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Modelling of an Air Compressor Using Energetic Macroscopic Representation

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Modeling of a Reciprocating Air Compressor using Energetic Macroscopic Representation

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ABSTRACT

This article introduces the energetic macroscopic representation (EMR) as approach for the dynamic nonlinear modeling of a reciprocating air compressor. EMR has been introduced recently for research development in complex electromechanical systems. It is based on action reaction principle, which organizes the system as interconnected subsystems according to the integral causality. The graphical modeling based on EMR has advantages such as readability, modularity, structural and functional characteristics. The EMR is the first step toward model-based control structure development. The air compressor system containing: slider-crank mechanism, cylinder head and valves, is divided into multitude simple subsystems. Each subsystem describes an elementary step of the energy conversion, several of these blocks may occur in a single module. Calculations are carried out using two basic principles: mass and energy balances. Models are developed for different subsystems, which are assembled into a final overall system EMR. The EMR modeling presented here allows the modeling of multi-physics components and highlights the interactions of the electromechanical, thermodynamical, heat transfer and fluidic phenomena that occur simultaneously in an air compressor.

1. INTRODUCTION

Reciprocating compressors is one the most important machines used throughout various industries. Due to its importance considerable effort has been devoted by many researchers toward the development of mathematical models for computer simulation of this type of machine. (See (Hamilton, 1974) and its refs which give a representative description of this type of work.)
In general, a cycle of operation of a high speed positive displacement compressor can be described as a number of complicated phenomena, interacting and taking place in a short period of time.

As regards the graphical modeling tools, their interest is double: firstly during the modeling phase itself and secondly as the easiest transfer of knowledge to the other users (Borutzky, 2010).
In some literatures graphical models of air compressor can be found (Engja, 1985 and Karnopp, 2000). They are based on the same equations and theoretical backgrounds than the previous ones but the graphical methodology highlights the understanding of the device behavior in a significant extent. To describe the modeling of thermodynamic system modeling, (Thoma, 2000) uses Bond Graph approach. Using the same tool, Karnopp (2000) proposes an overall picture of the considered system in a synthetic approach. Even if Bond Graph tool is able to describe complex systems, Energetic Macroscopic Representation (EMR) has its own strengths. In fact, beyond its modularity and readability, EMR reveals the structural and functional characteristics to provide a good model in agreement with physical (e.g. integral) causality (Hissel, 2008). Both Bond Graphs and EMR are obviously able to model multiphysical systems, but there are nevertheless different features between them. The former created by Bouscayrol is based on energy and data flow and uses a uniform representation for all types of physical systems; while with the latter specific pictograms are used and associated to each power component increasing its readability. As a matter of fact, it shows the different physical domains crossed by the energy flows. The Bond Graphs discriminate between the accumulation of potential and kinetic energies in the system whereas EMR doesn't

make any difference. Furthermore, in Bond Graphs, all subsystems are usually represented even if it leads to derivative relationships which violate the causality principle. The EMR description is more functional and is oriented towards control. Then any derivative relationship is avoided and some rules can be applied to respect this as concatenation for instance (Bouscayrol, 2005). EMR has been developed since 2000 to analyze systems from a macroscopic point of view; it allows the design of a control structure thanks to its control features like Maximum Control Structure (MCS) deduced from EMR using inversion rules. The use of Practical Control Structure (PCS) by applying simplification rules leads to a control and an energy management that can be easily implemented in real time.

Therefore, a complex system of several energy sources and several energy conversion units can be accurately modeled by the EMR tool with a clear readability. Moreover, the control of this system can after easily be implemented, thanks to the appropriate control tools.

As in other graphical representation methodologies, the product of the causal parameter pair (here called the action and reaction pair) is the instantaneous power exchange between the blocks (Chrenko, 2009) to which they are connected. Though EMR has been firstly developed to describe electromechanical system, Chrenko (2008) made it possible to extend it to new energy domains. Expanded to electrochemical, thermodynamical, thermal and fluidic domains, EMR has allowed describing many devices such as fuel cells systems, electromechanical systems, electrical or hybrid vehicle systems.

