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Jian Li
lijianxjtu@gmail.com

Feilong Liu

Quanke Feng

Weifeng Wu

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Jian LI[1], Feilong LIU[1], Quanke FENG[1], Weifeng WU[1]*

[1]School of Energy and Power Engineering, Xi'an Jiaotong University
Xi'an, 710049, China
86-29-82675258, Fax: 86-29-82663783, weifengwu@mail.xjtu.edu.cn

* Corresponding Author

ABSTRACT

The wear of star wheel teeth is an importance issue in the single screw compressor. In order to enhance the wear-resistance of star wheel teeth, some new profiles of the meshing pair were proposed to replace the currently used straight line envelope profile, such as single column envelope, multi-straight line (or multi-column) envelope. In this paper, a curved surface of the tooth flank is suggested. The groove flank is the envelope surface corresponding to the curved surface of the tooth flank. Section profile of the new tooth flank is a curved line, which could be elliptical, hyperbolic or involute. During the tooth meshing with the groove, the contact line between the tooth flank and the groove flank could move back and forth in the whole area of the tooth flank.

This design is expected to improve the lubricating between the tooth flank and the groove flank, and to prolong the service life of the single screw compressor. Geometric and kinematic investigation of the star wheel and the screw rotor is established. And a new designed tooth flank of the meshing pair is introduced in this paper. Numerical control high-velocity cutting technology is suggested to fabricate the new star-wheel and the screw rotor.

1. INTRODUCTION

The single screw compressor was first developed by Bernard Zimmern (1965) in the early 1960s. Initial sales were of machines compressing air, but refrigeration models followed (Haselden, 1985). Right now it has been used in many fields, such as power, refrigeration, air conditioning, petroleum and chemistry plants (Wu and Tao, 2006). In China, the domestic production of single screw compressors is about 14000, at the same time this number of twin screw compressors is around 60000 in 2010 (Sui, 2011).

The structure of a typical single screw compressor is shown in Figure 1, whose primary components are a screw and two star wheels. Compared with other kinds of compressors, the single screw compressor should have many advantages due to its symmetrical structure and well balanced radial gas pressure on the screw rotor (Zimmern *et al.*, 1972). However, its discharge capacity decreases sharply after several hundred hours running. This behavior is no doubt attributable to rapid wear of star wheel tooth flank surface meshing with the screw groove (Zimmern, 1990). Applying high wear resistance materials to the star wheel such as polyetheretherketone (PEEK) and polytetrafluoroethylene (PTFE) helps to prolong the life of single screw compressors, but it doesn't resolve this issue efficiently. The machining accuracy of the meshing pair was also an important issue, especially for the indexing accuracy. Now the indexing accuracy of a common CNC machine tool can reach 10 seconds, which means the maximum machining error is around 0.01mm for a 200mm diameter, while the design precision of the meshing pair in single screw compressors is around 0.05mm. Besides the material and precision, the meshing pair profile is the basic factor affecting the wear resistance (Zimmern, 2000). Therefore, some researchers proposed different profiles of the meshing pair to reduce the abrasion of the meshing pair.

The original profile of the single screw compressor meshing pair is a straight line envelope, invented by Zimmern in 1960s. The contact line of the meshing pair on a star wheel tooth flank is a fixed straight line, which is obviously easy to be worn. Thus, Zimmern (1976) developed a column (frustum) envelope meshing pair, so that the contact line moves on the star wheel tooth flank. And this type of meshing pair was modified by Jensen (1998, 2000) and Wu (2009) to improve its machinability. A straight line double envelope meshing pair and a column (frustum) double envelope meshing pair were introduced in the 1980s (Jin, 1982; Jin and Tang, 1985). The double envelope means using the straight line or column (frustum) enveloped surface of the screw rotor groove flank as a generating tool to envelop the star wheel tooth flank. Feng (2005) proposed a multi-straight-line envelope profile with the purpose of dispersing friction area to prolong the operating life. There are more than two straight lines on the tooth flank, and these straight lines mesh with the screw groove flank alternately. Wu (2009) deduced a multi-column envelope meshing pair, which was based on the column envelope and multi- straight line envelope meshing pairs. The star wheel tooth flank consisted of more than two parts of cylindrical segments, which mesh with the screw rotor groove flank alternately. Between these cylindrical segments are transition sections, which don't participate in meshing process. Essentially, the meshing area of a multi-column envelope star wheel tooth is not a continuous surface. Thus, during the meshing progress the connect line jumps from one cylindrical segment to another. That will cause lubricant film fluctuate and impact load, and then increase the wear.

