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ABSTRACT

A semi-empirical mathematical model is proposed to simulate the unsteady behavior of mass flow rate and power of reciprocating compressors. The model is based on thermodynamic equations fitted to calorimeter data sets of two compressors. The curve fitting suggests linear correlations between the measured values and the thermodynamic equations. Comparisons of computed and measured values of mass flow rate and power, in transient regime, were conducted for two fitted compressor curves. A good agreement of results was found for both compressors in start-up tests. One can conclude that the proposed semi-empirical model can be safely applied to dynamic simulations of the whole refrigeration system.

1. INTRODUCTION

The reciprocating compressor used in domestic refrigeration is responsible for most of the system required energy, if not all, and therefore, its performance enhancement is one of the main issues of system designers. Prediction of the compressor behavior can help designers to find out alternative solutions to reduce the refrigerator energy consumption. The modeling of reciprocating compressors for refrigeration appliances have been addressed by several authors. Such mathematical models can be divided into three categories: polynomial fits, semi-empirical and detailed models. In the first case, the correlations are fitted to calorimeter data, but they do not have any physical meaning and cannot be applied out of the test range (ARI, 1999, ASHRAE, 1993). The second approach, on the other hand, is based on simple thermodynamic correlations fitted to experimental data (Popovic and Shapiro, 1995, Jähnig et al., 2000, Kim and Bullard, 2002, Winandy et al., 2002, Hermes and Melo, 2006, Navarro et al., 2007a, Navarro et al. 2007b, Duprez et al. 2007). The third class is used to study details of the compressor design such as, valve flows, cylinder heat transfer, cylinder-piston leakage, bearing losses, etc. but requires large amount of data, e.g. valve geometry and stiffness, bearing gaps, which are only available to manufacturers (Dufour et al., 1995, Srinivas et al., 2002, Longo and Gasparella, 2003, Elhaj et al. 2008). CFD models are included in the last class, in which fluid flow and heat transfer within the cylinder and through the valves are considered three dimensional and turbulent (Yasar and Koças, 2007, Pereira et al., 2007).
Although the household refrigeration compressors work most of the time in transient condition, such as in on-off cycle and pull-down tests, the above models were developed for steady state situations as compressor design is based in a single condition. The compressor transient operation is usually useful when the whole refrigeration system is considered. The complexity of fluid expansion, two phase flows, etc. and time changes of properties make the system transient simulations very time consuming. Detailed models demand high computing times and are generally avoided in system simulations. The polynomial fits, on the other hand, should not be used as they do not have any physical meaning. Some authors (Koury et al., 2001, Ding, 2007, Hermes and Melo, 2008) have applied

semi-empirical models to dynamic simulations of the whole refrigeration system. Nevertheless, few works (Porkial, et al. 2002) have been conducted to verify the transient accuracy of such models.
The current work proposes a semi-empirical model to predict the performance of reciprocating compressor in transient regime. The model is based on thermodynamic equations fitted to manufacturer data by using linear correlations. Comparisons with experimental data show that these simple fitted correlations are quite accurate on the prediction of compressor mass flow rate and power.

2. MATHEMATICAL MODEL

The current model considers the compression process as a quasi-steady one, because of the compressor high rotational speed, the refrigerant compression is quite instantaneous in comparison to the response time of the whole refrigeration system. In other words, the compressor mass flow rate and power are instantaneously affected by any change of the boundary conditions.

2.1 Mass Flow Rate

The mass flow rate of an ideal compressor with clearance is (Gosney, 1992):

$$\dot{m}_i = \frac{\dot{V}_{sw}}{v_s}\eta_{vi} \quad , \tag{1}$$

where $\dot{V}_{sw}$ is the compressor swept volume rate, v_s is the specific volume at the suction port. v_s is computed as a function of pressure and temperature by employing the refrigerant superheating properties. η_{vi} is the volumetric efficiency of the ideal compressor, given by:

$$\eta_{vi} = 1 - c\left[\left(\frac{p_d}{p_s}\right)^{1/k} - 1\right] . \tag{2}$$

c is the clearance fraction, p_d and p_s are, respectively, the discharge and suction pressures and k is the refrigerant isentropic coefficient.
By analogy to the ideal compression, the actual mass flow rate can be defined as

$$\dot{m}_a = \frac{\dot{V}_{sw}}{v_s}\eta_{va} \quad , \tag{3}$$

where η_{va} is the volumetric efficiency of the real compressor. However, due to compression irreversibility, piston-cylinder leakages, suction and discharge valve throttling, suction gas heating and gas-to-cylinder wall heat transfer, η_{va} will be smaller than the ideal counterpart and consequently, the actual mass flow rate, $\dot{m}_a$.

