A Study of the Piston Cylinder Interface of Axial Piston Machines

Daniel Mizell
Purdue University

Follow this and additional works at: https://docs.lib.purdue.edu/open_access_dissertations

Recommended Citation
https://docs.lib.purdue.edu/open_access_dissertations/1876

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
A STUDY OF THE PISTON CYLINDER INTERFACE OF AXIAL PISTON MACHINES

A Dissertation
Submitted to the Faculty
of
Purdue University
by
Daniel Mizell

In Partial Fulfillment of the
Requirements for the Degree
of
Doctor of Philosophy

May 2018
Purdue University
West Lafayette, Indiana
THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF COMMITTEE APPROVAL

Dr. Monika Ivantysynova, Chair
Department of Agricultural and Biological Engineering

Dr. Andrea Vacca
Department of Agricultural and Biological Engineering

Dr. Farshid Sadeghi
Department of Mechanical Engineering

Dr. Karthik Ramani
Department of Mechanical Engineering

Approved by:
Dr. Jay P. Gore
Head of the Graduate Program
To my wife Ashby and son Troy, my refuge and inspiration.
ACKNOWLEDGMENTS

I would like to first thank Dr. Ivantysynova for offering me the opportunity to be a part of the Maha Fluid Power Research Center team. Her guidance and example have been an invaluable influence and inspiration to me and my research. Through my circuitous journey of discovery her advice and encouragement have been unfailing.

I also need to thank my many coworkers at the Maha lab. Each and every one has contributed in some way to the wonderful experience I have had. Special thanks to Matteo Pelosi whose PhD work formed my starting point, and to Andrew Schenk whose extensive tutoring enables me to use and maintain the Maha computer network. Thanks also to the pump modeling group, particularly Lizhi Shang, Ashley Busquets (Wondergem), and Meike Ernst for their help in testing the model, discussion, and suggestions. Thanks as well to all those who assisted me with the Tribo test rig, especially Anthony Franklin, Anna Garcia Teruel, and Cory Raizor. Finally, a special thanks to Susan Gauger and Connie McMIndes who do so much to brighten the lives of the Maha family.

Most of all I must thank my family for their support and encouragement throughout. Thanks to my wife Ashby for encouraging me to make the most of this opportunity, and for her constant love and support throughout my studies. Thanks to my parents and parents-in-law for their support and guidance. You have all made this possible.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>SYMBOLS</td>
<td>xi</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xv</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. STATE OF THE ART</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Modeling of the Piston Cylinder Interface</td>
<td>3</td>
</tr>
<tr>
<td>2.1.1 Piston Dynamics</td>
<td>5</td>
</tr>
<tr>
<td>2.1.2 Calculating Fluid Pressure Build Up</td>
<td>7</td>
</tr>
<tr>
<td>2.1.3 Surface Wear Profiles</td>
<td>7</td>
</tr>
<tr>
<td>2.1.4 Force Balance</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Experimental Investigation of the Piston Cylinder Interface</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Modeling of Tribological Point and Line Contacts</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Research Goals</td>
<td>21</td>
</tr>
<tr>
<td>3. THE NOVEL PISTON CYLINDER INTERFACE MODEL</td>
<td>23</td>
</tr>
<tr>
<td>3.1 Pressure Field Model and Calculation</td>
<td>24</td>
</tr>
<tr>
<td>3.2 Surface Wear Profiles</td>
<td>26</td>
</tr>
<tr>
<td>3.2.1 The Fluid Grid</td>
<td>30</td>
</tr>
<tr>
<td>3.3 Solid Body Temperature Distribution</td>
<td>33</td>
</tr>
<tr>
<td>3.4 Solid Body Pressure Deformation</td>
<td>33</td>
</tr>
<tr>
<td>3.5 Preventing Fluid Film Collapse</td>
<td>36</td>
</tr>
<tr>
<td>3.5.1 Linear Method</td>
<td>38</td>
</tr>
<tr>
<td>3.5.2 Iterative Method</td>
<td>40</td>
</tr>
<tr>
<td>4. SIMULATION RESULTS AND MEASUREMENT COMPARISON</td>
<td>42</td>
</tr>
<tr>
<td>4.1 The Tribo Test Rig</td>
<td>42</td>
</tr>
<tr>
<td>4.2 Tribo Test Rig Data Processing</td>
<td>44</td>
</tr>
<tr>
<td>4.3 Measured Operating Conditions</td>
<td>48</td>
</tr>
<tr>
<td>4.3.1 500rpm 80bar</td>
<td>48</td>
</tr>
<tr>
<td>4.3.2 500rpm 120bar</td>
<td>53</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Tribo test rig operating condition.</td>
</tr>
<tr>
<td>4.1</td>
<td>Operating condition information 500rpm 80 bar.</td>
</tr>
<tr>
<td>4.2</td>
<td>Operating condition information 500rpm 120 bar.</td>
</tr>
<tr>
<td>4.3</td>
<td>Operating condition information 500rpm 150 bar.</td>
</tr>
<tr>
<td>4.4</td>
<td>Operating condition information 500rpm 190 bar.</td>
</tr>
<tr>
<td>7.1</td>
<td>EHD test rig operating condition.</td>
</tr>
<tr>
<td>7.2</td>
<td>EHD pressure sensor model parameters.</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>6</td>
</tr>
<tr>
<td>2.3</td>
<td>11</td>
</tr>
<tr>
<td>2.4</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>14</td>
</tr>
<tr>
<td>2.7</td>
<td>18</td>
</tr>
<tr>
<td>2.8</td>
<td>19</td>
</tr>
<tr>
<td>2.9</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>24</td>
</tr>
<tr>
<td>3.2</td>
<td>28</td>
</tr>
<tr>
<td>3.3</td>
<td>29</td>
</tr>
<tr>
<td>3.4</td>
<td>32</td>
</tr>
<tr>
<td>3.5</td>
<td>35</td>
</tr>
<tr>
<td>3.6</td>
<td>37</td>
</tr>
<tr>
<td>3.7</td>
<td>39</td>
</tr>
<tr>
<td>3.8</td>
<td>41</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>4.1</td>
<td>43</td>
</tr>
<tr>
<td>4.2</td>
<td>45</td>
</tr>
<tr>
<td>4.3</td>
<td>46</td>
</tr>
<tr>
<td>4.4</td>
<td>47</td>
</tr>
<tr>
<td>4.5</td>
<td>47</td>
</tr>
<tr>
<td>4.6</td>
<td>50</td>
</tr>
<tr>
<td>4.7</td>
<td>51</td>
</tr>
<tr>
<td>4.8</td>
<td>54</td>
</tr>
<tr>
<td>4.9</td>
<td>52</td>
</tr>
<tr>
<td>4.10</td>
<td>55</td>
</tr>
<tr>
<td>4.11</td>
<td>56</td>
</tr>
<tr>
<td>4.12</td>
<td>58</td>
</tr>
<tr>
<td>4.13</td>
<td>60</td>
</tr>
<tr>
<td>5.1</td>
<td>65</td>
</tr>
<tr>
<td>5.2</td>
<td>66</td>
</tr>
<tr>
<td>5.3</td>
<td>70</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>5.4</td>
<td>Example of superposition of two rectangular areas of applied pressure on half space deformation model.</td>
</tr>
<tr>
<td>5.5</td>
<td>Pressures applied to deformation model in refined areas.</td>
</tr>
<tr>
<td>5.6</td>
<td>Sample <em>HD</em> analysis output for single analysis region. Color indicates fluid pressure in <em>bar</em>, contours indicate film thickness in <em>µm</em>.</td>
</tr>
<tr>
<td>5.7</td>
<td>Sample <em>HD</em> analysis cumulative output. Circle size indicates required load support. Tribo Test Rig, 500<em>rpm</em>, 120<em>bar</em>.</td>
</tr>
<tr>
<td>6.1</td>
<td>Nonuniformity of film thickness <em>Uₘ</em> in one dimensional grid.</td>
</tr>
<tr>
<td>7.1</td>
<td>Detail view of pressure sensor relationship to lubricating gap.</td>
</tr>
<tr>
<td>7.2</td>
<td>Temperature field of EHD cylinder surface measured (top) and simulated (bottom).</td>
</tr>
<tr>
<td>7.3</td>
<td>Pressure field of EHD fluid film simulated at 45° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).</td>
</tr>
<tr>
<td>7.4</td>
<td>Pressure field of EHD fluid film simulated at 90° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).</td>
</tr>
<tr>
<td>7.5</td>
<td>Pressure field of EHD fluid film simulated at 135° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).</td>
</tr>
<tr>
<td>7.6</td>
<td>Pressure field of EHD fluid film simulated at 270° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).</td>
</tr>
<tr>
<td>7.7</td>
<td>Simulated correction forces for HD piston cylinder model (top) and standard definition model (bottom).</td>
</tr>
<tr>
<td>7.8</td>
<td>Simulated axial friction forces for HD piston cylinder model and standard definition model.</td>
</tr>
<tr>
<td>7.9</td>
<td>Simulated power losses for HD piston cylinder model and standard definition model.</td>
</tr>
<tr>
<td>7.10</td>
<td>Simulated leakage flow for HD piston cylinder model and standard definition model.</td>
</tr>
<tr>
<td>7.11</td>
<td>Simulated leakage flow for HD piston cylinder model and standard definition model.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>7.12</td>
<td>Simulated film thickness at 90° for HD piston cylinder model (top) and standard definition model (bottom).</td>
</tr>
<tr>
<td>7.13</td>
<td>Absolute difference in simulated film thickness between HD piston cylinder model and standard definition model.</td>
</tr>
</tbody>
</table>
SYMBOLS

