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ABSTRACT

The robustness evaluation of compressor components must take into account the variations in the mechanical properties of the material (such as Young's modulus and mass density), geometric parameters (as radius, thickness, length) and loads. Sometimes, the combination of these factors could result in the component failure. The correct way to assess the robustness of the component (considering the parameters variation) is through the statistical analysis using probabilistic design techniques. One methodology is the application of Monte Carlo method to generate random combinations of geometrical, loads and physical parameters that produce the same real component variation. This work shows the application of probabilistic analysis (Monte Carlo method) to estimate the scatter of resonance modes frequency of a discharge tube. The input parameters are the geometry, material properties and damping spring. The alternatives are analyzed by finite element method.

1. INTRODUCTION

Nowadays, the designs are becoming more and more challenging to engineers. Requirements are more complex and contradictory. According to Vlahinos (2002), the main requirements are related to: cost, performance, safety, quality, time to market, short life cycle, environmental impacts and aesthetics.

The question that arises is how to design a compressor, mainly related to mechanical components, which meet the aforementioned requirements. Despite all the scientific progress achieved in recent decades, the recommended scientific methodology follows the basic principles cited by Polya (1947) and adapted by Kececioglu *et al*. (1967) for mechanical components, which are:

- first step: state the problem
- second step: understand the problem
- third step: devise a plan of attack
- fourth step: carry out the plan
- fifth step: examine the solution - look back.

Simply put, this is nothing more than the famous PDCA (plan-do-check-act or plan-do-check-adjust), showed by Deming, that recently has become widely known through Six Sigma methodology, but which had its origins more than 60 years ago.

Figure 1 shows the development workflow of a mechanical component or system. The workflow follows the requirements mentioned above and the methodology suggested. Certainly, all the concepts involved in the workflow are available for a long time. What has changed dramatically in recent decades were the resources (mainly computers and equipment in general), and by extension, the CAX computational tools, that increased their performance exponentially. Today, the CAX tools make things that were impossible, or not feasible, to execute by the end of the last century.

In the past, when only simplified analytical models were available, the experimental played an important role in the development of projects. Typically, the process was time consuming, expensive, and the analysis scope was limited, since it was not possible to explore all the opportunities available and to assess their impact. Sometimes it is difficult to imagine how the development of a new housing for a compressor would be done, using just the experimental approach. It is common, during the development of a new housing, to create and test some dozens of virtual models (Bortoli, 1992). Knowing the cost and time needed to build a compressor housing stamping tool (bottom and cover), it is not difficult to evaluate the gains that the CAX tools had brought for this topic. At that time, experimental tools, like DOE, were fundamental to evaluate the most important factors and interactions among them. The popularization of CAE tools, like finite element and finite differences, caused a revolution in the analysis, increasing substantially its importance in the development of projects. These numerical tools are able to analyze more detailed and refined models, where alternatives can be evaluated accurately and fast.

When the optimization tools began to be used in numerical models, it was difficult to update the model geometry. It was common to choose only the input parameters for optimization with little impact on the geometry, such as shell thickness or beam section (Bortoli and Puff, 1998). The total integration of CAD and CAE environments was the remarkable fact in the last decade, which opened a new universe in design development, with extensive use of optimization tools and statistics. The solution approach changed from the deterministic to stochastic way, because the world is not nominal neither worst case.

2. ROBUST DESIGN

The design of any mechanical component is strongly influenced by its geometry and dimensions, mechanical properties of the material used, and the loads acting on it. In real application, there are not two components exactly the same, and the impact of variations on the mechanical component must be evaluated. Figure 2 shows a hypothetical example of one function F versus a design variable X (F could be the stress level, energy consumption, natural frequency value, etc). The engineer mission is, other than finding the optimal point, to evaluate the solution robustness. In the above example, the variation of the design variable, with nominal value at the optimal point, could lead to design failure. Another effect of the design variable variation is to increase the dispersion the function F. It is easy to observe that a small shift to the right of the design variable reduces the F function dispersion. It is important to notice that the variable dispersion stays the same. There is just a shift of its nominal value. This is a good example to illustrate that sometimes the optimum could not be robust.

Robust design could be defined as a design of a product so that its functionality varies minimally despite of disturbing factor influences. The goal of robust design (Vlahinos, 1999) is to deliver customer expectations at profitable cost, regardless of: customer usage, degradation over product life and variation of manufacturing, supplier, distribution and installation.