2.2 Compressor Work

The isentropic compression work of an ideal compressor with clearance can be expressed as (Gosney, 1992):

$$w_i = p_s v_s \frac{k}{k-1}\left[\left(\frac{p_d}{p_s}\right)^{\frac{k-1}{k}} - 1\right] . \tag{4}$$

The actual compressor work, which is always higher than its isentropic counterpart because of thermodynamic and mechanical losses, can be simply calculated as

$$w_a = \frac{\dot{W}_a}{\dot{m}_a} \quad , \tag{5}$$

where $\dot{W}_a$ is the actual compressor power (measured).

3. CALORIMETER TESTS

By analyzing the isentropic model presented above, one can see that the variables affecting the compressor performance are: evaporating and condensing pressures (or temperatures), and gas specific volume at the suction port (indirectly, the suction pressure and temperature). Additionally, ambient conditions (air speed and temperature) also affect the performance of the real compressor because of heat losses.

In order evaluate the performance of the two compressors manufactured by EMBRACO (nominated here X and Y), thirteen calorimeter tests were conducted by controlling condensing, evaporating, suction line, ambient and compressor shell temperatures, as well as the air speed. Nine tests were performed at the ambient temperature of 32°C, combining three evaporation temperatures (-35, -25, -15°C) and three condensing temperatures (45, 55 and 65°C). Table 1 shows the measured values of mass flow rate, compressor power, shell, discharge line and suction line temperatures for those nine tests. The refrigerant employed was the R134a. The compressor shell temperature was measured at the shell middle height. The suction and discharge line temperatures were measured 100mm away from the compressor shell.

Table 1 – Calorimeter test results for compressors X and Y. Ambient temperature = 32ºC. Refrigerant R134a.

			Compressor X			**Compressor Y**		
			Condensing temperature (°C)			**Condensing temperature**		
		Measured Variable	**45**	**55**	**60**	**45**	**55**	**60**
Evaporating temperautre (°C)	**-35**	Mass flow rate (kg h^{-1})	2.21	1.92	1.75	2.48	2.01	1.79
		Compressor power (W)	102.68	102.53	100.65	92.0	87.0	83.5
		Compressor shell temperature (°C)	64.4	65.9	66.8	63.9	64.5	60.0
		Discharge line temperature (°C)	66.2	67.3	67.1	63.6	64.2	60.3
		Suction line temperature (°C)	39.4	40.6	39.3	40.5	44.5	41.2
	-25	Mass flow rate (kg h^{-1})	4.41	3.90	3.95	4.64	4.28	4.03
		Compressor power (W)	145.9	148.9	150.0	126.4	130.8	130.7
		Compressor shell temperature (°C)	67.4	70.1	72.3	63.5	65.1	66.0
		Discharge line temperature (°C)	75.1	81.4	82.5	72.9	74.8	76.6
		Suction line temperature (°C)	35.6	39.8	40.5	38.8	40.0	41.0
	-15	Mass flow rate (kg h^{-1})	7.69	7.04	6.74	7.77	7.38	7.16
		Compressor power (W)	192.5	202.5	210.1	162.4	175.0	179.8
		Compressor shell temperature (°C)	68.1	71.9	74.1	62.3	65.9	66.5
		Discharge line temperature (°C)	80.9	87.0	91.6	71.2	78.9	80.6
		Suction line temperature (°C)	30.9	32.5	34.8	29.9	34.6	34.0

Admitting the shell temperature depends directly on the ambient condition (air temperature and convection coefficient) and that such condition is different from that the whole refrigeration system is tested, the shell temperature was considered to be an independent variable. To verify the effect of the ambient condition on the compressor performance, three other tests were performed by varying the compressor shell temperature and the results. An additional test with a 43°C ambient temperature was conducted to check if the shell temperature could be really used as an independent variable. These last two set of results are not shown for lack of space.