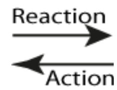
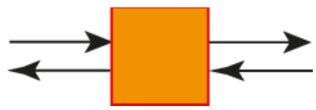
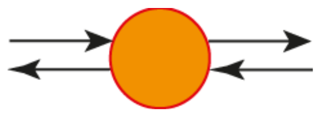
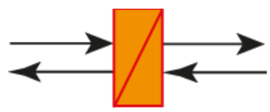
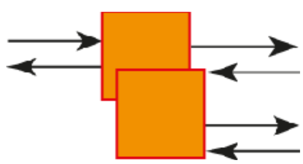
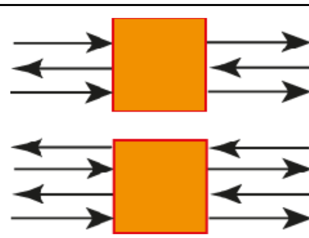
Until now, EMR modeling of the air compressor is scarce. This paper is organized as follows. In Section 2: some basic knowledge on EMR is recalled. Some considerations regarding EMR are made to apply it to thermo-pneumatic problems. Then, each physical phenomenon taken into account is described and its EMR representation is provided. In Section 3, the air compressor behavior including the elements: slider-crank mechanism, cylinder-piston assembly and valves is described. The model is detailed in this section. Hence, putting together all EMR subparts, the whole compressor Energetic Macroscopic Representation is proposed. Finally, the last section (Section 4) is devoted to the simulation results and their comparisons with Finite element model and also to the discussion of the obtained results.

2. EMR DESCRIPTION

2.1 Short Presentation of the Considered Graphic Modeling Tool

As mentioned before, EMR is used for the modeling of this multi-physics compressor. The product of an action variable with the corresponding reaction variable provides a value that is always consistent to an instantaneous power. Thus, it has been possible by Chrenko (2007, 2009) to extend EMR beyond its initial aims in order to describe all multi-physics systems. The EMR model inversion element by element allows obtaining straight the control structure; it is called the Maximum Control Structure (MCS) and implies the assumption (of course theoretical) that all values are measurable. In practice, it is not always the case. So, a Practical Control Structure (PCS) can be deduced from the MCS by applying simplification rules. For more details, please refer to Bouscayrol(2000) and Chrenko (2007, 2009). In this paper, only the EMR modeling part is presented. Table 1 presents the generalized EMR pictograms used in the presented model.

Table 1: Extended Energetic Macroscopic Representation blocs

Energetic Macroscopic Representation (EMR) blocs		
Pair of action a reaction		Two parallel black arrows pointing in opposite directions
Source of energy		Light green oval with dark green rim
Energy conversion Same domain		Orange square with red rim
Energy conversion Different domains		Orange circle with red rim
Energy accumulation		Orange rectangle with red rim and red diagonal bar
Coupling device (distribution of energy) Same domain		Overlapping orange squares with red rim
Thermo-fluidic causalities (three and four variables both for gas and liquid)		Orange squares with red rim

2.2. Adaptation of EMR to a thermofluidic system

The EMR methodology is based on the assumption that every energy exchange can be described using one action and one reaction parameter. However, Chrenko(2009) argues that in the case of a multiport this does not have to be true all the time.

2.2.1. Causality between action and reaction parameter pair: As in other graphic representation methodologies, the product of the causal parameter pair (here called action and reaction pair) gives the power exchanged between the blocks (Bouscayrol, 2005). This principal is reasonable as long as there is a relation between action and reaction leading to the power. This relation has not necessarily to be a product. This approach might be called pseudo-EMR, in analogy to pseudo-Bond Graph (Thoma, 1993). It gives the possibility to handle the parameter pair in some areas more freely. For example, Chrenko(2009) uses pressure p and mass flow $\dot{m}$ or either Temperature T and Enthalpy flow $\dot{E}$ as the parameters of EMR.

2.2.2. Representation of multiports: The basic ideas of EMR contain the fact that all systems can be subdivided into small parts, which follow either a simple conversion, a Kirchhoff law or a first order differential equation in integral form. For most systems this subdivision can be obtained. However, in some cases the link between energetic

domains cannot be detached, they can only be described in matrix form. In this case the energetic domains inside the gas flows cannot be detached. The element has two parameter pairs upstream and downstream. According to Thoma (2006), such an element can be called EMR-multiport in analogy to Bond Graph-multiports.
Chrenko(2009) proposes that as the equation cannot be separated into the two energetic domains a multiport using both parameter pairs is used, see Figure. 1.