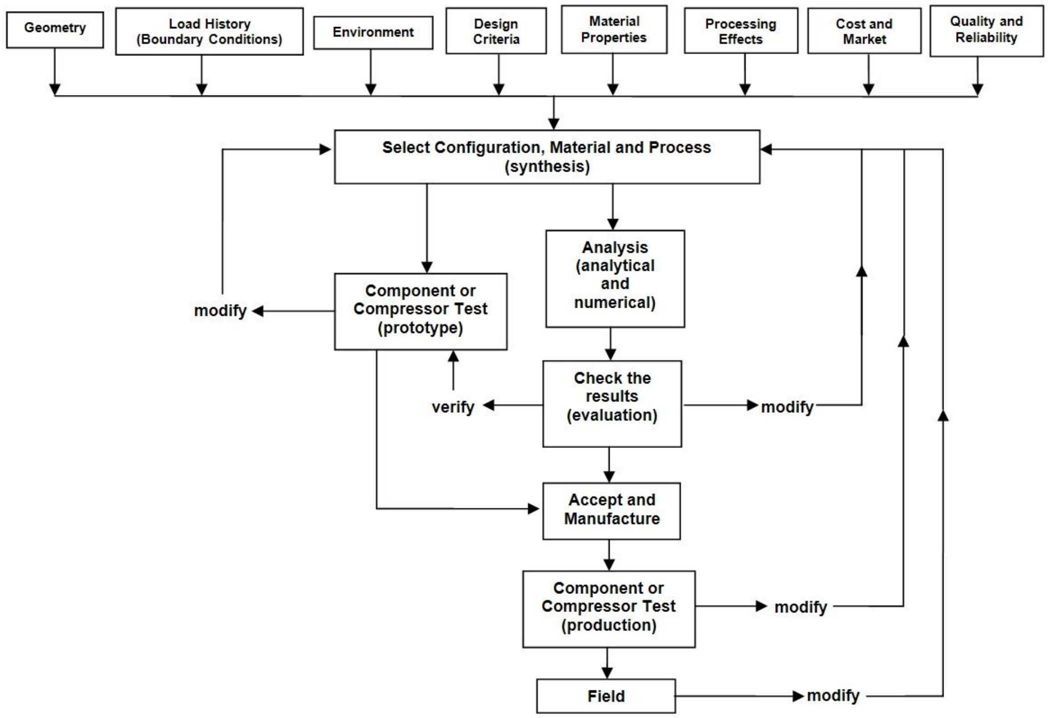


Figure 1: Component/system design workflow

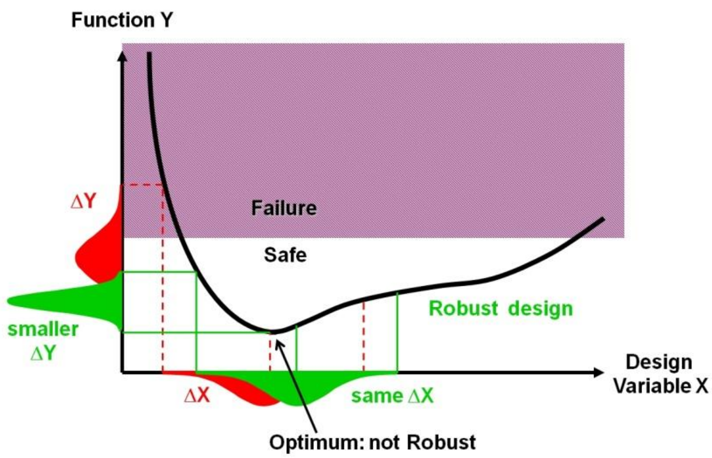


Figure 2: Robust design (from Noesis, 2012)

2.1. Variations (geometric, material properties and loads)

Although the definition of the nominal geometric dimensions, the real dimensions are stochastic, and usually contained within a tolerance range. The tolerance range is related mainly with the manufacturing process used, and other factors involved, as, labor and environment. Figure 3 shows a relationship between the various manufacturing process, with their respective tolerance capabilities, and the cost. For the development of designs, it is necessary to know the tolerance range, and the distribution (statistical) of the values on that. When the components exist, and manufactured in a controlled process, the recommended procedure is to evaluate statistically by collecting samples. When the components do not exist, the recommendation is to obtain data through other similarly manufactured components. In most cases the distribution has a normal behavior.