This paper presents a curved surface of the star wheel tooth flank. The groove flank is the envelope surface corresponding to the curved surface of the tooth flank. Section profile of the new tooth flank is a curved line, which could be elliptical, hyperbolic or involute. During the tooth meshing with the groove, the contact line between the tooth flank and the groove flank could move continuously in the whole area of the tooth flank. Meanwhile, the meshing area of the star wheel increases significantly. This design is expected to improve the lubricating between the tooth flank and the groove flank, and to prolong the service life of this meshing pair.

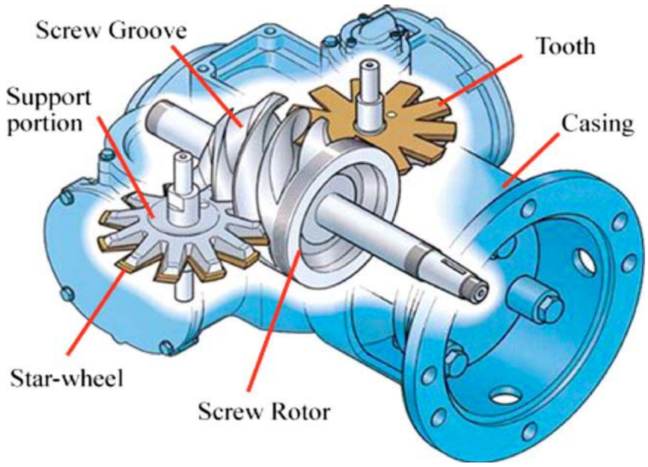

Figure 1: Basic components of a typical single screw compressor

2. MATHEMATICAL MODEL

2.1 Geometry and Kinematics of the Meshing Pair

The geometric and kinematic relations between the star wheel and the screw rotor in the meshing process are shown in Figure 2. Four right-handed Cartesian coordinates are introduced to describe the meshing process. $S_1(X_1, Y_1, Z_1)$ and $S_3(X_3, Y_3, Z_3)$ coordinates are used for expressing the starting positions of the star wheel and the screw rotor. And thus S_1 and S_3 coordinates are stationary. The origin O_1 and O_3 are chosen at the horizontal plane, the Z_1 axis (the rotation axis of the star wheel) is vertical and Z_3 axis is the axis of the screw rotor. $S_2(X_2, Y_2, Z_2)$ and $S_4(X_4, Y_4, Z_4)$ are used for describing the rotation of the star wheel and the screw rotor respectively.

In Figure 2, Φ_{sw} and Φ_{sr} are rotation angles of the star wheel and the screw rotor. According to the gear transmission principle, the ratio of Φ_{sw} and Φ_{sr} is set to be N_{sw}/N_{sr}, where N_{sw}, N_{sr} are the numbers of screw grooves and star wheel teeth,

$$\frac{\omega_{\mathrm{sw}}}{\omega_{\mathrm{sr}}}=\frac{\phi_{\mathrm{sw}}}{\phi_{\mathrm{sr}}}=\frac{N_{\mathrm{sw}}}{N_{\mathrm{sr}}}=P \tag{1}$$

Where ω is rotational velocity, subscript "*sw*" and "*sr*" represent star wheel and screw rotor respectively. P is usually chosen as11/6.

$$\omega_{sw}=(2\pi n_{sr}/60)\cdot P \tag{2}$$

Where n_{sr} (r/min) is the rotational speed of a screw rotor.