As expected and shown in Table 1, both mass flow rate and compressor power are more sensitive to the evaporating than to the condensing temperature, because low pressures affects more significantly the pressure ratio. On the other hand, the mass flow rate varies slightly and the compressor power does not vary with the shell temperature.

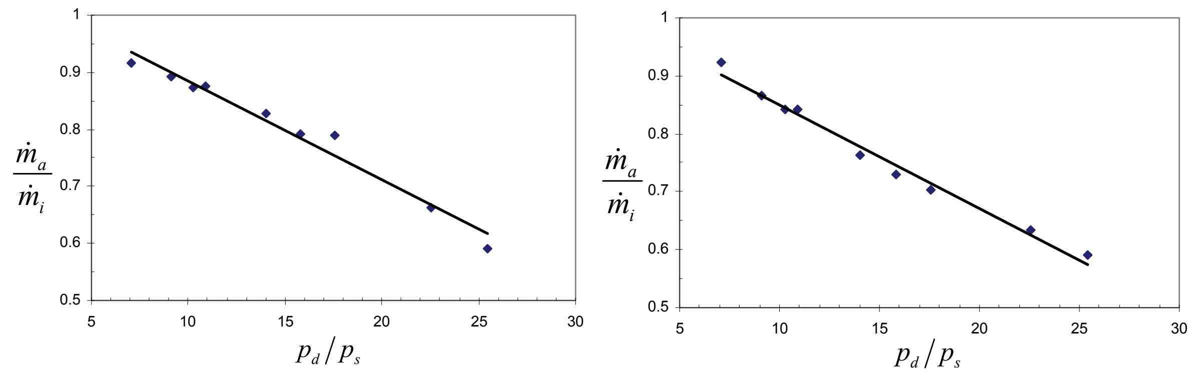
Besides the mass flow rate is as much insensitive to the ambient temperature as to the shell temperature. Additionally, both compressor powers do not change with either ambient or shell temperatures. As the product of equations (1) and (4) shows that the compressor power is independent of the specific volume, the compressor shell temperature should not affect it.

4. MODEL CALIBRATION

The difference between equations (1) and (3) is the volumetric efficiency. As already mentioned, the value of the actual volumetric efficiency is lower than its ideal counterpart because of thermodynamic losses and gas leakages that depend on operating conditions. As heat transfer and leakages are intensified with the increase of the discharge-to-suction pressure ratio, one may suggest that the actual-to-ideal volumetric efficiency ratio is also pressure ratio dependent. Figure 1, for example, shows the ideal-to-actual mass flow rate ratio as a function of the discharge-to-suction pressure ratio, for compressors X and Y. The clearance fractions of compressor X and Y needed for the calculation of the ideal volumetric efficiency were provided by EMBRACO. As can be seen, the higher the pressure ratio the lower the mass flow rate ratio and its value is always less than one. The coefficients of determination (R^2) of the straight lines of Figures 1a and 1b are, respectively, 0.985 and 0.971. As the suction port temperature is not usually measured in calorimeter tests, the compressor shell temperature was used to evaluate the gas specific volume in equation (1). This implies the mass flow rate ratio is a linear function of the pressure ratio.
Thus, from Figure 1, a straight line can be proposed as a correlation between the mass flow rate ratio and the pressure ratio:

$$\frac{\dot{m}_a}{\dot{m}_i} = \overline{\eta}_v = a + b\left(\frac{p_d}{p_s}\right) , \tag{6}$$

where $\overline{\eta}_v$ is the volumetric efficiency ratio ($= \eta_{va}/\eta_{vi}$), a and b are the linear and the angular coefficient of the straight line which are fitted to calorimeter data. The a and b values for compressors X and Y are shown in Table 4. According to equation (6), $\overline{\eta}_v$ gets close to one as the discharge pressure approaches the suction pressure. This would be expected as the losses and leakages decrease with the drop in the pressure ratio. Table 4 also shows the coefficient of determination and the largest difference between the measured and calculated mass flow rates. Note that the largest difference lies within the 5% uncertainty estimated for the calorimeter tests.