\begin{align*}
a & \quad \text{Length of area of applied pressure} & [m] \\
b & \quad \text{Width of area of applied pressure} & [m] \\
B & \quad \text{Temperature gradient interpolation matrix} & [-] \\
C & \quad \text{Proportionality constant} & [\text{Pa m}] \\
C_T & \quad \text{Constitutive matrix} & [-] \\
C_{\rho T} & \quad \text{Empirical constant} & [-] \\
D_c & \quad \text{EHD test rig capillary diameter} & [m] \\
D_o & \quad \text{Outer diameter of gap region which impacts EHD pressure sensor} & [m] \\
D_s & \quad \text{EHD test rig pressure sensor diameter} & [m] \\
dF & \quad \text{Piston force imbalance} & [N] \\
dv & \quad \text{Search direction unit vector} & [-] \\
e & \quad \text{Piston position} & [m] \\
e & \quad \text{Piston shifting velocity} & [\text{m s}^{-1}] \\
e_{\text{trial}} & \quad \text{Trial piston shifting velocity} & [\text{m s}^{-1}] \\
E' & \quad \text{Equivalent stiffness of piston cylinder contact} & [\text{Pa}] \\
E_K & \quad \text{Young's modulus of piston} & [\text{Pa}] \\
E_Z & \quad \text{Young's modulus of cylinder} & [\text{Pa}] \\
F_{aK} & \quad \text{Force acting on piston/slipper center of mass due to axial acceleration} & [N] \\
F_{cK} & \quad \text{Sum of correction forces acting on piston} & [N] \\
F_{DK} & \quad \text{Resultant force of displacement chamber pressure} & 
\end{align*}
acting on piston \([N]\)

\[F_e\] Sum of external forces acting on piston \([N]\)

\[F_f\] Sum of fluid forces acting on piston \([N]\)

\[F_{SK}\] Reaction force between slipper and swashplate \([N]\)

\[F_{SKy}\] Side loading component of reaction force between slipper and swashplate \([N]\)

\[F_{TG}\] Viscous drag force acting between slipper and swashplate \([N]\)

\[F_{TK}\] Viscous friction acting on piston within the lubricating gap area \([N]\)

\[F_{\omega K}\] Force acting on piston/slipper center of mass due to centripetal acceleration \([N]\)

\[G_0\] Empirical constant [-]

\[h\] Fluid film thickness \([m]\)

\[h_b\] Fluid film thickness contribution of bottom surface \([m]\)

\[h_K\] Fluid film thickness contribution that moves with piston surface \([m]\)

\[h_{max}\] Maximum film thickness \([m]\)

\[h_{min}\] Minimum film thickness \([m]\)

\[h_s\] EHD test rig sensor volume height \([m]\)

\[h_T\] Heat transfer coefficient \([\frac{W}{m^2\cdot K}]\)

\(i\) Index [-]

\(K\) Fluid bulk modulus \([Pa]\)

\(K_{cd}\) Thermal conduction matrix within body \([\frac{C}{W}]\)

\(K_{cv}\) Thermal convection matrix at body surface \([\frac{C}{W}]\)

\(K_T\) Thermal conduction matrix \([\frac{C}{W}]\)

\(l_c\) EHD test rig capillary length \([m]\)

\(l_F\) Cylinder guide length \([m]\)

\(\dot{m}_c\) Mass flow through EHD pressure sensor capillary \([\frac{kg}{s}]\)
\( m_K \) Mass of piston slipper assembly \([kg]\)

\( n \) Normal vector \([-]\)

\( N \) Natural coordinate matrix \([-]\)

ODC Outer dead center \([-]\)

\( p \) Fluid pressure \([Pa]\)

\( P \) Dimensionless pressure \([-]\)

\( p_{DC} \) Displacement chamber pressure \([Pa]\)

\( p_g \) Pressure in EHD test rig gap surrounding pressure sensor \([Pa]\)

\( p_s \) Pressure in EHD test rig sensor volume \([Pa]\)

\( q_b \) Heat flux from lubricating gap \( \left[ \frac{W}{m^2} \right] \)

\( Q_b \) Heat load from lubricating gap \([W]\)

\( Q_{cv} \) Convective load \([W]\)

\( R_i \) Inner radius of gap region which impacts EHD pressure sensor \([m]\)

\( R_K \) Piston radius \([m]\)

\( R_o \) Outer radius of gap region which impacts EHD pressure sensor \([m]\)

\( R_Z \) Cylinder radius \([m]\)

\( S \) Surface \([m^2]\)

\( S_a \) Empirical constant \([-]\)

\( t \) Time \([s]\)

\( T \) Temperature \([C]\)

\( T_0 \) Reference temperature \([C]\)

\( T_i \) Nodal temperature \([C]\)

\( T_\infty \) Ambient temperature \([C]\)

\( u_a \) Velocity in x direction of top surface \(\left[ \frac{m}{s} \right]\)

\( u_b \) Velocity in x direction of bottom surface \(\left[ \frac{m}{s} \right]\)

\( U_h \) Nonuniformity of film thickness \([-]\)
\( \hat{u}_K \)  Piston circumferential surface velocity \([\text{m/s}]\)

\( V \)  Volume \([\text{m}^3]\)

\( v_a \)  Velocity in y direction of top surface \([\text{m/s}]\)

\( v_b \)  Velocity in y direction of bottom surface \([\text{m/s}]\)

\( \hat{u}_K \)  Piston axial surface velocity \([\text{m/s}]\)

\( V_s \)  EHD test rig pressure sensor volume \([\text{m}^3]\)

\( \hat{w} \)  Half space surface deformation \([\text{m}]\)

\( w_a \)  Normal squeeze velocity of top surface \([\text{m/s}]\)

\( w_b \)  Normal squeeze velocity of bottom surface \([\text{m/s}]\)

\( Z_0 \)  Empirical constant \([-\text{]}\)

\( \Delta t \)  Simulation time step \([\text{s}]\)

\( \Delta v \)  Line search magnitude \([\text{m/s}]\)

\( \mu \)  Fluid viscosity \([\text{Pa} \cdot \text{s}]\)

\( \nu_K \)  Piston Poisson ratio \([-\text{]}\)

\( \nu_Z \)  Cylinder Poisson ratio \([-\text{]}\)

\( \rho \)  Fluid density \([\text{kg/m}^3]\)

\( \rho_0 \)  Reference density \([\text{kg/m}^3]\)

\( \sigma \)  Correction stress \([\text{Pa}]\)

\( \phi \)  Shaft angle \([\text{rad}]\)

\( \phi_K \)  Angular position within lubricating gap \([\text{rad}]\)

\( \omega \)  Shaft rotational velocity \([\text{rad/s}]\)

\((x, y, z)\)  Global coordinate system of axial piston machine \([\text{m}]\)

\((\hat{x}, \hat{y}, \hat{z})\)  Coordinate system in unwrapped gap \([\text{m}]\)

\((x_K, y_K, z_K)\)  Piston local Cartesian coordinate system \([\text{m}]\)

\((X, Y)\)  Coordinate of calculated pressure \([\text{m}]\)

\((X_1, Y_1)\)  Central coordinate of applied pressure \([\text{m}]\)
ABSTRACT

Mizell, Daniel PhD, Purdue University, May 2018. A Study of the Piston Cylinder Interface of Axial Piston Machines. Major Professor: Monika Ivantysynova, School of Mechanical Engineering.

The piston cylinder interface of axial piston machines of swash plate type is one of three critical lubricating interfaces that are responsible for proper machine operation. The interface must simultaneously bear large time changing external loads while preventing excessive leakage or friction. For long term machine reliability, a full fluid film must be maintained between the piston and cylinder surfaces. The goal of this work is to further the understanding of the phenomena contributing to fluid film behavior. A novel multi-body non-isothermal fluid-structure-thermal-interaction piston cylinder interface model is introduced that considers compressible fluid flow, squeeze due to transient deformation, as well as realistic surface profiles based on profilometer measurements. Piston force balance and correction forces are examined in instances where the fluid pressure build up numerically calculated on the standard coarse grid does not fully support the required load. Results of the piston cylinder model are verified by comparison to measurements made using a special purpose test pump. Small areas of collapsed film due to insufficient calculated load support are further investigated through a novel High Definition model that individually refines the analysis of each area of collapsed film. Dynamic grid refinement and a linear half space pressure-deformation model are employed to show the potential for full film load support in these areas. Combining the developed dynamic grid refinement
method with the full piston cylinder model and comparing to measurements confirms full film lubrication is occurring.
1. INTRODUCTION

The transmission of power through fluid power systems is useful in many applications. Fluid power systems present a high power density and contain fewer hazardous materials compared to electrical solutions, and are more flexible in control and configuration than purely mechanical powertrains. For these reasons, fluid power systems are found in widely varying applications ranging from earth moving equipment to aerospace. Power in such systems must first be converted from mechanical to fluid power, typically by means of a hydraulic pump. Power is transmitted by the flow of pressurized fluid through pipes and hoses to a location potentially remote from the prime mover. Fluid energy can be used immediately by an actuator such as a linear acting cylinder, rotary actuator, or motor. Some systems are able to store hydraulic energy for later use by compressing gas or a spring in a hydraulic accumulator.

The axial piston hydraulic machine is found in many applications, where it can function as a pump and motor. The principle advantage of axial piston machines is their variable displacement, giving them the ability to vary the volume of fluid displaced per shaft rotation. Among pump and motor designs which achieve variable displacement, axial piston machines of swash plate type are a popular choice. This results from their relative simplicity, high efficiency, and resilience.

Axial piston hydraulic machines rely on hydrodynamic lubrication for satisfactory operation. There are three main lubricating interfaces as shown in Figure 1.1, where machine parts must remain separated by a thin film of oil despite large and time-changing loads pressing the parts together. These interfaces exist between the cylinder block and valveplate, between the slipper and swashplate, and between the piston and cylinder. In addition to preventing solid contact between the machine
parts, the fluid films also function to prevent excessive leakage of the high pressure fluid from the displacement chambers into the case volume. This combination of requirements distinguishes the lubricating interfaces of axial piston machines from simpler hydrodynamic interfaces, which typically perform a single sealing or bearing function.