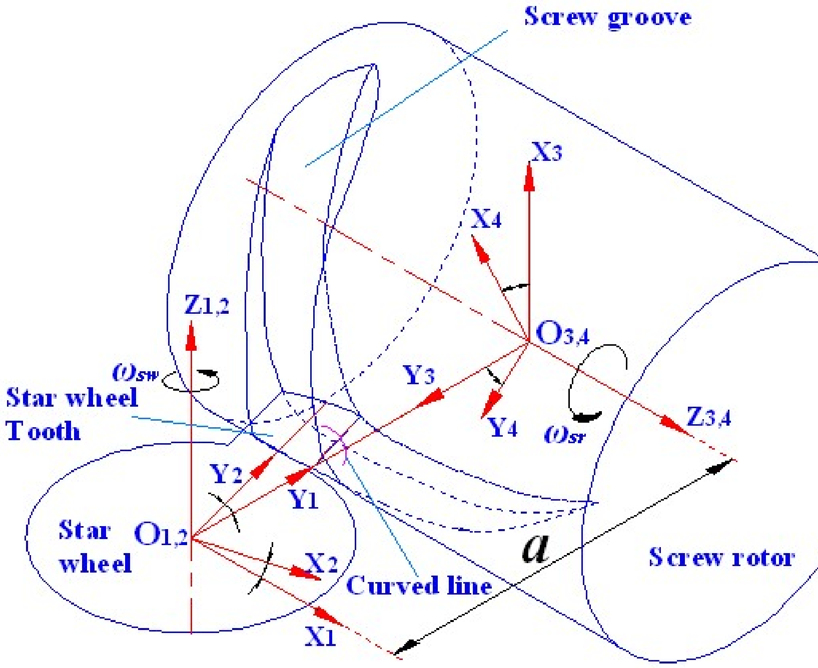

Figure 2: Geometric and kinematic relations between the star wheel and the screw rotor

2.2 Relative Velocity and Meshing Conditions

2.2.1 Relative Velocity in S2 Coordinate: Firstly, we derive the expressions of relative velocity in S1 coordinate. In the coordinates, showed in Fig 2, we add a same angular velocity-ω_{sr} to the star wheel and the screw rotor. The mutual moving relationships remain unchanged; meanwhile the screw rotor is stationary and the star wheel does a complex movement. This complex movement contains a carrier velocity v_e through -ω_{sr} rotating around Z2 axis, and a relative velocity v_r through ω_{sw} rotating around Z1 axis. According to the principles of kinematics, when a point does a complex movement, its absolute velocity v equals the vector sum of its carrier velocity v_e and its relative velocity v_r:

$$v=v_e+v_r=(-\omega_{sr})r_e+\omega_{sw}r_1 \tag{3}$$

In static S1coordinate,

$$r_1=X_1 i+Y_1 j+Z_1 k \tag{4}$$

$$O_1O_2=aj \tag{5}$$

$$r_e=r_1-O_1O_2=X_1 i+(Y_1-a)j+Z_1 k \tag{6}$$

Substituting Eq. (4) and Eq. (6) into Eq. (3), the following equation is obtained:

$$\begin{aligned} v&=-\omega_{sr}\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ X_1 & Y_1-a & Z_1 \end{vmatrix}+\omega_{sw}\begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ X_1 & Y_1 & Z_1 \end{vmatrix} \\ &=-\omega_{sw}Y_1 i+(\omega_{sw}X_1+\omega_{sr}Z_1)j-\omega_{sr}(Y_1-a)k \end{aligned} \tag{7}$$

This equation also can be written as:

$$\begin{cases} v_{X_1} = -\omega_{sw} Y_1 \\ v_{Y_1} = \omega_{sw} X_1 + \omega_{sr} Z_1 \\ v_{Z_1} = -\omega_{sr}(Y_1 - a) \end{cases} \tag{8}$$

Through the coordinates transform, the relative Velocity in S2 coordinate is expressed as:

$$\begin{cases} v_{X_2} = \omega_{sw}(-Y_2 + PZ_2 \sin\phi_{sw}) \\ v_{Y_2} = \omega_{sw}(X_2 + PZ_2 \cos\phi_{sw}) \\ v_{Z_2} = \omega_{sw} P(a - Y_2 \cos\phi_{sw} - X_2 \sin\phi_{sw}) \end{cases} \tag{9}$$

2.2.2 Meshing Conditions: A curved line of a star wheel tooth is show in Figure 3. The curved line of the star wheel tooth lies in a plane that is perpendicular to the star wheel plane. Point A is any point on the curved line. In the coordinate S2, this curved line could be expressed as:

$$\begin{cases} X_2 = x \\ Y_2 = y \\ Z_2 = z(x, y) \end{cases} \tag{10}$$

It could be a continuous curve such as elliptical, hyperbolic, involute, etc.