(a) (b)

Figure 1 – Ideal-to-actual mass flow rate ratio as a function of the pressure ratio. (a) Compressor X and (b) compressor Y.

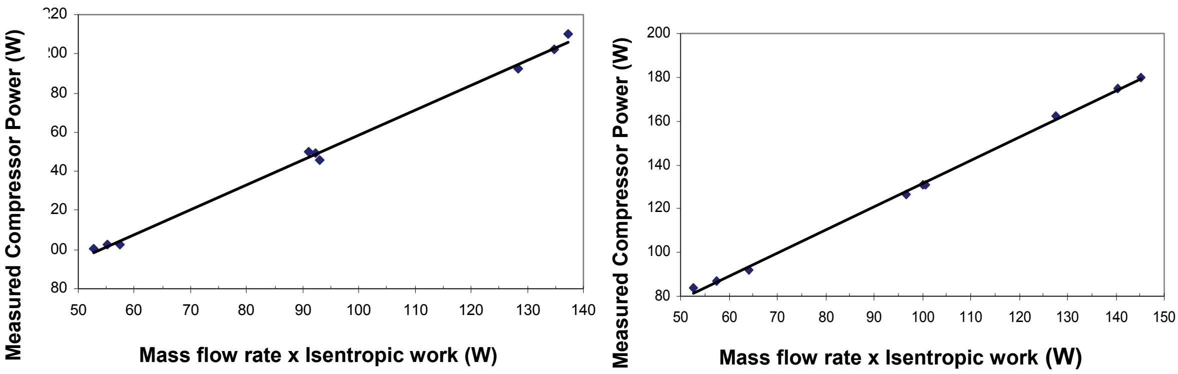
Despite losses and thermodynamic irreversibility, the compressor power must be related to the thermodynamic compressor work. In order to check the correlation, the measured compressor power was plotted against the product of mass flow rate and isentropic compression work for both compressors X and Y, as presented in Figure 2. Straight lines are fitted to the points of Figure 2a and 2b and the coefficients of determination are, respectively, 0.997 and 0.999. The following correlation was thus proposed for the calculation of compressor power:

$$\dot{W}_a = \dot{m}_a w_a = \dot{W}_l + \frac{\dot{m}_a w_i}{\eta_g}, \qquad (7)$$

Table 4 – Fitted constants of equations (6) and (7) for compressor X and Y.

Compressor	**Mass flow rate**				**Compressor Power**			
	a (-)	b (-)	R^2	Largest Difference [%]	$\dot{W}_l$ (W)	η_g (-)	R^2	Largest Difference [%]
X	1.0282	-0.01781	0.985	-2.5	31.59	0.7860	0.997	2.8
Y	1.0579	-0.01733	0.971	4.6	25.09	0.9398	0.999	1.5

where $\dot{W}_l$ and $1/\eta_g$ are, respectively, the linear and angular coefficients of the straight line. $\dot{W}_l$ is suggested to be the power consumption for the unloaded compressor. The unloaded compressor still consumes energy to overcome the losses even if the refrigerant is not been compressed. On the other hand, η_g is a thermodynamic efficiency of the compression process. The values of $\dot{W}_l$ and $1/\eta_g$ are also shown in Table 4, together with the coefficient of determination and the largest difference between the measured and calculated power for compressors X and Y.
The linear functions (6) and (7) suggest that only two conditions would be necessary to calibrate either the mass flow rate or the power model. Note that the maximum and the minimum mass flow rate took place at the lowest and the highest pressure ratio, respectively. On the other hand, the minimum power occurred at the highest pressure ratio but the maximum power was found at the highest evaporation-highest condensation pressure ratio. Despite the linear behavior, extrapolation is not advised and therefore the two extremes of mass flow rate and power should be taken into account in the calibration. In order to assure a good fit, four conditions are suggested for the calibration: the maximum and the minimum pressure ratios, the highest evaporation-highest condensation pressure ratio and an intermediate pressure ratio.