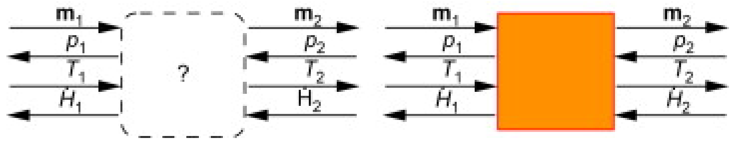


Figure. 1. Representation of a multiport in EMR.

3. DESCRIPTION OF AIR COMPRESSOR BEHAVIOR

A schematic of the compressor in question is shown in Fig. 2. For modeling purposes the compressor is divided into three subsystems, namely:
A. slider-crank mechanism,
B. cylinder head,
C. valves,
Each subsystem is modeled independently and the submodels are then combined into a complete model of the total system.

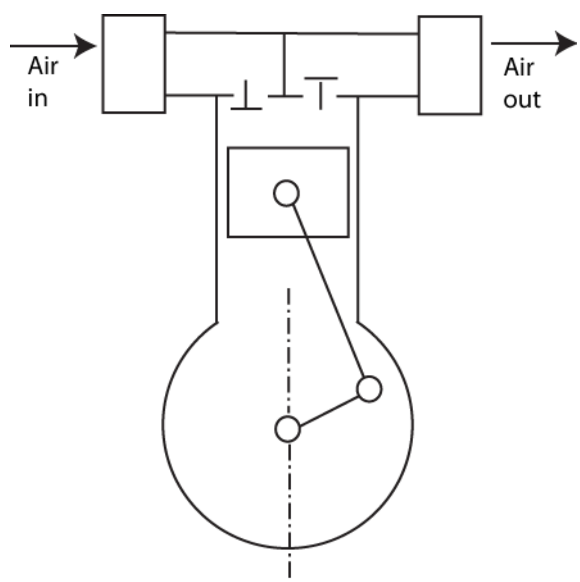


Figure 2: Compressor schematic

3.1 Slider-crank mechanism

The slider-crank relationship, $m(\theta)$, was derived as:

$$m(\theta) = \left[R\sin\theta - \frac{R^2 \sin\theta\cos\theta}{\left(L^2 - R^2\sin^2\theta\right)^{1/2}} \right], \qquad (1)$$

This transformer modulus shows the relationship between ω and the piston speed V, as well as the relation between torque τ, and piston force F.

$$m(\theta) = \frac{V}{\omega} = \frac{\tau}{F} \qquad (2)$$

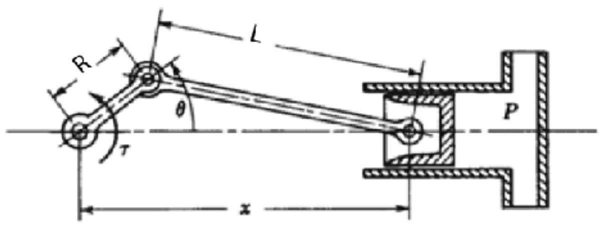


Figure 3: Crank and piston mechanism

Plus, the relation between the volume change and piston displacement is:

$$\dot{V} = -A_p \dot{x}_p \qquad (3)$$

One may represent the device by the EMR of Figure 3.

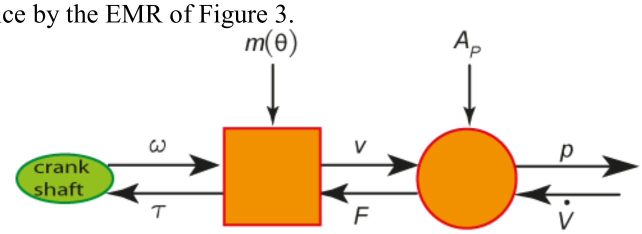


Figure 4: EMR of Crank and piston mechanism

3.2 Cylinder head

Considering the cylinder head volume as a control volume that can transport mass and energy through the "in" and "out" port, but flow can go in either direction at either port. Finally we can obtain work from the control volume by volume expansion. Notice that we assume that one pressure, one temperature, one density, etc. characterize the entire internal volume of the control volume.