Because the star wheel tooth flank moves tangentially with the groove flank, the relative velocity of the tooth flank to the groove flank must be orthogonal to the normal vector of the tooth flank at the contact point (Kang et al. 1996). The meshing condition ($v \perp n$) at the connect point can be expressed as:

$$v \bullet n = 0 \tag{11}$$

In the coordinate S2, the normal vector n is:

$$n_2 = -\frac{\partial z}{\partial x} i - \frac{\partial z}{\partial y} j + k \tag{12}$$

Substituting Eq. (12) and Eq. (9) into Eq. (11), the following equation is obtained:

$$P(a - Y_2 \cos\phi_{sw} - X_2 \sin\phi_{sw}) + \frac{\partial z}{\partial x}(Y_2 - PZ_2 \sin\phi_{sw}) - \frac{\partial z}{\partial y}(X_2 + PZ_2 \cos\phi_{sw}) \ = 0 \tag{13}$$

Substituting Eq. (10) into Eq. (13), x can be solved as:

$$x = f(y, \phi_{sw}) \tag{14}$$

If the curve function $z(x,y)$ is given, the connect point at different rotation angles of the star wheel (Φ_{sw}) and different height of the tooth (y) can be solved.

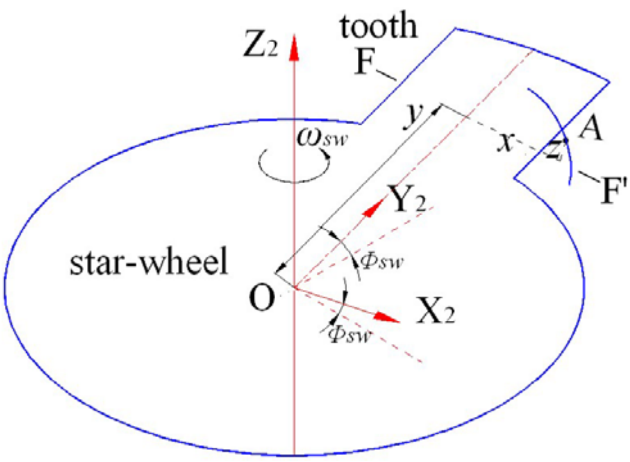

Figure 3: A curved line of a star wheel tooth

3. DESIGN OF A NEW TOOTH PROFILE

As shown in Figure 4, the angle α between v_a (component velocity of the relative velocity v in X2-O2-Z2 surface) and horizontal plane (X2-O2-Y2) could be obtained from Eq. (9):

$$\alpha = \mathrm{a}\tan[\frac{P(a - Y_2 \cos\phi_{sw} - X_2 \sin\phi_{sw})}{Y_2 - PZ_2 \sin\phi_{sw}}] \tag{15}$$

According to the structure characteristics of the single screw compressor, the angle α has the widest variation section [$\alpha_{\min}$ ~ $\alpha_{\max}$] at the top of the star wheel tooth. Usually, α_{max}-α_{min} is around 15º. From the top to the root of the tooth, this variation section becomes smaller to zero. At the root of the tooth, α_{max} equals $\alpha_{min.}$

A new designed tooth flank of the meshing pair in single screw compressor is shown in Figure 4. The meshing area of the tooth flank is a continuous curved surface, in which the curvature changes continuously along with the thickness direction (z-axis). The meshing curved surface can be distributed to the whole area of the tooth flank, and also can be distributed to a middle part of the tooth flank. Between the meshing area and the up surface (or down surface) is a side surface, which is the flat portion of the tooth flank. After determining the tooth flank, the screw groove flank can be calculated by the above meshing conditions.