(a) (b)

Figure 2 – Compressor power as a function of the product of the mass flow rate and the isentropic compressor work. (a) Compressor X and (b) compressor Y.

5. MODEL VALIDATION

In order to validate the model, start-up tests were carried out in a 300 liter vertical freezer: The freezer was tested with both compressors X and Y at ambient temperature of 32°C. The characteristics of the refrigeration system are described in Table 5. Two pressure transducers were installed, respectively, at the compressor suction and discharge lines. In order to measure temperatures, T type thermocouples were placed at the compressor shell and at the suction

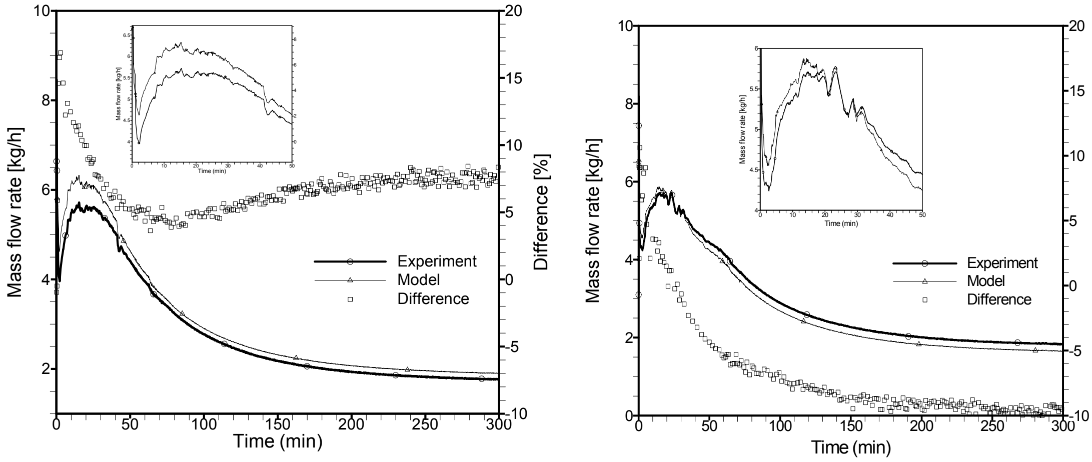
and discharge lines. The suction line thermocouple was located 100mm away from the compressor shell and the discharge line one, 150mm away. A Coriollis flowmeter was installed at the compressor discharge line and was thermally isolated from the ambient. To check the influence of the flowmeter on the system performance, the freezer was tested without and with it. A reduction on the condenser superheating region was observed, which did not affect significantly the system performance as a whole.
The compressor electric power was measured by a wattmeter. A data acquisition system was used and all variables were recorded every 4 seconds. The instruments uncertainties are shown in Table 6.
Comparisons of measured and computed values of mass flow rate and compressor power were conducted for both compressors X and Y. The computed variables were based on measured boundary conditions such as, suction and discharge pressures and compressor shell temperature.
Figure 3 shows comparisons of the measured and computed mass flow rates of both compressors for the start-up test. Not only the results are quite close – the differences lie within -4 to +18% for compressor X and within -10 to 12% for compressor Y - but also the curve shapes are very similar (see the zoom at the upper part of the figure), meaning the quasi-steady state hypothesis is adequate.
Figure 4 shows the comparison of the computed and the measured power. The differences lie under +10% for compressor X and -5% for compressor Y.

Table 5 – Characteristics of the refrigeration system.

Refrigeration system	Vertical Freezer
Capacity	300 liters
Evaporator type	Roll-bond
Condenser type	Wire-and-tube
Capillary tube-suction line heat exchanger	Concentric counter-flow
Fluid refrigerant	R134a

Table 6 – Measuring instruments and their uncertainties.