Piston motion is driven by the rotation of the cylinder block around the shaft axis. The pistons are forced to move with the cylinder bores, which causes the slippers to move across the surface of the swashplate. When the swashplate surface is inclined with respect to the pump axis, a reciprocating motion is induced in the piston slipper assemblies. This reciprocating motion causes a periodic change in displacement chamber volume. The rotation of the cylinder block relative to the valve plate also causes each displacement chamber to be connected at appropriate times to the inlet and outlet ports. In this way, the increase and decrease in volume, respectively, causes a net fluid flow from the inlet port to the outlet port.
2. STATE OF THE ART

2.1 Modeling of the Piston Cylinder Interface

Many researchers have worked to understand the operation of the piston cylinder interface through numerical modeling. Yamaguchi [1] used the Reynolds equation to perform a stability analysis on pistons of varying shape. The model included the expansion of the cylinder bore due to high pressure in the displacement chamber, but did not consider piston micro-motion within the lubricating gap. Analysis presented in the paper indicate that an axially tapered piston displays improved stability compared to cylindrical designs. Yamaguchi [2] included force equilibrium of the piston in a later work and concluded that metal to metal contact was almost invariably present. Sadashivappa et al. [3] developed a simple mathematical model of the piston cylinder interface to investigate out-of-round and tapered pistons. A comparison to experiment showed the model under predicted the measured leakage. Gels and Murrenhoff [4] developed a FEM based model considering an isothermal, isoviscous piston cylinder interface. Piston deformation was approximated using beam bending. The model was used to optimize the lubricating gap length and clearance as well as investigate tapered shapes for the piston and bushing. Kumar and Bergada [5] showed piston grooves to be beneficial for small diameter pistons using a rigid body CFD based model. Agarwal et al. [6] presented a model of the piston cylinder interface in radial piston pumps considering the Fluid Structure Interaction (FSI) between the lubricating film and its bounding surfaces. Kuzmin et al. [7] introduced a rigid body model investigating the relative rotation between piston and cylinder at low speed conditions.
Surface contact and mixed friction are incorporated in some models for conditions where the imposed load cannot be carried with a full lubricant film. Fang and Shirakashi [8] investigated the incidence of mixed friction between the piston and cylinder at low operating speeds. The model accounted for axial and rotational motion of the piston, and assumed a perfectly smooth, cylindrical, rigid piston and cylinder. An iteration scheme was used to find a piston position in which the external and fluid pressures balance. In the event of contact between the piston and cylinder a contact force was implemented to balance the remaining external load. Fatemi et al. [9] presented a model considering a simplified pressure deformation for the piston cylinder interface. The model used a modified form of the Reynolds equation to account for the surface roughness of the piston and cylinder.

Development of the model which forms the basis of the current work began with Berge [10] (now Ivantysynova) who developed a numerical model to calculate load support at the piston cylinder interface of axial piston machines. The model considered a rigid piston and cylinder and both axial and circumferential motion as well as a non-isothermal temperature field in the lubricating gap. Though load support was calculated and compared with external loads, the model did not adjust piston motion to achieve force balance. Wieczorek [11] built on this foundation a model for all three lubricating interfaces of axial piston machines. His model balanced external forces with fluid pressure buildup by calculating a piston micro squeeze motion within the cylinder bore. Ivantysynova and Huang [12] added Elastohydrodynamic deformation to the model using an influence matrix approach and an external commercial FEM program. Pelosi and Ivantysynova [13] implemented a thermal model which calculates the temperature distribution within the piston and cylinder block based on the heat flux from the lubricating gap. Using this model, Pelosi and Ivantysynova [14] showed that thermal deformations are critical to the operation of axial piston machines at high pressures. Pelosi [15] considered as well the effects of thermal stresses on the deformation of the piston and cylinder block.
The model used in this research is adapted from the work of Pelosi [15]. The primary considerations and shortcomings of Pelosi’s model are discussed below. Later chapters will describe additions and other improvements which have been made to the model which is used in the present work.

2.1.1 Piston Dynamics

Figure 2.1 shows the forces acting on the piston during machine operation. The largest source of external load comes from pressure in the displacement chamber, $p_{DC}$. The pressure in each displacement chamber changes with time as the machine operates as shown in Figure 2.2, as each chamber is connected alternately to the inlet and outlet ports. This pressure acting on the face of the piston causes an axial force $F_{DK}$. This axial force, along with the inertial force due to piston acceleration $F_{aK}$ and viscous friction from the lubricating gap $F_{TG}$ must be reacted by a normal force between the slipper and swashplate which is transferred to the piston as $F_{SK}$. Because the swashplate may be inclined, $F_{SK}$ may not act along the axis of the piston, resulting in a side-loading component $F_{SKy}$. The lubricating interface between piston and cylinder must carry the radial load on the piston resulting from the sum of $F_{SKy}$, the centrifugal acceleration force of the piston $F_{ωK}$, and the viscous drag force generated by the fluid film between the slipper and swashplate $F_{TG}$.

Figure 2.1. Forces acting on the piston cylinder interface.
Figure 2.2. Measured displacement chamber pressure from Tribo test rig, 500 rpm.
2.1.2 Calculating Fluid Pressure Build Up

The piston cylinder model developed by Pelosi [15] calculated fluid pressure build up using the incompressible form of the Reynolds equation:

\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( h^3 \frac{\partial p}{\partial y} \right) + 6\mu \left( \dot{u}_K \left( 2 \frac{\partial |h_b|}{\partial x} - \frac{\partial h}{\partial x} \right) + \dot{v}_K \left( 2 \frac{\partial |h_b|}{\partial y} - \frac{\partial h}{\partial y} \right) + 2 \frac{\partial h}{\partial t} \right) = 0
\]

(2.1)

In Pelosi’s model, the \( \frac{\partial h}{\partial t} \) term comes only from the rigid motion of the piston relative to the cylinder. The impact of changing pressure deformations with respect to time are neglected. Pelosi separates the various contributions to film thickness into components that affect only the bottom surface \( h_b \) and the total film thickness \( h \). This approach is adequate for rigid surfaces, however because the surfaces deform under pressure loading further considerations must be made. As the deformable piston moves relative to the fluid grid, its surface also moves relative to the pressure which is calculated within the fluid grid. Because pressure deformation is a direct result of pressure loading, the piston pressure deformation component of film thickness does not move with the piston, and instead should be considered stationary with respect to the fluid grid.

2.1.3 Surface Wear Profiles

Surface to surface contact can occur in isolated situations such as machine start up and shut down, as well as during an initial wearing in period. During the initial wear in period, material is often worn away from the softer material, typically near the ends of the cylinder bore. The resulting smooth tapering near the ends of the cylinder bore is capable of improved load support relative to the nominal cylindrical shape. Eventually, the bore wear accumulates enough to create a new surface shape that is capable of generating sufficient load support to prevent further contact during steady state operation. In this way, wear in the piston cylinder interface tends to be self-stabilizing.
In practice, the piston and cylinder surfaces deviate from their nominal cylindrical shape by up to tens of microns. This deviation in surface shape is due to both the wear discussed previously and manufacturing variability. The magnitude of these surface deviations can easily be larger than the thickness of the fluid film in critical areas of the lubricating gap and therefore must be considered in the model. In the model developed by Pelosi [15] only axisymmetric surface profiles are considered. This is typically adequate only for the piston profile, but the cylinder bore profile is normally more complex with the wear profile changing in shape and depth at various points around the circumference of the cylinder bore.

\subsection*{2.1.4 Force Balance}

In Pelosi’s model, the force balance calculation includes only fluid forces $F_f$ and external forces $F_e$ as shown in Figure 2.3. In cases where the fluid forces are not adequately capable of balancing the external forces, a method of preventing fluid film collapse is needed. Pelosi implemented a method referred to here as velocity correction to prevent collapse of the fluid film.

In the velocity correction method correction forces are only calculated after the force balance loop has converged. This arrangement presents minimal computational expense but has significant drawbacks. A correction stress $\sigma$ is first calculated for each fluid cell whose height is less than the specified minimum. The correction stress is linearly proportional to the difference between its film thickness and the minimum specified film thickness as shown in Figure 2.4. Because the surfaces being simulated are not ideal, the minimum film thickness is defined according to the surface micro-roughness of the piston and cylinder.

\[
\sigma (i) = \begin{cases} 
C (h_{\text{min}} - h (i)) & h (i) \leq h_{\text{min}} \\
0 & \text{otherwise}
\end{cases}
\] (2.2)
Choice of the proportionality constant $C$ is critical to simulation stability, and is typically chosen as

$$C = 2 \frac{E'}{R_K} \quad (2.3)$$

$E'$ is the equivalent Young’s modulus of the piston and cylinder surfaces in contact.

$$E' = \frac{1}{\frac{1-\nu_K^2}{E_K} + \frac{1-\nu_Z^2}{E_Z}} \quad (2.4)$$

The correction stress field is then integrated to the control points using:

$$\begin{align*}
F_{cK1} &= \int_0^{l_F} \int_0^{2\pi} \sigma \cos(\phi_K) \left(1 - \frac{z_K}{l_F}\right) d\phi_K dz_K \\
F_{cK2} &= \int_0^{l_F} \int_0^{2\pi} \sigma \sin(\phi_K) \left(1 - \frac{z_K}{l_F}\right) d\phi_K dz_K \\
F_{cK3} &= \int_0^{l_F} \int_0^{2\pi} \sigma \cos(\phi_K) \frac{z_K}{l_F} d\phi_K dz_K \\
F_{cK4} &= \int_0^{l_F} \int_0^{2\pi} \sigma \sin(\phi_K) \frac{z_K}{l_F} d\phi_K dz_K
\end{align*} \quad (2.5)$$