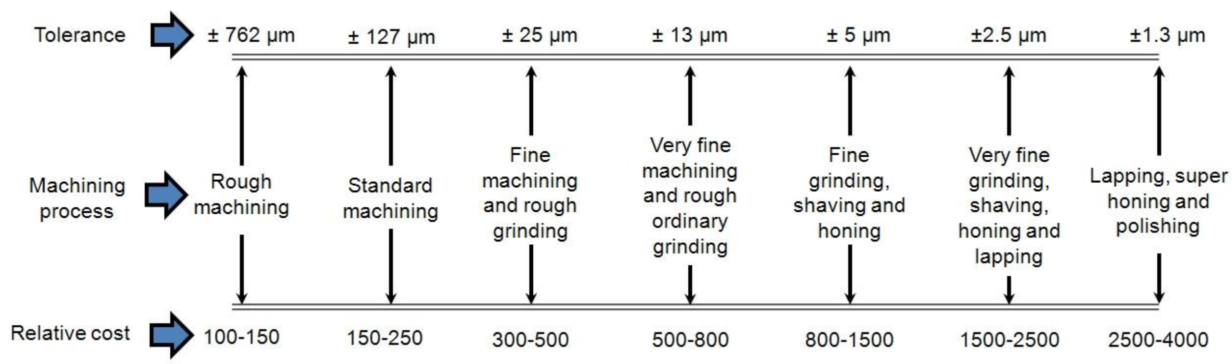


Figure 3: Relative cost x machining process x tolerance (from Rao, 1992)

Variations also occur in the mechanical properties of material, due to the uncertainties in the chemical composition, manufacturing processes and procedures, and heat treatments. Different from what happens with the geometric variations, obtaining the distribution of mechanical properties is not a simple task. There is little information available in the literature, and even from qualified suppliers of raw materials. Table 1 shows some information about this topic. A stochastic variable can be described using the mean (μ) and the standard deviation (σ), or by using the mean and the coefficient of variation (C.o.V.). Coefficient of variation is the rate between standard deviation and the mean (Shigley and Mischke, 1989).

Table 1: Parameters for modeling data uncertainties (from Capiez-Lernout *et al*., 2006)

Element/Material Type	Property	C.o.V. (σ/μ)	Probability Distribution
Isotropic Material	Young's modulus Poisson's ratio Shear modulus Mass density	8% 3% 12 % 4%	truncated Gaussian
Orthotropic Shell Element Material	Young's modulus Poisson's ratio Shear modulus Mass density	8% 3% 12% 4%	truncated Gaussian
Solid Element Anisotropic Isotropic Material	Mat. property matrix Mass density	12% 4%	truncated Gaussian
Simple Beam	Section dimension Non-structural mass	5% 8%	truncated Gaussian
Layered Composite Material	Non-structural Mass Thickness of plies Orientation angle	8% 12% $\sigma = 1.5$	truncated Gaussian
Spring element property	Elastic prop. value	8%	truncated Gaussian
Shell element	Membrane Thickness Non-structural Mass	4% 8%	truncated Gaussian
Spring element	Stiffness	10%	truncated Gaussian
Concentrated mass	Mass	3%	truncated Gaussian
Damping	Modal Damping Structural Damping	40% 25%	lognormal

Certainly, the evaluation of the load on the component as well as its variation is the most difficult to quantify because it is subject to the applications in the field, with an infinite combination of factors. The reproduction in the laboratory and tests in the field run the risk of not considering the entire operation that the component could be subjected on the life. Often, the alternative is overtaxing the loads to reduce the risk of failure in the field. Since there is no free lunch, the side effect is an increase in the cost of the component.

2.2. Analysis deterministic versus probabilistic

Table 2 presents the main differences between the two ways to do the designs. The first one is considered the classical way and is based on deterministic analysis. The second one, is more contemporary, is more adapted to recent times and is based on probabilistic analysis, which considers effectively all the existing dispersion on the process.