Variable	**Instrument**	**Uncertainty**
Temperature	T-type thermocouple	0.2 ^oC
Discharge pressure	Absolute transducer	0.03 bar
Suction pressure	Absolute transducer	0.03 bar
Mass flow rate	Coriollis	0.012 kg h^{-1}
Compressor power	Wattmeter	0.2% of the measuring value



(a) (b)

Figure 3 – Comparison of the measured and computed mass flow rate for the start-up test. (a) Compressor X and (b) compressor Y.

7. CONCLUSIONS AND DISCUSSIONS

In the current work, a semi-empirical model to predict the transient mass flow rate and the power of domestic refrigeration compressors was presented. The model was based on thermodynamic equations and they were fitted to

calorimeter test data sets of two compressors. The analysis showed that the actual-to-ideal volumetric efficiency ratio is linearly dependent on the discharge-to-suction pressure ratio and that the compressor power is a linear function of the product of mass flow rate and compressor isentropic work.
The start-up transient experiment was used to validate the current model. The mass flow rate differences were found within -4% to +18% for the compressor X and -10 to 12% for compressor Y in the transient period. In steady state, the observed differences for compressors X and Y were 8% and -10%, respectively.
The measured and computed curves are quite with all differences lying within 0 to +10% and 0 to 5% for compressors X and Y, respectively.
Considering the good agreement with experimental values, one concludes that the proposed semi-empirical model can be applied to dynamic simulations of reciprocating compressors. Therefore, the process can be considered quasi-steady because the pressure changes affect quite instantly the mass flow and the compressor power. The model can be used to dynamic evaluations of the whole refrigeration system performance.
In refrigeration systems tests, the compressor electric power is usually measured but not the mass flow rate. Therefore, the mass flow rate can be obtained from the power correlation. This estimation can be quite accurate, as it is better fitted to calorimeter tests than the mass flow correlation. Consequently, the dynamic system capacity can be continuously computed based on the compressor measured electric power.

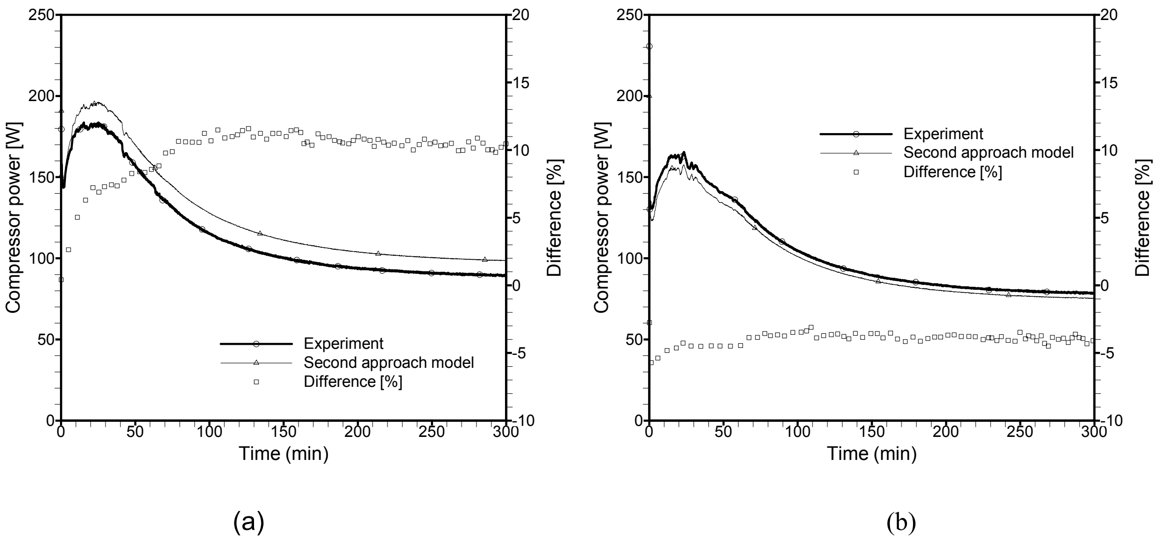


Figure 4 - Comparison of the measured with the calculated compressor power by employing the computed mass flow rate. The start-up test. (a) Compressor X and (b) compressor Y.

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