The position of the piston defined as the eccentricity of the piston axis at each end of the lubricating fluid film $e = \{e_1, e_2, e_3, e_4\}$ as shown in Figure 2.5 is then updated for the next time step:

$$e(t + \Delta t)_i = e(t)_i + \dot{e}(t)_i \Delta t + \frac{F_{cK,i}}{2m_K} (\Delta t)^2 \quad (2.6)$$

This equation assumes inertial acceleration of the piston resulting from the correction forces. There are several problems with this approach. First, because the correction is applied after the force balance loop has completed, the correction is not reflected in the fluid pressure build up. Neglecting the change in position when calculating the pressure build up in the next time step implies a breakdown of mass conservation because the cell volumes have changed without any change to the surrounding flows or pressures. Further, the presented method lacks the stabilizing influence of the viscous fluid film and can easily become unstable.
One approach to remedying the lack of stability is to limit the areas in which contact forces can be applied. In practice it is generally acceptable to limit the calculation of contact forces to the outermost several millimeters of cells on either end of the lubricating gap. This approach is applicable only to cases of cylindrical geometry. As the cylinder surface wears, or if a non-cylindrical piston design is simulated, fluid film breakdown tends to occur further away from the ends of the fluid film. Because the correction region is limited to small areas near the ends of the lubricating gap, large areas of collapsed film can occur before any correction force is calculated. This is demonstrated in Figure 2.6 where there can be a considerable area of penetration before correction stresses are calculated if only areas to the right of the green line are considered.
Figure 2.3. Force balance at the piston cylinder interface in the model developed by Pelosi [15].
Figure 2.4. Calculation of correction stress for velocity correction method.
Figure 2.5. Piston position in the model developed by Pelosi [15].
Figure 2.6. Area of collapsed film occurring with no correction forces calculated when wear profiles are introduced. Correction stress only considered to the right of green line.
2.2 Experimental Investigation of the Piston Cylinder Interface

The numerical modeling work above is complemented and verified by experimental work exploring various aspects of piston cylinder operation. Dowd and Barwell [16] designed a highly simplified test rig to investigate the impact of different material pairs on the tribological performance of the piston cylinder interface. They were able to detect contact between the piston and cylinder by monitoring the electrical resistance between the parts. The test rig was also able to measure friction forces acting on the cylinder. Construction details of their test rig prevented accurate loading conditions from being applied to the piston however. Hooke and Kakoullis [17] investigated the influence of slipper ball socket friction on piston rotation relative to the cylinder bore by observing piston rotation. The numerical model developed by Fang and Shirakashi [8] was tested using a specially modified pump. Contact between the piston and cylinder was again measured by monitoring the electrical resistance between the parts. Experiment showed that their model over predicted the amount of contact compared to the experimental data. Manring [18] used a simplified piston cylinder interface to measure axial friction forces and compared the results to the Strubeck friction curve. Tanaka et al. [19] investigated run-in wear of various piston profiles and stiffness values using a simplified two piston pump capable of measuring friction forces and metal to metal contact. Treuhaft et al. [20] also studied pump run-in wear behavior using radioactive tracer technology.

A special purpose test rig known as the Tribo was designed and constructed by Lasaar and Ivantysynova [21] which is capable of measuring the friction forces between the piston and cylinder. The cross section of the Tribo test pump is shown in Figure 2.7. A specially designed replaceable bushing is separated from the rotating cylinder block by a series of hydrostatic bearings. The bushing is held in place only through a mechanical connection to a force sensor. Any net friction force acting on the bushing will therefore be measured by the force sensor. Comparing the measured friction forces to those calculated by the piston cylinder model allows verification of
the modeling approach. A pressure sensor is also installed in the same displacement chamber for the purpose of measuring instantaneous displacement chamber pressure.

Signals from the friction force sensor as well as the pressure sensor are first amplified, and then transmitted through a frequency modulated radio telemetry system. The radio signals are received by an antenna near the pump, and the signal is demodulated and recorded using a Data Acquisition (DAQ) system. Due to the design of the radio telemetry system, only a single sensor channel can be transmitted at one time. It is therefore necessary to align separately measured data in post-processing according to a shaft trigger which trips at a specific shaft position once per revolution.

In the test hydraulic circuit implemented by Lasaar and Ivantysynova [21] the Tribo pump was connected to two other swash plate type machines on a common shaft. One unit recirculated flow in a test circuit between the Tribo pump and itself, while the other operated as a secondary controlled motor which provided power to make up for the losses in the test rig. Friction force measurements made by the test rig in this configuration contain a high noise level as shown in Figure 2.8. Sources for the noise can include mechanical vibrations induced in both the rotating group and also in the test stand itself by the three hydraulic machines. Additionally, because the majority of the fluid flowing through the Tribo pump was recirculated, control of the inlet fluid temperature was difficult to maintain. For these reasons a redesign of the Tribo test rig is desirable. The redesigned test rig described later in section 4.1 is designed to minimize the number of hydraulic machines located on the test stand, and also simplify the temperature control of the incoming fluid.

Looking again at Figure 2.8, the match between the axial friction profile simulated by the model of Pelosi [15] and the measured friction leaves much to be desired. Although the simulated friction line falls within the range of the measured friction forces, the shape of the simulated friction is poorly reflected in the measurement data. In the high pressure stroke between $0 - 180^\circ$ the simulated data is rather noisy, and matches the measurement poorly. In the low pressure stroke between $180 - 360^\circ$ the simulated friction profile is nearly flat, while the measurement shows
sustained activity. From this example it is clear that the model experiences difficulty in matching measurements. A solution to this problem must address both the quality of measurements and the accuracy of simulation.

Another special purpose test rig known as the EHD pump was designed by Everth [22, 23] to measure the temperature and pressure fields within the piston cylinder lubricating gap during operation. A cross section of the EHD test pump is shown in Fig. 2.9. The EHD is a reverse kinematic pump with a single piston located in a stationary cylinder block while a rotating “wobble plate” generates the pumping action. Fluid flow in the displacement chamber is controlled by check valves. Nine pressure sensors and nine thermocouples are arranged about the lubricating surface of the cylinder bore. Each thermocouple is diametrically opposite the location of a 0.3mm hole connecting to a pressure sensor. Every thermocouple and pressure sensor pair is located in a unique axial position. By rotating the block about its axis in 2° increments it is possible to measure steady state pump behavior around the entire circumference of the cylinder surface with each sensor. Data from each sensor and each angular position of the block are assembled into a measurement grid accounting for the location of each sensor during every measurement run. A shaft trigger that fires once per shaft revolution allows the synchronization of data from every measurement run, allowing the time changing pressure field to be visualized.

Table 2.1. Tribo test rig operating condition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Speed</td>
<td>1000 rpm</td>
</tr>
<tr>
<td>Inlet Pressure</td>
<td>20 bar</td>
</tr>
<tr>
<td>Outlet Pressure</td>
<td>170 bar</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>60°C</td>
</tr>
<tr>
<td>Outlet Temperature</td>
<td>64°C</td>
</tr>
<tr>
<td>Drain Temperature</td>
<td>72°C</td>
</tr>
</tbody>
</table>
Figure 2.7. Cross section view of Tribo test rig.
Figure 2.8. Sample Tribo test rig measurement compared to a simulation using the model developed by Pelosi [15].

Figure 2.9. Cross section view of EHD test rig.
2.3 Modeling of Tribological Point and Line Contacts

The study of tribological line and point contacts operating in the regime of elastohydrodynamic lubrication (EHL) faces many of the same challenges seen in the lubricating gaps of axial piston machines. Therefore a review of EHL models and techniques provides guidance in improving the piston cylinder interface model. Dowson and Higginson [24] solved the problem of EHL line contact numerically utilizing a linear half-space deformation model. An empirical model for fluid density and viscosity as functions of pressure and temperature was developed by Roelands [25]. Cheng [26] modeled EHL using a two dimensional solution for elliptical contacts, and compared the developed model to measurement data. Taylor and O’Callaghan [27] applied a FEM approach to the EHL problem. Brandt [28, 29] presented an adaptive multi-level approach to solving boundary value problems in which the grid is refined concurrently with the numerical solution of the problem. Hamrock and Dowson [30] presented a linear half space model for the calculation of elastic surface deformation under Hertzian contact stresses. Brewe et al. [31] utilized a grid with variable spacing in the investigation of EHL in point contacts, with additional grid refinement in the area of maximum pressure. Houpert and Hamrock [32] presented a solution to the compressible EHL problem using an analytical solution to surface deformations. Lubrecht et al. [33] applied the multigrid method to the elastohydrodynamic line contact problem. Kim and Sadeghi [34, 35] investigated the three dimensional thermal problem in a rolling and sliding point contact. Hu and Zhu [36] presented a solution for numerically calculating the load support in areas of mixed lubrication in point contacts by modifying the Reynolds equation in areas of asperity contact. Goodyer [37] explored transient effects in elastohydrodynamic lubrication. Habchi [38] achieved perfect agreement between EHL models with loose coupling between fluid and solid domains and a fully coupled model. For highly loaded contacts, the loosely coupled models were shown to be more computationally efficient. Each of
these resources inspires a closer look at load support in critical areas of the piston cylinder interface using the novel high definition model described in chapter 5.

2.4 Research Goals

It is believed that during steady state operation there is negligible contact between the piston and cylinder bore. The goal of this thesis is to achieve a piston cylinder model which accurately reflects experimentally observed behaviors while demonstrating full film lubrication. To achieve this the following shortcomings of the existing model must be addressed:

- Axisymmetric surface profiles are insufficient to realistically describe the shapes of cylinder bores after pump run-in.
- Pressure buildup calculations within the fluid film neglect key physical effects relating to squeeze motion of the gap surfaces.
- Handling of conditions of insufficient fluid load support is nonphysical and potentially unstable.
- Grids used for calculation of pressure buildup and surface deformations are too coarse to capture fine details of fluid film behavior.