Table 2: Differences between deterministic and probabilistic analysis

Deterministic Analysis	Probabilistic Analysis
provides only nominal answers	provides a probability and reliability (design for reliability)
uses safety factors (leads to costly over-design)	takes uncertainties into account in a realistic fashion, and leaves the analysis more realistic
only the extreme nominal analysis is considered	manufacturing tolerances are included (design for manufacturing)
it is not possible to analyze the sensitivity variables	analysis of dispersion (sensitivity)
does not allow the correlation between variables	allows correlation between variables

2.3. Monte Carlo approach

The Monte Carlo simulation method is the most common and traditional method for a probabilistic analysis (ANSYS, 2011). This method allows the simulation of how virtual components behave the way they are built. One simulation loop represents one manufactured component that is subjected to a particular set of loads and boundary conditions. Figure 4 illustrates the method. The number of simulation loops that are required for a Monte Carlo simulation does not depend on the number of random input variables. The required number of simulation loops only depends on the amount of the scatter of the output parameters and the type of results that are expected from the analysis. The number of simulations that are necessary in a Monte Carlo analysis to provide good results is usually about 50 to 200.

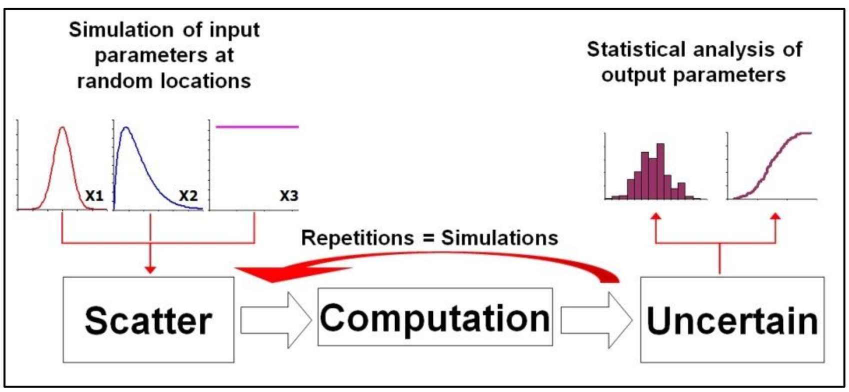


Figure 4: Monte Carlo simulation method scheme

3. APPLICATION

Figure 5 presents an example of a discharge tube. It connects the cylinder head with the compressor housing, conducting the compressed refrigerant from the first to the latter. Normally the tube is made of steel, and it has a special shape (a sequence of arcs and straight segments) to reduce the vibration generated during the compressor operation to the housing (Puff *et al.*, 2006 and Gaertner, 2008). Sometimes, a thin spring is put on the tube to increase the damping. One point which is very important for the discharge tube design is to avoid that the tube natural frequencies be coincident with the operation ones. The robust design methodology is applied to evaluate the behavior of the natural frequency (three modes) of a discharge tube, considering variations in geometry, material and damping spring.

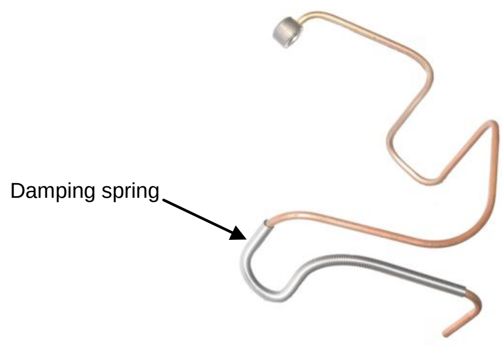
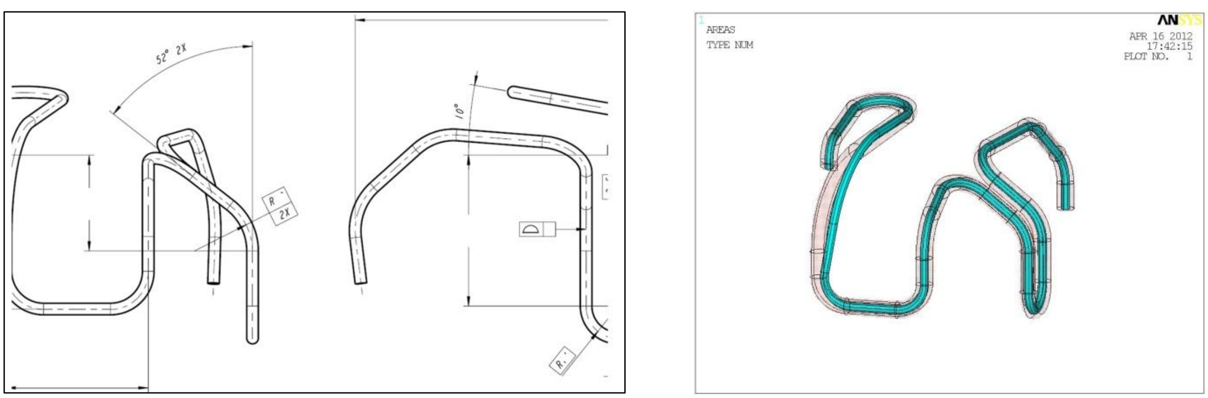


Figure 5: The discharge tube

3.1. Problem definition

Many dimensions are needed to define the discharge tube geometry. Figure 6(a) shows an example of a part of the draw, where is possible to observe the complexity of the task. A profile of a surface tolerance is used to control the entire surface as a single feature. Figure 6(b) shows a tube, where a part of this geometry does not respect the profile of a surface tolerance.