Writing the mass and energy equations for the mentioned control volume, we have:

$$\frac{d}{dt}E = h_i \dot{m}_i - h_e \dot{m}_e - p\frac{dv}{dt} - \dot{E}_w, \qquad (4)$$

$$\frac{d}{dt}m = \dot{m}_i - \dot{m}_e, \qquad (5)$$

Here $E$is the internal energy of the control volume, and we assume that the gas obeys the gas law.

Even though the EMR representation is not a true one, but as the causality indicates it possesses all integral causality and therefore accepts flow input on all arrows. It then integrates these flows to produce the state variables E, m, and V. And finally the accumulator operates on these variables through appropriate constitutive laws to produce the outputs P and T. The constitutive relations are

$$T = \frac{1}{c_v}\frac{E}{m}, \qquad (6)$$

$$T = \frac{mRT}{V} = \frac{mR}{V}\frac{1}{c_v}\frac{E}{m} = \frac{R}{c_v}(\frac{E}{V}). \qquad (7)$$

Thus, if $\dot{E}$, $\dot{m}$ and $\dot{V}$ are prescribed(causally), then the thermodynamic accumulator will output T and P via the constitutive laws mentioned. Considering the principals described in section 2 The EMR representation of the cylinder head is shown in Figure. 5.

The rate of heat transfer between the gas and the cylinder head wall is modeled by a general approach:

$$\dot{E}_w = H_c A_c (T_w - T_c), \quad (8)$$

Where H_c is the overall heat transfer coefficient, A_c the cylinder control volume surface area, T_w the surface area temperature and T_c the instantaneous gas temperature. Regarding the heat transfer one approach to use may be that given by Eichelberg (1939)

$$H_c = 2.47\sqrt{P_c T_c}\sqrt[3]{V_{pm}}, \qquad (9)$$

Where P_c and T_c are instantaneous cylinder gas pressure and temperature and V_{pm} the mean piston velocity.

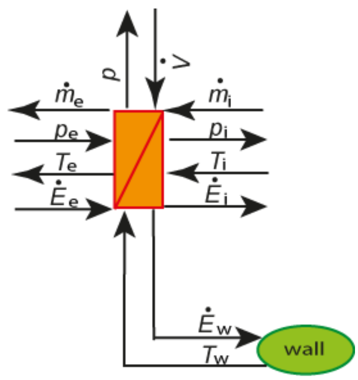


Figure 5: EMR of Cylinder head

3.3 Suction and discharge valves

The valves are assumed to be treated instantaneously as a simple orifice with effective cross-sectional area, A, and isentropic flow. The mass flow rate, m_v, through the valve as a function of valve area, upstream pressure P_u, temperature T_u, and downstream pressure, P_d, can be computed using the well known relation

$$\dot{m}_v = A_v p_u \sqrt{\frac{2k}{(k-1)RT_u}}\sqrt{(p_r)^{2/k} - (p_r)^{(k+1)/k}}, \qquad (10)$$

Where k is the ratio of specific heats and R is the ideal gas constant.

And for

$$p_r \le p_{r\ crit} = (\frac{2}{k+1})^{k/(k-1)}, \qquad (11)$$

The flow is choked. And the mass flow is independent of the downstream pressure.

The transported energy $\dot{E}_h$ associated with the mass flow $\dot{m}$ is then

$$\dot{E}_h = c_p T_u \dot{m}\ , \quad (12)$$

Considering the principals described in section 2, the EMR representation of the valve is shown in Figure. 6.