It is believed that these shortcomings can be addressed by the following actions:

- Implement realistic surface profiles for all simulations.
- Improve pressure field calculations by including additional physical effects to the Reynolds equation.
- Improve handling of situations requiring additional load support.
- Implement an adaptive multigrid approach to refine critical areas of the fluid grid.
• Implement a linear half-space deformation model to calculate deformations with higher resolution than allowed by FEM grid.

The following key points are to be addressed with the Tribo test rig:

• Simplify the hydraulic test circuit to eliminate unnecessary mechanical vibrations.

• Improve control over the inlet oil temperature.
3. THE NOVEL PISTON CYLINDER INTERFACE MODEL

This chapter describes the development of a novel piston cylinder model which addresses the shortcomings detailed in the previous chapter. Model advances which address the shortcomings detailed previously are:

- Physical effects added to the Reynolds Equation:
  - Elastohydrodynamic (EHD) squeeze is added.
  - Accurate surface velocities are considered with respect to translational squeeze.
  - Compressible flow is considered.

- Realistic measured surface profiles in one or two dimensions are implemented for all simulated operating conditions.

- New methods for preventing fluid film collapse are introduced.

- All calculated forces acting on the piston are considered in the force balance.

- Simulation speed and efficiency improved by implementing Finite Element Method for the thermal analysis.

Figure 3.1 gives an overview of the modeling approach for the piston cylinder model. A finite volume method is used to calculate pressure build up in the lubricating gap. This pressure is applied to the surfaces of the solid parts, which results in deformations of the solid parts calculated using the Finite Elements Method (FEM). The resulting surface deformations are fed back into the fluid model as updated film thickness boundary conditions. Viscous dissipation in the lubricating film results
in heat generation which is conducted into the solid parts. This heat conduction is calculated using a FEM thermal analysis of each part. The calculated surface temperature distribution is also used to update the boundary conditions for the fluid model. The calculated temperature distribution within the solid parts is further used in a FEM analysis accounting for the impact of thermal stresses on the deformation of the solid parts. The resulting deformations are also fed back to the fluid model as updated film thickness boundary conditions. The following sections expand on modeling advances in each section of the model.

3.1 Pressure Field Model and Calculation

To achieve steady state operation, the radial load acting on the piston must be supported in the lubricating interface entirely through hydrodynamic and hydrostatic
pressure buildup. Unlike the other lubricating interfaces, the majority of the piston cylinder load must be carried by hydrodynamic pressure build up in the fluid film. Due to the cylindrical shape of the piston cylinder interface any hydrostatic pressure is distributed approximately equally around the circumference of the piston, providing little net load support. In some circumstances an imbalance of hydrostatic pressure can increase the load that must be carried by hydrodynamic build up. The Reynolds equation used in the present model is adapted from the general form found in Hamrock et al. [39]:

\[
\frac{\partial}{\partial \hat{x}} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \hat{y}} \right) = \frac{\partial}{\partial \hat{x}} \left( \frac{\rho h (u_a + w)}{2} \right) + \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h (v_a + v_b)}{2} \right) + \rho (w_a - w_b) - \rho u_a \frac{\partial h}{\partial \hat{x}} - \rho v_a \frac{\partial h}{\partial \hat{y}} + h \frac{\partial \rho}{\partial t} \tag{3.1}
\]

To adapt this general equation to the situation of the piston cylinder, two modifications are made. First, the substitution \( w_a - w_b = \frac{\partial h}{\partial t} \) is made which includes the change of film thickness due to both rigid motion and transient pressure deformation. Second, it is noted that \( u_b = v_b = 0 \) because the cylinder surface is stationary in the \((\hat{x}, \hat{y}, \hat{z})\) coordinate system.

The term \( h_K \) captures those components of the fluid film thickness that move along with the piston, contributing to the pressure build up in an effect known as translational squeeze. Specifically, these components are the rigid film thickness, the thermal deformation of the piston surface, and the wear profile of the piston surface. Note that pressure deformation of the piston does not contribute to the translational squeeze effect. This is because the fluid grid is fixed to the bushing. As such, the calculated pressure and by extension the pressure deformation of the piston are also non-moving with respect to the bushing. Should the pressure distribution within the fluid grid change over time, a corresponding change in piston pressure deformation will be seen. The impact of this effect on pressure build up is properly considered within the \( \frac{\partial h}{\partial t} \) squeeze term. The final form of the Reynolds equation used is shown below.
\[ \frac{\partial}{\partial \hat{x}} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial \hat{y}} \right) = \hat{u}_K \frac{\partial (\rho h)}{2\partial \hat{x}} + \hat{v}_K \frac{\partial (\rho h)}{2\partial \hat{y}} + \rho \frac{\partial h}{\partial t} \]

\[ - \rho \hat{u}_K \frac{\partial h_K}{\partial \hat{x}} - \rho \hat{v}_K \frac{\partial h_K}{\partial \hat{y}} + h \frac{\partial \rho}{\partial t} \]  

(3.2)

In some areas the fluid film thickness may drop below a specified minimum allowable height. In such cases, pressure build-up is calculated assuming the gap surfaces are parallel and separated by the minimum allowable height. To implement this assumption, the Reynolds equation is simplified by substituting zero for any film thickness gradient terms. The resulting simplified Reynolds equation is then:

\[ \frac{\partial}{\partial \hat{x}} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h^3}{12\mu} \frac{\partial \rho}{\partial \hat{y}} \right) = \hat{u}_K h \frac{\partial \rho}{2\partial \hat{x}} + \hat{v}_K h \frac{\partial \rho}{2\partial \hat{y}} + h \frac{\partial \rho}{\partial t} \]  

(3.3)

### 3.2 Surface Wear Profiles

As described in section 2.1.3, wear profiles are critical to the proper understanding of the lubricating interfaces. There must therefore be a robust method of simulating various surface shapes arising from surface wear, manufacturing processes, and various combinations of these effects. These profiles may be axisymmetric as is usually the case with pistons or non-axisymmetric which is typical of bushing surfaces.

Because the lubricating fluid film is very thin relative to the diameter of the piston, the curvature of the interface can be safely neglected. The fluid film can then be unwrapped as shown in Figure 3.2 where the rectilinear (\(\hat{x}, \hat{y}, \hat{z}\)) coordinate system is introduced. Using this coordinate system, a surface profile is constructed which defines the deviation from nominal for every (\(\hat{x}, \hat{y}\)) point for both the piston and cylinder. Because the piston translates axially and rotates about its axis during machine operation, these motions must be accounted for when interpolating the surface shapes to the fluid geometry.

The surface profile is specified over a user defined grid point by point, so the model developed in the current work can simulate any measured or theoretical surface shaping. In the case of a measured profile, a surface profilometer is used to trace the
profile of the piston or cylinder surface in the axial direction. By combining several traces spaced at intervals around the circumference of the part, a complete profile of the surface can be constructed as shown in Figure 3.3. Piston surface profiles are typically axisymmetric, and in such cases a single profilometer trace is sufficient to define the entire surface. While Pelosi’s model only considered axisymmetric shapes applied to the piston and cylinder, the present model accepts both one-dimensional (axisymmetric) and two-dimensional (non-axisymmetric) surface profile inputs for both the piston and cylinder surfaces.
Figure 3.2. The unwrapped fluid grid.
Figure 3.3. Experimental surface profile measurement of a bushing from the Tribo test rig.
3.2.1 The Fluid Grid

Figure 3.4 compares the rectangular, structured grid used for the fluid properties calculations with the triangular, unstructured grid that forms the surface of the solid bodies of the piston and cylinder. Both of these mesh types are chosen for convenience and in accordance with the different challenges facing each domain. The solid mesh must conform to complex and irregular geometry that is present in the piston and cylinder block solid volumes. For this purpose an unstructured tetrahedral mesh is well suited. An additional benefit is that such a mesh can be quickly generated by software using CAD files as input. The generation of solid meshes is accomplished using an external commercial software. Therein lies the largest disadvantage of the unstructured tetrahedral mesh. If more detail is desired in a specific region of the solid body meshes, the entire mesh must be recreated using the external software. Although this process is relatively fast, it is vastly slower than the generation or refinement of a structured grid which can be accomplished within the model during the course of an analysis.

A structured rectangular grid is used for the fluid domain to take advantage of these strengths. The fluid domain in the piston cylinder interface can be modeled as an unwrapped rectangular space as described previously, therefore a rectangular grid serves as a natural subdivision of the domain. The structured grid also maintains the cell boundaries parallel to the axes used in the discretization of the Reynolds equation, significantly simplifying the required calculations. Further, because the grid is defined and constructed within the piston cylinder model at run time, changing the dimensions or refinement of the grid is minimally expensive.

The finer the solid and fluid grids become, the greater detail is available in the model. Because the model calculates a fluid structure interaction (FSI), the resolution of physical effects is limited to the coarsest grid in use. Additional computational resources are consumed by every grid cell, so a mismatch in grid sizes between fluid and solid meshes results in additional computational resource consumption for minimal
additional resolution. The best configuration is approximately shown in Figure 3.4 where the grid sizes are comparable.