In Figure 4, the plane γ is perpendicular to Y2 axis through a point (0, $y2$, 0). A line is the intersection line of the plane γ and the tooth flank, which includes three sections of line (L_u, L_r, L_d). Among these three sections of line, L_r is the intersection line of the plane γ and the meshing curved surface. When the meshing curved surface distributed to the whole area of the tooth flank, there is only L_r. Point A is any point on line L_r and α_A is its slope angle. If A moves on L_r form up to down, α_A will change from $\alpha_{min}(y2)$ to $\alpha_{max}(y2)$. The slope angle of line L_u should less than or equal to $\alpha_{min}(y2)$ and the slope angle of line L_d should more than or equal to $\alpha_{max}(y2)$. In Figure 5, L_r is a part of an elliptic and L_u & L_d are tangent lines.

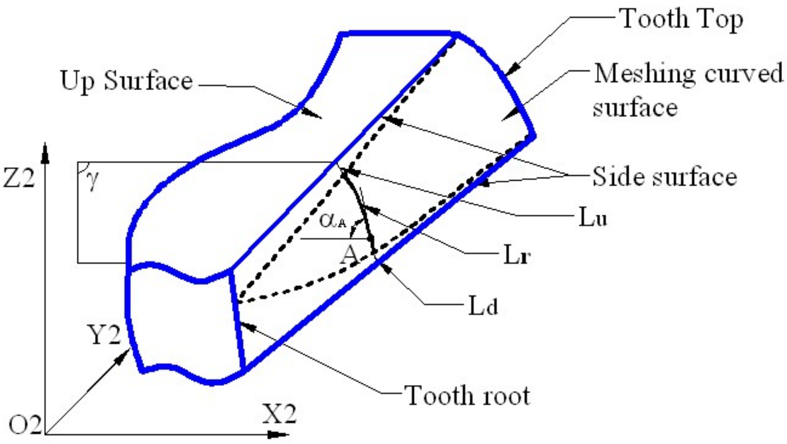

Figure 4: A new designed tooth flank

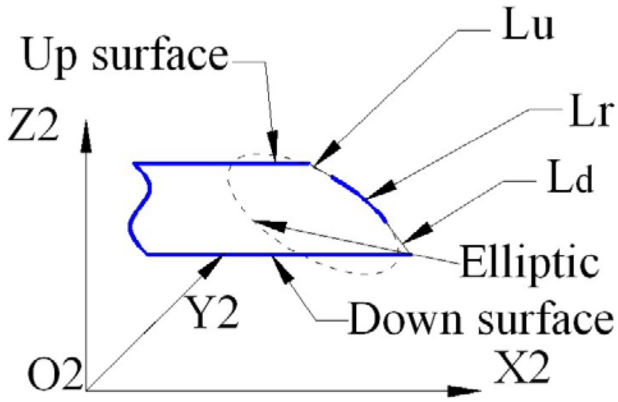

Figure 5: An elliptical tooth flank

4. CONCLUSIONS

This paper presents a new designed tooth flank of the meshing pair in single screw compressor. Section profile of the new tooth flank is a curved line, which could be elliptical, hyperbolic or involute. Geometric and kinematics investigation of the meshing air is established in the paper. During the tooth meshing with the groove, the contact line between the tooth flank and the groove flank could move continuously in the meshing area of the tooth flank. Meanwhile, the meshing area of the star wheel increases significantly, which can be distributed to the whole area of the tooth flank. This design is expected to improve the lubricating between the tooth flank and the groove flank, and to prolong its service life.

Numerical control milling machine is suggested to fabricate the new star-wheel teeth. And the screw grooves could be processing by numerical control milling machine or numerical control lathe.

NOMENCLATURE

Φ	rotation angle	(º)	a	center distance	(mm)
N_{sw}	the numbers of screw grooves	(–)	n	normal vector	(–)
N_{sr}	the numbers of star wheel teeth	(–)	α	slope angle	(º)
ω	rotational velocity	(rad/s)	α_{max}	maximum slope angle	(º)
n_{sr}	rotational speed of the screw rotor	(r/min)	α_{min}	minimum slope angle	(º)
v	absolute velocity	(m/s)			
v_e	carrier velocity	(m/s)			
v_{r}	relative velocity	(m/s)			
P	ratio of tooth number	(–)			
X	coordinate	(–)			
Y	coordinate	(–)			
Z	coordinate	(–)			

Subscripts

sw	star wheel
sr	screw rotor
1	coordinate system S1
2	coordinate system S2

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