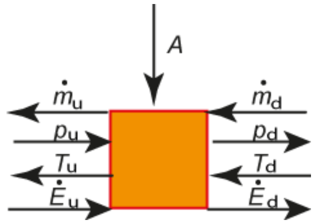


Figure 6: EMR of Valves

3.4 Complete EMR Model

The three previously developed submodels are assembled to form the complete model shown in Figure. 7. This EMR displays in one simple diagram the intricate mechanical-thermal interaction of the single stage compressor. The basic lumping process and the system structure are clearly indicated. The EMR shows all mass and energy flow directions and causalities. The advantages of the EMR techniques are clearly illustrated in that relatively complicated mechanical machines can be modeled by "assembling" a general set of building blocks.
Finally the area modulations A_i for the inlet and A_e for the exhaust are needed to be specified by a logic expression. For example, for A_i :

$$\text{If } \theta \geq 180° \text{ and } p \leq p_i, \quad A_i = A_{i\ max} \tag{13}$$
$$\text{Otherwise,} \quad A_i = 0$$

This will keep the inlet open as long as pressure inside the cylinder is less than the atmospheric pressure. The similar logic can be defined for exhaust valve.
The equations mentioned until now can be delivered to any explicit equation solver in any order desired. In this case we have used MATLAB Simulink for simulation. An integration algorithm will march the solution from one time step to next.

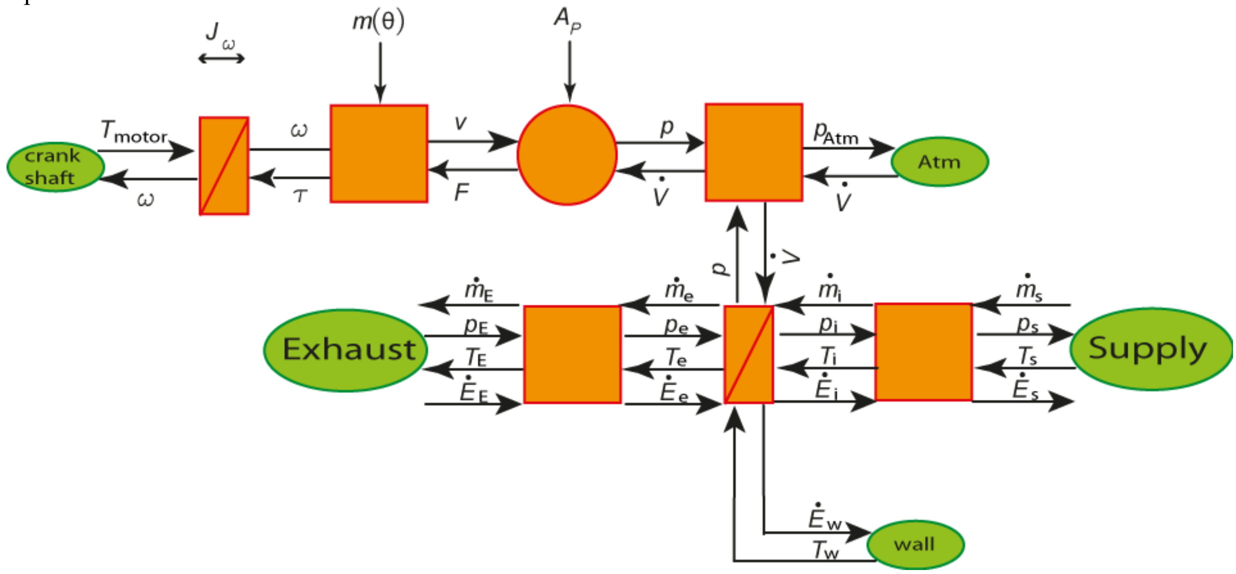


Figure 7: Complete system EMR

4. SIMULATION RESULTS

The model equations were programmed in a Matlab Simulink® environment for simulation.
Figures 8 and 9 show pressure and temperature in the compressor cylinder. Also, Figures 10 and 11 show mass and volume in the compressor cylinder.

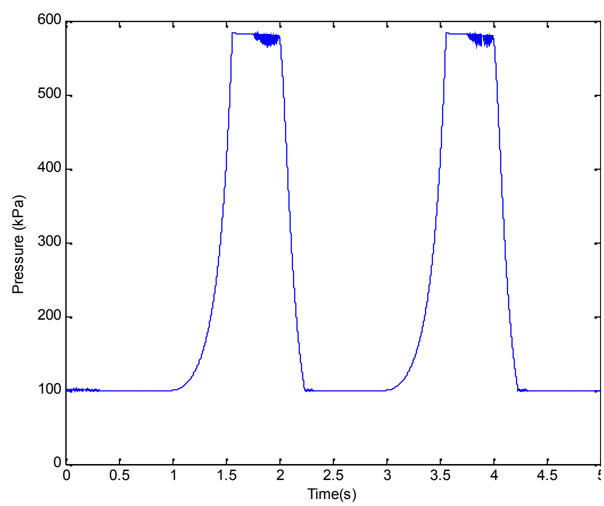


Figure 8: Simulated Pressure in the cylinder.