Typically the most interesting behavior within the fluid film occurs in the areas of low film thickness. It would be beneficial to refine the grid in these areas to a finer resolution so that additional behaviors and phenomena could be explored. These areas are transient however, and occur in different locations in the fluid film throughout the operation of the machine. It is therefore impossible to predict where additional refinement is needed at the time when the solid meshes are being generated. Chapter 5 details the construction of a model incorporating dynamic grid refinement to further explore these issues.
Figure 3.4. Rectangular fluid grid cells (red) overlaid upon triangular solid body mesh faces (blue).
3.3 Solid Body Temperature Distribution

It has been noted that the Finite Volume Method (FVM) temperature field model implemented by Pelosi [15] required a disproportionately long time to reach convergence when compared with the Finite Element Method (FEM) used to calculate deformations due to thermal stresses. In an effort to reduce overall simulation times that in some cases can reach up to a week, a faster method was sought. A new FEM solver has been implemented inspired by the work of Zecchi [40]. Thermal conduction within the solid part forms the conduction matrix $K_{cd}$. Mixed boundary conditions are used for areas where convection occurs from the solid part into a fluid volume. For these boundaries, the terms $K_{cv}$ and $Q_{cv}$ together define the boundary. Finally, the heat flux occurring from the lubricating gap is applied using the heat flux vector $Q_b$. The final result saves a significant amount of simulation time while maintaining accuracy of the simulation results.

\[ K_T T_i = Q_{cv} + Q_b \]  
(3.4)

\[ K_T = K_{cd} + K_{cv} = \int_V B^T C_T B dV + h \int_S N^T N dS \]  
(3.5)

\[ Q_b = \int_S N^T q_b n^T dS \]  
(3.6)

\[ Q_{cv} = h_T \int_S N^T T_\infty dS \]  
(3.7)

3.4 Solid Body Pressure Deformation

Displacement chamber pressure varies as a function of shaft angle, and acts on a significant area within the cylinder block. In the model developed by Pelosi [15] all displacement chambers are subjected to a single uniform pressure equal to that within the reference bore. However, during machine operation each displacement
volume experiences pressures that can at times vary significantly from the pressure in the neighboring displacement volumes. This effect is most pronounced near the transitions between the high and low pressure ports when one displacement volume is connected to high pressure and the other volume is connected to the low pressure port. During these transitions it is important to consider the pressure deformation caused by each unique pressure field. The pressure within each displacement volume can then be considered as a lumped parameter. Due to the uniform pressure field acting on all surfaces surrounding the displacement volume, a single Influence Matrix (IM) is defined which corresponds with the application of such a pressure. One such IM is defined for each separate displacement chamber (DC) volume, as shown in Figure 3.5. An influence matrix is composed of influence vectors. Each vector describes the effect or influence over the entire FEM grid surface of a reference pressure applied to a given portion of the FEM grid. Using the principle of linear superposition, the deformation of the entire surface given an arbitrary pressure field can be calculated.

Also shown in Figure 3.5 is the Gap surface for which a separate IM is generated for each triangular face. This discretization allows the non-uniform pressure field from the lubricating gap to be applied. IMs constitute the main memory requirement during model execution. Therefore judicious use of IMs is necessary to limit memory requirements to achievable levels. The model developed by Pelosi [15] neglected the pressure deformation resulting from neighboring cylinder bores. To approximate the deformation caused by the pressure fields in neighboring cylinder bores (marked Cyl), the model developed within this thesis research applies a uniform pressure to these surfaces which is equal to half the associated displacement chamber pressure. Because this applied pressure is uniform across the surface, a single IM can be used for each cylinder bore.

Not shown in Figure 3.5 is the pressure loading on the remaining faces which experience case fluid pressure. Because the case volume experiences a uniform fluid pressure, a single IM is used to capture the effect of case pressure over these faces.
Figure 3.5. Cylinder block mesh showing face definitions for pressure loading.
3.5 Preventing Fluid Film Collapse

In some cases the simulated lubricating fluid film is not capable of supporting a sufficient amount of the external loads. If nothing is done in these cases to supplement the simulated load support the piston squeeze motion will continue to move the piston toward the cylinder surface. If this condition is not resolved, the film will collapse in the area which lacks support. Eventually, the numerical solution of the fluid film will diverge, causing the simulation to fail. It is therefore beneficial to implement some method of preventing extensive fluid film collapse so that the simulation may converge and produce a complete set of results for analysis. Several methods for preventing film collapse are presented in this section. All methods presented are founded upon the correction force vector $F_{cK} = \{F_{cK1}, F_{cK2}, F_{cK3}, F_{cK4}\}$. Simulation stability, performance, and accuracy all depend heavily on how $F_{cK}$ is calculated and how it affects piston motion.

The proposed solution to the problems described above is to consider the correction forces $F_{cK}$ directly in the force balance loop as depicted in Figure 3.6. In this way, the action of the correction forces is appropriately accounted for in the motion of the piston and the fluid pressure build up at all times. With the correction forces now included in the force balance loop, the calculation of the correction forces becomes critically important to the stability and accuracy of the simulation results.

Two methods for calculating $F_{cK}$ are presented in the following sections. Both methods are implemented and tested in the following chapter and their results compared with measurements.
Figure 3.6. Force balance at the piston cylinder interface.
3.5.1 Linear Method

The first method considered is inspired by the work of Wieczorek [11]. This method calculates the correction stress $\sigma$ according to Equation (2.2) and Figure 3.7. Correction forces $F_{cK}$ are then calculated according to Equation (2.5). Because the correction stress is simplified to a linear relationship with respect to surface penetration, a coefficient must be defined to relate the two quantities. Such a coefficient has no physical basis, and must be chosen to achieve a combination of desirable performance and stability. The value given by Equation (2.3) is found to provide satisfactory results in most situations. A significantly lower value is required for situations with unusually thin cylinder walls, to account for the resulting increased compliance of the surface.

An implicit algorithm is used to calculate the shifting motion of the piston. As the piston shifting velocity is iterated during the force balance loop, its position is updated resulting in a change in the amount of surface penetration and therefore correction forces. Because this method relies on surface penetration to calculate correction forces, the fluid film thickness is inherently unrealistic within and immediately surrounding the collapsed area.
Figure 3.7. Calculation of linear force balance correction stress.
3.5.2 Iterative Method

The second method builds on the force correction linear method and attempts to calculate a correction stress that results in the elimination of surface penetration. To accomplish this, the correction stress $\sigma$ is included in the pressure load used to calculate solid body pressure deformations. As $\sigma$ increases in a given area, the resulting deflection of the gap surfaces away from one another tends to reduce the amount of penetration as shown in Figure 3.8. The $\sigma$ field is updated at each iteration using:

$$\Delta \sigma (i) = C (h_{min} - h (i))$$

subject to:

$$\sigma \geq 0$$

Correction forces $F_{cK}$ are then calculated using Equation (2.5). The coefficient $C$ in Equation (3.8) no longer affects the final magnitude of the correction forces, and is instead tuned for speed and stability of convergence. The magnitude of the forces is entirely a result of the stresses needed to separate the surfaces from one another. This stress is in turn a result of the amount of rigid penetration, the material properties, and the geometry of the machine components. Because the correction forces depend on deformations rather than relative positions, an explicit ODE algorithm is used to solve for piston motion. This method typically results in smaller collapsed areas compared to the force correction linear method. Film geometry is typically more realistic in the areas near a collapsed region as well, because unrealistic surface penetrations are not permitted.
Figure 3.8. Calculation of iterative force balance correction stress.
4. SIMULATION RESULTS AND MEASUREMENT COMPARISON

The intent of any modeling effort is to predict or to help understand a physical phenomenon. The piston cylinder model is aimed at both of these goals. For a new pump or motor design, the model should give a reasonable prediction of performance and reliability. For an existing design, the model should give insight into the root cause of problems and predict the effects of design changes. A prerequisite for these uses is the ability to correctly predict the performance of an existing design. Below, simulation results are compared with measurements made using a specially built test rig.

One of the most important outputs of the piston/cylinder model is the energy dissipated in the lubricating gap. This energy dissipation comes from a combination of leakage flow and friction forces. A well performing model will predict friction forces similar in magnitude and shape to experimental values.

4.1 The Tribo Test Rig

To remedy the issues identified with the Tribo test rig in section 2.2 a redesign of the hydraulic circuit was made. Figure 4.1 shows the updated hydraulic circuit for the Tribo test rig. The heart of the test rig is the previously described Tribo pump/motor with its associated sensors and telemetry system. The pump/motor is now the only hydraulic machine on the test stand and is driven by a three phase electric induction motor controlled by a regenerative variable frequency drive. A drive enable circuit fulfills a safety function by disabling the electric motor in the event of any emergency stop button being pressed or a fault detection by the DAQ system.
Figure 4.1. Hydraulic circuit for Tribo test rig.

When the drive is actively controlling the electric motor, voltage is applied to an enabling valve which allows hydraulic oil from the lab hydraulic supply to reach the Tribo pump/motor. Fluid pressures at the inlet and outlet of the Tribo pump/motor are controlled by variable pressure relief valves. In this configuration, oil from the temperature controlled main power supply passes through the Tribo pump once and then returns back to the main reservoir for cooling. This arrangement vastly simplifies the requirement of maintaining the inlet temperature at the desired set point. Fluid pressures and temperatures are measured at the pump/motor inlet, outlet, and case drain ports.
4.2 Tribo Test Rig Data Processing

Processing the data from the Tribo test rig presents several challenges. The telemetry system can only transmit a single channel at a time. Because the three measurement channels cannot be recorded concurrently, it must be assumed that measurements are made in steady state operation. This assumption implies that data captured from every revolution should mirror every other revolution as is the case for the displacement chamber pressure signal shown in Figure 4.2. In practice small differences exist in the friction data from each revolution as shown in Figure 4.3. For this reason, data is collected across many revolutions, and the typical behavior is compared against simulation.

Once friction measurements for every revolution have been aligned according to the shaft trigger, a histogram is taken for each angle as shown in Figure 4.4. All of the individual histograms are then oriented vertically and combined into a time varying histogram contour plot as shown in Figure 4.5. The color scale of the plot represents the occurrence frequency of each friction value at a given shaft angle. All occurrence frequencies are normalized such that they fall on the interval [0, 1]. As a result, shaft angles with more consistent data will generate higher occurrence frequencies relative to shaft angles with more measurement noise.