(a) The discharge tube drawn

(b) Profile of a surface tolerance

Figure 6: The geometry dimension control

The natural frequencies of the tube are calculated by finite element approach. Beam elements are used to represent the tube, and the spring is modeled by mass elements. The beam elements have a good performance to calculate the natural frequencies, that is, precise and fast (Bortoli, 2009 and Silva, 2011). They are very convenient for Monte

Carlo approach, as mentioned before, because a large quantity of loops is necessary to achieve precise results. Figure 7 shows the FEM model for the tube. A special routine was developed to check if the tube is according to the profile surface control tolerance. The tube that does not follow the tolerance surface profile is eliminated of the analysis.

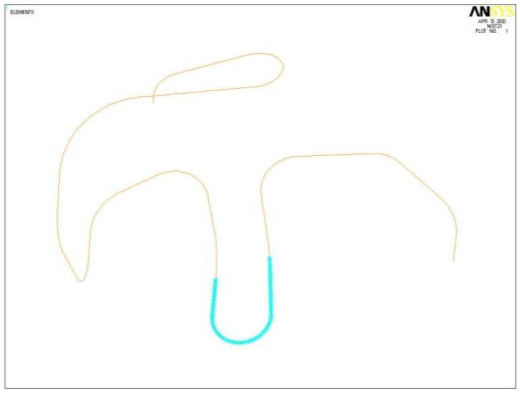


Figure 7: The FEM model

3.2. Input variables

Figure 8 shows the input variables for the problem. For each input variable, it is necessary to choose the distribution function, and the data necessary to define it. Sixty six input variables were defined to describe the geometry of the tube, the material properties, and the damping spring.

No.	Name	Type	Par1	Par2	Par3	Par4
1	DEXT_TU	TGAU	4.0000	2.66667E-02	3.9200	4.0500
2	ESPE_TU	TGAU	0.51000	2.66667E-02	0.43000	0.56000
3	E_TU	TGAU	1.94700E+08	1.55760E+07	1.71336E+08	2.18064E+08
4	RHO_TU	TGAU	8.37000E-06	3.34800E-07	7.86780E-06	8.87220E-06
5	CP_TU	TGAU	0.30000	9.00000E-03	0.28650	0.31350
6	LSPRI	UNIF	72.000	88.000		
7	FYMAM	UNIF	0.0000	2.0000		
8	FX2	UNIF	0.0000	2.0000		
9	FX3	UNIF	0.0000	2.0000		
63	R12_13	UNIF	18.000	22.000		
64	R13_14	UNIF	11.000	15.000		
65	R14_15	UNIF	13.000	17.000		
66	R15_16	UNIF	18.000	22.000		

Figure 8: Random input variable specifications

3.3. Output variables

Table 3 presents the statistics of random output parameters. The main information is the mean and standard deviation of the output parameters. Figure 9 shows the convergence of the analysis for the mean value of the first frequency. More than fifty loops are necessary to achieve accuracy results.

Table 3: Statistics of random output parameters

Name	Mean	Standard Deviation	Skewness	Kurtosis	Minimum	Maximum
FREQ1	67.38	2.490	3.5218E-02	-0.8571	63.10	72.91
FREQ2	91.32	3.584	8.4662E-02	-0.8128	85.29	99.03
FREQ3	134.7	5.202	0.1102	-0.7098	125.6	146.3