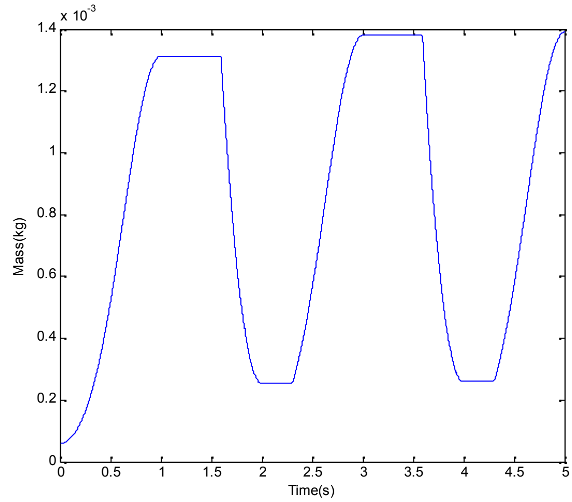


Figure 10: Simulated mass in the cylinder.

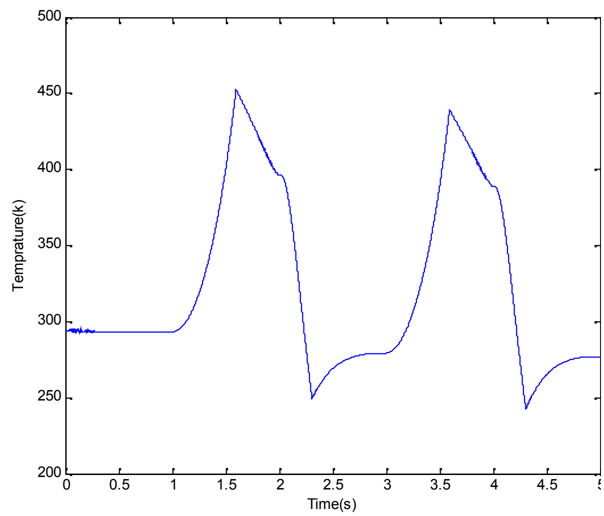


Figure 9: Simulated Temperature in the cylinder.

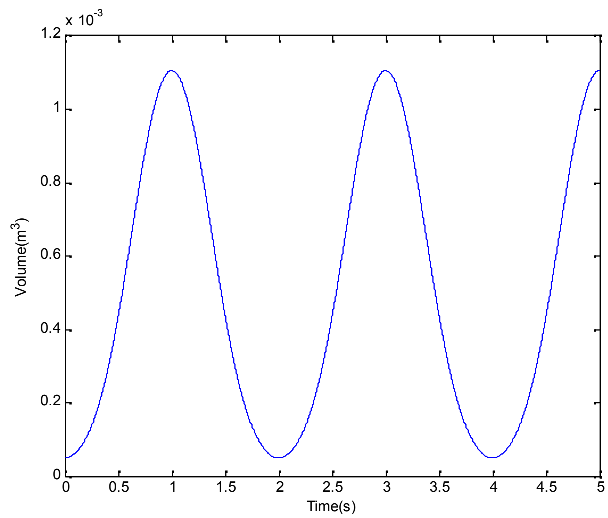


Figure 11: Simulated volume of the cylinder.

To see if the developed model of the complex thermodynamic processes is an “adequate” description of the real phenomenon, we have compared it to FEM model developed by COMSOL Multiphysics. The procedure of Finite Element modeling is described by the author in another paper. (Heidari and Barrade, 2011). The p-V indicator diagram of both models has been shown in Figure12. The generality of the models show a good agreement. The slight difference in the results comes from the difference in accuracy of heat transfer prediction and also difference in defining the nature of valves.
Since the FEM model is more complicated, it can better predict the heat transfer between air and the cylinder body.