To compare measurements with simulation results, the simulated friction curve is overlaid upon the contour plot. Because the force sensor used in the Tribo test rig is a piezoelectric type sensor, it is capable of measuring only dynamic forces. This means that the recorded data cannot indicate the exact magnitude of the force at a given instant of time, only the change in force with time. As a result, it is unknowable from the measured data when the friction curve transitions between positive and negative. To compensate for this unknown, the measured friction values are shifted vertically to find a best fit with the simulated curve. This shifting is accomplished automatically using a MATLAB comparison script.
Similarly, because the exact shaft angle which the shaft trigger indicates is unknown, a shift in the measured shaft angle axis is also permitted. This shift is again computed automatically using the MATLAB comparison script to ensure fair comparison between measurement and various different simulation results.

Figure 4.2. Aligned telemetry signals from Tribo test rig displacement chamber pressure sensor.
Figure 4.3. Aligned telemetry signals from Tribo test rig axial friction force sensor.
Figure 4.4. Creation of a histogram from a single shaft angle of multiple revolutions.

Figure 4.5. Creation of a time-varying histogram contour from individual histograms.
4.3 Measured Operating Conditions

Within each following section is presented a table containing information about the steady-state operating conditions during the measurement. Following the table is a figure comparing simulated and measured friction forces for several variations of the piston cylinder model. The various models are as follows:

- **Top Left**: Full model, including all physical effects discussed in Chapter 3. Linear force correction method is used.
- **Top Right**: Full model, except EHD Squeeze as described in Section 3.1 is neglected.
- **Middle Left**: Full model, with incompressible oil.
- **Middle Right**: Full model, with piston pressure deformation included in $h_K$ as described in Section 3.1.
- **Bottom Left**: Full model, using iterative force correction method as described in Section 3.5.2.
- **Bottom Right**: Full model, using velocity correction method as described in Section 2.1.4.

4.3.1 500rpm 80bar

This operating condition, defined in Table 4.1, was measured using the Tribo test rig with a previously unused bushing. Because of the initial condition of the bushing, and the relatively low power operating condition, it is assumed that minimal wear occurred prior to the measurement. Therefore, for the simulation of this operating condition the measured wear profile was scaled down as shown in Figure 4.6.

A comparison of the various models in Figure 4.7 shows that the full model, along with the model lacking EHD squeeze, and the model considering incompressible
flow give similar results that closely match measurements. When piston pressure deformation is included in the translational squeeze, the simulated friction force begins to diverge notably from the measured profile. The iterative force balance method results in a very low friction force compared to measurement. This is consistently seen throughout all comparisons, and is due to higher film thicknesses in the critical areas of the film resulting from the iterative method of force correction. This phenomenon is shown in Figure 4.8 The velocity correction method performs similarly to the full model.
Figure 4.6. Modified wear profile for the bushing used in the presented measurements.
Figure 4.7. Simulated friction vs. measurement. Tribo test rig, 500rpm 80bar 42C. From top left: full code, without transient squeeze, with incompressible Reynolds, without translational squeeze, iterative force correction, velocity correction.
Figure 4.8. Simulated film thickness for full code (top) and change in film thickness due to iterative force correction (bottom). Tribo test rig, 500rpm 80bar 42°C.
The operating condition defined in Table 4.2 was measured by the Tribo test rig after a significant running time. Therefore, the wear on the bushing is assumed to be overall similar to the wear measured upon the completion of measurements shown in Figure 4.9. Comparison against measurements in Figure 4.10 again shows close agreement for the first three model variations. Simulated friction forces again diverge significantly from measurement in the case of including piston pressure deformation in the translational squeeze term. In this case, both the iterative force correction and the velocity correction methods perform quite well. This is primarily due to the combination of low load and a favorable bushing surface profile that aids fluid load support. This can be seen in Figure 4.11 where the correction forces are entirely below 0.25% of the external load. With such a small demand for correction forces, the details of the correction algorithm do not strongly impact the simulation results. It can also be seen that the correction forces in the case of incompressible fluid are significantly higher.

Table 4.2. Operating condition information 500rpm 120bar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Speed</td>
<td>503rpm</td>
</tr>
<tr>
<td>Inlet Pressure</td>
<td>20.bar</td>
</tr>
<tr>
<td>Outlet Pressure</td>
<td>119bar</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>42°C</td>
</tr>
<tr>
<td>Outlet Temperature</td>
<td>44°C</td>
</tr>
<tr>
<td>Drain Temperature</td>
<td>46°C</td>
</tr>
</tbody>
</table>
Figure 4.9. Measured wear for the bushing used in the presented measurements.
Figure 4.10. Simulated friction vs. measurement. Tribo test rig, 500rpm 120bar 42C. From top left: full code, without transient squeeze, with incompressible Reynolds, without translational squeeze, iterative force correction, velocity correction.
Figure 4.11. Correction forces normalized to external forces for full model (top) and model with incompressible fluid (bottom).
4.3.3 500rpm 150bar

As in the previous operating condition, the operating condition defined in Table 4.3 is assumed to have a fully developed wear profile on the bushing surface. Comparing measurements to simulation results in Figure 4.12 shows that all models reflect quite closely the measured friction. It can be noted that neglecting either EHD squeeze or compressible flow leads to numerical instabilities in the predicted friction. All other model variations for this operating condition show negligible differences relative to one another.

Table 4.3. Operating condition information 500rpm 150bar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Speed</td>
<td>503rpm</td>
</tr>
<tr>
<td>Inlet Pressure</td>
<td>21bar</td>
</tr>
<tr>
<td>Outlet Pressure</td>
<td>150.bar</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>41°C</td>
</tr>
<tr>
<td>Outlet Temperature</td>
<td>44°C</td>
</tr>
<tr>
<td>Drain Temperature</td>
<td>47°C</td>
</tr>
</tbody>
</table>
Figure 4.12. Simulated friction vs. measurement. Tribo test rig, 500rpm 150bar 42°C. From top left: full code, without transient squeeze, with incompressible Reynolds, without translational squeeze, iterative force correction, velocity correction.
4.3.4 500rpm 190bar

As previously, the operating condition defined in Table 4.4 considers a fully developed wear profile on the bushing surface. Comparing measurements to simulation results in Figure 4.13 shows that all models again reflect quite closely the measured friction. It is again the case that neglecting either EHD squeeze or compressible flow leads to numerical instabilities in the predicted friction. All other model variations for this operating condition show negligible differences relative to one another.

Table 4.4. Operating condition information 500rpm 190bar.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Speed</td>
<td>502rpm</td>
</tr>
<tr>
<td>Inlet Pressure</td>
<td>23bar</td>
</tr>
<tr>
<td>Outlet Pressure</td>
<td>194bar</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>42°C</td>
</tr>
<tr>
<td>Outlet Temperature</td>
<td>46°C</td>
</tr>
<tr>
<td>Drain Temperature</td>
<td>50°C</td>
</tr>
</tbody>
</table>
Figure 4.13. Simulated friction vs. measurement. Tribo test rig, 500rpm 190bar 42°C. From top left: full code, without transient squeeze, with incompressible Reynolds, without translational squeeze, iterative force correction, velocity correction.
4.4 Evaluating the Models

The impact of each modification to the model can be identified from the results presented in the previous section. The iterative force correction method does not consistently match measured friction values. Although the friction predictions made by the velocity correction code are satisfactory in the examples presented, the method violates the conservation of mass and introduces numerical instabilities as discussed previously. The models neglecting EHD squeeze produces increased numerical instabilities during the high pressure stroke. This problem is exacerbated at higher speeds. Incorrectly including piston pressure deformation in the translational squeeze term results in predicted friction forces which do not match measurements as reliably as the full model. Modeling the fluid as incompressible begins to produce numerical instabilities as the operating pressures rise, though the overall predicted friction force agrees well with measurement. Additionally, the incompressible fluid leads to dramatically increased correction forces. For these reasons, the full model is chosen for continued development.

In all the examples presented the full model is able to satisfactorily match the measured axial friction values while requiring relatively small correction forces. However the full model continues to predict areas of collapsed fluid film where full film lubrication is assumed.
5. THE HIGH DEFINITION LUBRICATION MODEL

As discussed in Section 3.5, much work has been done to solve an array of problems centered around collapsed regions of the lubricating film. It has been noted that swash plate type axial piston machines that operate successfully in practice continue to show collapsed film regions when simulated. Efforts by Pelosi [15] to include thermal and pressure deformations, along with the addition of the effects described in chapter 3 have improved simulation accuracy, but have not resulted in elimination of such collapsed regions. The most likely causes of the continuing occurrence of these collapsed regions are:

- Excessive coarseness of the fluid grid.

- Excessive coarseness of the FEM solid body deformation analysis.

Typical fluid grid and FEM element edge dimensions used for this work are approximately 3% of the piston diameter. This scale is chosen to:

- Reduce simulation time requirements. A typical simulation takes from two to five days to complete.

- Reduce simulation memory requirements. A typical piston cylinder simulation requires approximately 2GB of memory.

As grid spacing becomes finer, simulation times increase and fewer simulations can be run concurrently due to hardware memory limitations. To maintain adequate computational throughput, a sacrifice in model fidelity must be made in favor of computational time and resource conservation. This chapter details the construction
of a High Definition (HD) model to investigate the assumption that finer scales in both fluid and solid domains will improve calculated load support. Key improvements implemented in this model are:

- Adaptive multigrid applied to pressure calculations.
- Linear half-space deformation model to calculate surface deformations in high definition.
- A fluid properties model extended to high pressures.