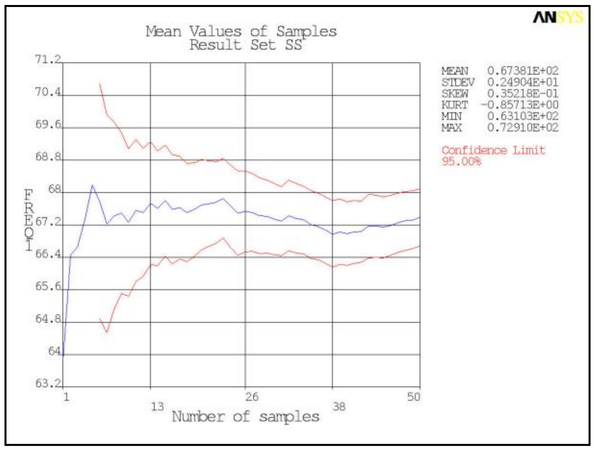


Figure 9: The convergence of mean value for the first natural frequency

3.4. Results analysis

Figure 10 presents the correlation sensitivities for the first frequency. The significant factors are Young's modulus, mass density, and one specific region of the geometry of the tube. It is important to notice, that in this case, the damping spring is robust for the process. Other important aspect to be evaluated is the correlation between variables (figure 11). It is easy to observe that the Young's modulus has a good correlation with the natural frequency. The same behavior is not observed for the mass density. The same analysis can be done for the other output variables.

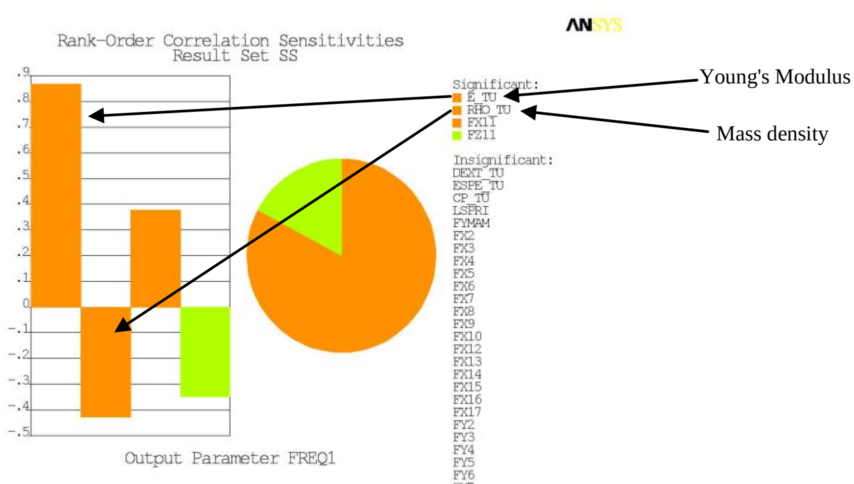
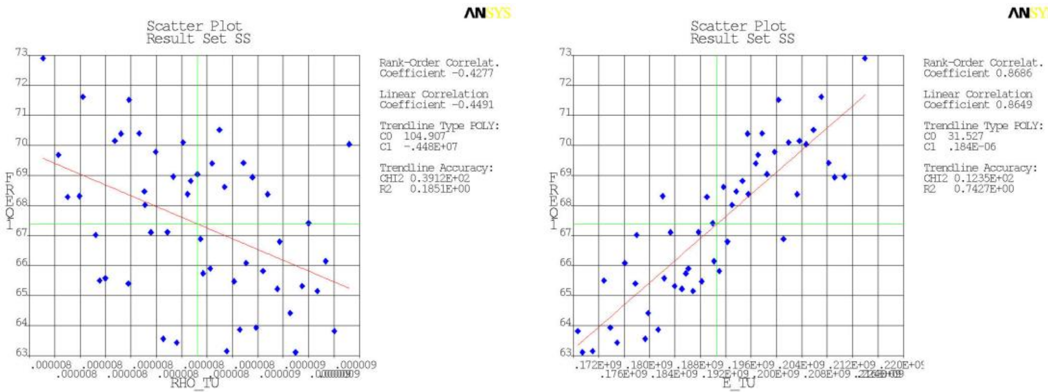


Figure 10: Correlation sensitivities between variables



(a) Mass density x first natural frequency (b) Young's modulus x first natural frequency

Figure 11: Correlation between variables

4. CONCLUSIONS

The conclusions of this work are:

- The probabilistic analysis helps to understand products under real conditions of manufacturing and application;
- The robust design methodology is an excellent tool to improve the quality of compressors;
- The Monte Carlo method is an efficient way to do probabilistic analysis;
- It is very important to have lean finite element models to make it feasible to apply the Monte Carlo methodology (CPU time);
- The success of the robust design methodology is to have precise information about the variations about geometry, material and load.

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