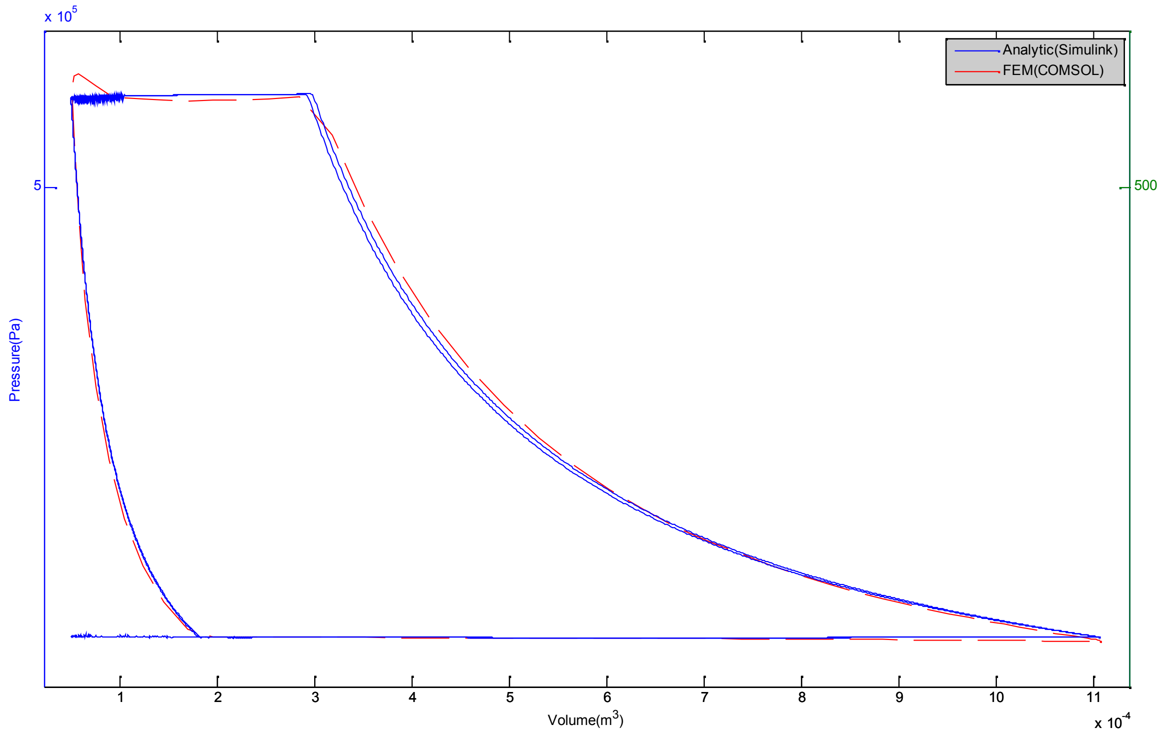


Figure 12: P-V diagram of FEM and analytical model.

5. CONCLUSIONS

In this paper the use of pseudo EMR approach for representing a reciprocating compressor is presented. It is shown that pseudo EMRs are a very powerful resource for the modeling of convective (open) thermodynamic systems. Pseudo EMRs in combination with EMRs have pushed physical modeling a big step forward. The number of complex machines or systems which can be modeled in a straightforward manner using EMR techniques are greatly enhanced. Besides, the rather simple model developed here shows a good agreement with the more sophisticated and computationally expensive Finite Element Model.

NOMENCLATURE

The nomenclature should be located at the end of the text using the following format:

$\dot{m}$	Mass flow	(kg/s)	**Subscripts**	
$\dot{E}$	Enthalpy flow	(J/K)	s	suction
$\dot{V}$	Volume flow	(m^3/s)	i	inlet
T	Temperature	(k)	e	exit
p	Pressure	(Pa)	E	Exhaust
ω	Rotational speed	(Hz)		
τ	Torque	(N.m)		
A	Surface area	(m^2)		
c_p	Isobaric Heat Capacity	(N.m/kg.K)		

k	Heat capacity ratio	(-)
J_ω	Flywheel rotary Inertia	(kgm^2)
R	crank radius	(m)
L	connecting rod length	(m)

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