in Section 4.2. The ejector outlet pressure was a model output, and the flow from the ejector diffuser outlet to the compressor suction port was assumed to be isobaric. The number of parameters and conditions varied was limited to isolate the effects of varying certain parameters on ejector performance and geometry. The ratios of motive nozzle throat diameter to suction nozzle throat diameter and mixing section area to the diffuser outlet area were kept constant. Furthermore, the ejector component efficiencies were kept constant at nominal values with the motive nozzle at 0.7, suction nozzle at 0.6, and mixing section at 0.85, as motivated by the literature (Fang Liu, 2014).

Figure 10 illustrates the variation of ejector efficiency with gas cooler pressure and evaporating temperature. In general, ejector efficiency increases as the gas cooler pressure increases with a varying degree at different evaporating temperatures until almost becoming constant at an upper value. It would appear that this nearly-constant value is an optimum with a fairly broad plateau, after which the ejector efficiency decreases at a slow rate, as shown by the -15 °C evaporation temperature line. However, a broader gas cooler range would need to be applied to confirm this. The reason for the chosen gas cooler pressure upper limit is the outlet state of the motive nozzle would approach, and occasionally cross, the saturated liquid line, striking a numerical discontinuity (Barta et al., 2021). The gas cooling pressure at which the optimum occurs is directly proportional to the evaporation temperature.

Figure 11 shows the ejector system COP difference compared to a four-component system. To isolate the effect of the ejector, the performance of the system with an ejector is compared to a standard four-component system operating with the identical compressor at the same gas cooler and evaporator conditions. The COP benefit of the ejector has a maximum value associated with a certain gas cooling pressure, which is a result of the combined effects of change in the compressor as well as ejector performance at different conditions. As the gas cooler pressure increases past this maximum, the added benefit of the ejector on the system COP decreases. Moreover, at higher evaporating temperature conditions the ejector system performs poorer than the normal four-component system. This agrees with the well-reported concept of ejectors being beneficial in systems with higher temperature lift due to additional available expansion work. Additionally, four test points at low evaporation temperatures and high gas cooling pressures were outside the bounds of the compressor map utilized and hence were removed from this study.

Figure 12 shows the variation of design motive nozzle throat and mixing section diameter with gas cooler pressure and evaporating temperature for the same parameters used for Figure 10 and Figure 11. As the gas cooler pressure increases, the motive nozzle throat diameter decreases with more sensitivity to the variation at lower gas cooler pressures, which can be mainly attributed to the change in motive nozzle inlet conditions and motive nozzle mass flow rate due to the compressor volumetric efficiency change. The change in mixing section diameter shows a similar behavior at higher evaporating temperatures. However, the variation in mixing section diameter with gas cooler
pressure is less significant at the lower evaporating temperatures. One reason for this is the relatively small variation in motive nozzle mass flow rate, which is also the compressor mass flow rate, with a change in gas cooler pressure at lower evaporating temperatures. This highlights that the characteristics of other components in a system can greatly affect the ejector design and the need to carefully consider these characteristics in the design process in order to have optimum system performance at various operating conditions. A similar study can be extended to other operating conditions, such as ambient temperature, and with a more detailed heat exchanger model or variable speed compressor.

Figure 12: Ejector motive nozzle throat and mixing section diameter variation with operating conditions.

4.4 Effects of Geometry on Cycle Efficiency

In Section 4.3, the geometric parameter ratios of motive to suction nozzle throat diameter and mixing section to diffuser outlet diameter ratios were kept constant. In this section, the effect of these two ratios on the ejector as well as system performance are studied. Here, a parametric study of these two geometric ratios at a single operating condition was conducted. In this study, the ejector component efficiencies were kept constant at the same nominal values used in Section 4.3. For the parametric study, the operating conditions were kept similar to the ones used in Section 4.2, and only one geometric ratio was varied at a time while the other was kept fixed at 0.33. Figure 13 shows the variation in system COP and ejector efficiency with geometric ratios. The ratio of motive nozzle throat to suction nozzle throat diameter does not have a significant impact up to a certain value, and increasing the ratio beyond that suddenly decreases the system and ejector performance. This is due to the decrease in suction nozzle mass flow rate and decrease in pressure rise between the ejector and evaporating pressure. Increasing the mixing section to diffuser outlet diameter ratio decreases the system as well as ejector performance in an almost quadratic correlation because of the corresponding decrease in the diffuser pressure lift coefficient.

Figure 13: Ejector efficiency and system COP variation with geometric ratios.
5. CONCLUSIONS

This paper presented a design tool for two-phase flow ejectors applied to vapor compression cycles. Governing equations were presented and discussed, as was sub-model validation against experimental data, which resulted in an MAE between 3% and 4%. The developed design tool was validated using experimentally-derived polynomials at air conditioning conditions. Then, constant efficiencies for nozzles and the mixing section were used as inputs to the model to broaden the analysis to study a transcritical CO₂ system with an ejector operating in the evaporating temperature range of -15 °C to 20 °C and gas cooler pressure in the range of 80 bar to 110 bar. The ejector model was then integrated in a cycle model where the effects of varying ejector component efficiencies and operating conditions on ejector performance and geometric parameters were assessed. Novelty was achieved through primary consideration of geometry for varying operating conditions instead of efficiency for a fixed geometry, as well as through the insights provided into the tradeoff of performance versus geometry that can be used to inform design decisions. Meaningful trends were obtained, and physical explanations were provided. The gas cooling pressure where the maximum COP benefit from an ejector relative to a four-component cycle occurred was found to be lower than the gas cooling pressure where the maximum ejector efficiency occurred. It was also found that the cycle COP at higher evaporation temperatures could be hurt by applying an ejector. To quantify these observations, taking the ejector geometric ratio parametric study as an example, increasing the ejector efficiency from 19.9% to 20.8% at a given condition would require a diffuser length increase of 5.1 mm. To further increase the ejector efficiency from 20.8% to 21.7% would require a diffuser length increase of 17.1 mm. Therefore, the analysis conducted herein can offer quantification as to the diminishing returns on performance with geometry variation to provide sound background for decisions regarding ejector design. Future work should develop more comprehensive sub-models that can capture efficiency variation over a broad range of operating conditions, as well as to assess the results of the model using various two-phase speed of sound definitions to broaden the applicability of the model. Additionally, experimentally validating optimized designs through testing of a prototype is a next step in this work.

NOMENCLATURE

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<th>Symbol</th>
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<th>Acronyms</th>
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<td>a</td>
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<td>(kg s⁻¹ m⁻²)</td>
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<td>Correction Factor</td>
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<td>h</td>
<td>Specific enthalpy</td>
<td>(kJ kg⁻¹)</td>
<td>MAE</td>
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<tr>
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<td>Mass flow rate</td>
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<td>V</td>
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Greek symbols

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<th>Symbol</th>
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References


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