5.1 Modeling Approach

Although the best strategy in terms of fidelity, refining the fluid and solid domain meshes in the piston cylinder analysis is prohibitively expensive in terms of computational time. The analysis of each time step must complete before the model can move on to the next time step. This constrains the piston-cylinder simulation to a minimal amount of parallelization, and a long simulation time. To take advantage of available parallel computing resources, a non-sequential approach must be adopted. Accordingly, the HD model is intended to revisit the areas of collapsed film from an analysis of the standard piston cylinder model. Boundary and loading conditions taken from a single collapsed region form the basis for each HD analysis. Within each HD analysis both the fluid grid and the pressure deformation calculations are dynamically refined to a much finer grid where additional detail is required. The result is a much greater insight into the behavior and load carrying ability of the fluid film in these areas that collapse in the standard analysis.

5.1.1 Generation of Inputs

MATLAB scripts are used to analyze the outputs of the piston cylinder model. These scripts search through every time step of the simulation results to find every area of collapsed film.
5.1.1.1 Analysis Region

First, an analysis region must be defined. In this case, regions where contact stress is calculated by the standard model are clearly of interest. It is desirable however to have a slightly larger domain over which the model can solve for the pressure distribution. Therefore all surrounding areas with film thickness less than a critical value of \(3h_{\text{min}}\) are also included for analysis.

It is likely that multiple areas of contact stress will be computed by the standard definition piston cylinder model during a single time step. In this circumstance a means of considering the individual areas in separate analyses is desirable. These goals can be accomplished by the following method. The results can be seen in Figure 5.1 and Figure 5.2.

1. Construct a matrix with each entry representing a cell in the rectangular fluid grid from the piston cylinder model.

2. Scan the entire grid, marking each cell that falls below the predetermined minimum film thickness.

3. Scan the marked cells for the first cell that is not assigned to an analysis patch, and assign a patch number.
   - Loop through the newly identified patch, and incorporate all bounding elements with low film thickness.
   - Continue adding bounding elements until the analysis patch no longer grows.
   - Repeat this step until all elements with low film thickness have been assigned to an analysis patch.
Figure 5.1. Film thickness (contours), fluid pressure (blue shading), and correction stress (red shading).
Figure 5.2. Regions of interest defined based on film thickness.
5.1.1.2 Boundary Conditions

The following properties are defined for each cell within the analysis patch:

- Location within the gap.
- Total film thickness.
- Translating portion of film thickness.
- Fluid pressure.
- Correction pressure.
- Bounding surface temperature.
- Squeeze motion due to rigid body motion and transient deformation.
- Connectivity to other cells in analysis patch.

The MATLAB script builds an input file for each analysis patch that defines the above information for each grid point.

**Pressure**

Pressure boundaries can be set according to the gap pressure field present in the standard piston cylinder model analysis.

**Load**

The required load support can be well defined based on standard piston cylinder model outputs. It consists of the load supported by fluid and contact pressure fields in the analysis region. These fields can be summed, and the resultant load force magnitude integrated throughout the area.

**Film Thickness**

The fluid film thickness and shape are critical in enabling hydrodynamic load support. As such, a realistic starting estimate of film thickness must be made. This can also be achieved by interpreting the results of the standard piston cylinder model.
The film thickness is taken from the standard piston cylinder model, before it is saturated to the specified minimum thickness.

If the load support calculated by the HD model is to balance the required load, there must exist a degree of freedom by which the calculated load support can be affected. To achieve this degree of freedom, a variable vertical offset is added to the film thickness. Iterations of the analysis are made, with the initial large offset incrementally decreasing until either the fluid film collapses or the load is fully supported. In cases where the fluid film ultimately collapses, the highest achieved load support is recorded.

5.1.2 Surface Pressure Deformation

In addition to the nominal surface shaping given by the boundary conditions, the surface shape will also depend on the pressure field calculated by the HD model. A method is needed to calculate the pressure deformation of the bounding surfaces in widely varying levels of detail. Some areas where the grid is dynamically resized to a fine spacing require surface deformations to be calculated at a matching level of detail. Other areas that are not refined can use a similarly coarse calculation of surface deformation without penalty. The influence method used by Pelosi [15], although well suited to the standard piston cylinder model, is not suitable for the HD model because the FEM mesh must be defined prior to the analysis. Because the level of refinement varies significantly throughout the analysis region, any method that requires a predefined mesh must provide sufficient resolution everywhere. This would provide excessive refinement throughout the majority of the grid, leading to prohibitive computational expense and memory requirements during the analysis.

A linear half-space model developed by Hamrock and Dowson [30] is chosen to meet these requirements. In this model, the analysis region is broken up into rectangles of constant pressure. The deformation of any point within the region can then be calculated using the principle of linear superposition. It is important to note here
that the surfaces in the analysis region have already been deformed by the pressure and contact stress fields calculated by the standard piston cylinder model. Therefore, the pressure field used in the half-space calculations should be the difference between the pressure calculated by the HD model and the pressure and contact stress fields calculated by the standard piston cylinder model.

Referring to Figure 5.3, deformation of the elastic half space is calculated using:

$$\bar{w} = \frac{2P}{\pi} \int_{-a}^{a} \int_{-b}^{b} \frac{1}{\sqrt{(Y - Y_1)^2 + (X - X_1)^2}} dX_1 dY_1$$  \hspace{1cm} (5.1)

Here, the dimensionless pressure $P$ is calculated:

$$P = \frac{P_{\text{fluid,HD}} + P_{\text{contact,HD}} - P_{\text{fluid,Standard}} - P_{\text{contact,Standard}}}{E'}$$  \hspace{1cm} (5.2)

$E'$ is the equivalent stiffness of the piston and cylinder surfaces as calculated in Equation (2.4). Figure 5.4 gives an example of the superposition of deformations resulting from two rectangular areas of pressure applied to the surface.

Pressure deformation calculations begin on the coarsest grid level, shown in red in Figure 5.5 with a unique pressure applied to the rectangular area of each fluid cell. As the grid is refined, new cell centroids (shown in blue) are created that fall midway between existing cell centroids. By introducing smaller cells during the grid refinement process, the pressure loading of the half space can become more finely detailed. The deformation resulting from the coarser grids is already applied to the surface before the finer grid loading is calculated. Therefore it is necessary to only consider the difference (shown in blue) between the pressure already accounted for and the newly added pressure (green). By nature of its construction, each new grid point is placed on the border of two or more cells existing in the coarser grid. So it is the difference between the finer cell’s calculated pressure and the linearly interpolated pressure from the coarser grid at the same location that is applied to the finer grid. This process repeats for each additional grid level.
Figure 5.3. Section of linear half space domain showing applied pressure in yellow rectangle centered at point \((X_1, Y_1)\), and deformation calculated at point \((X, Y)\).

Figure 5.4. Example of superposition of two rectangular areas of applied pressure on half space deformation model.
Figure 5.5. Pressures applied to deformation model in refined areas.
5.1.3 Adaptive Grid Refinement

A rectangular structured grid is utilized. This allows for significant simplification of calculations (e.g. a constant spacing between every cell), however a method of refining parts of the grid must be developed. Mesh refinement is an automated process within the HD model. This is accomplished using an adaptive multigrid approach in which the finer mesh levels are only defined within selected portions of the global mesh. The results from these refined areas are then used to update the coarse mesh as with a standard geometrical multigrid approach.

The simulation begins with its coarsest grid, as given from the standard piston cylinder model. The model solves this grid until convergence has been reached. A specified percentage of the grid with the lowest film thickness is then refined to form the next finer level. Bilinear interpolation is used to initialize the fluid and surface parameters for the newly created grid points. The model then solves the multigrid problem, using the original coarse grid to speed convergence for low frequency effects, and the higher resolution refined grids to calculate finer details. This refinement process continues, with the specified percentage of each level forming the grid for the next level, until the specified number of refinement levels have been created. When the fully refined system converges, the load support is compared with the required support to determine whether to repeat the analysis with a lower starting film thickness or terminate the simulation if sufficient load support is achieved.

5.1.4 Fluid Properties

The oil used in experiments at the Maha Fluid Power Research Center has been characterized over a range of temperatures from $25^\circ C$ to $120^\circ C$ and pressures from $100 kPa$ to $137.8 MPa$. Because the HD model may generate pressures well in excess of these measurements, a more general oil model is required. According to Roelands [25], the variation of viscosity with respect to pressure and temperature of a lubricating oil can be described as:
\[ \log (1000 \mu) = G_0 \left( 1 + \frac{\rho}{1.96133 \times 10^9} \right)^{Z_0} \left( 1 + \frac{P}{138} \right)^{S_a} - 1.2000 \] \tag{5.3}

This equation was fitted to the measured data for the oil in use at the Maha Lab.

Fluid density can be modeled as:

\[ \rho = \rho_0 \left( 1 + \frac{0.6p}{1 + 1.7p} \right) + c_{pT} (T - T_0) \] \tag{5.4}

This equation was also fitted to measured data from the oil used in the Maha Lab. Within the range of measured fluid property data, an empirical fit is used to more closely approximate the measured data. As the fluid pressure approaches the boundary of the measured range, the model smoothly transitions to the Roelands model described. The model developed in this research is capable of calculating fluid properties up to a pressure of 1 GPa.

#### 5.1.5 Pressure Build Up

As with the standard piston cylinder model, pressure build up is computed using the Reynolds equation. However because the HD model begins with different assumptions the derivation is somewhat different. Again, the starting point is the general form found in Hamrock et al. [39]:

\[ \frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial x} \left( \frac{\rho h (u_a + u_b)}{2} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h (v_a + v_b)}{2} \right) \]

\[ + \rho (w_a - w_b) - \rho u_a \frac{\partial h}{\partial x} - \rho v_a \frac{\partial h}{\partial y} + h \frac{\partial \rho}{\partial t} \] \tag{5.5}

Because the bottom cylinder surface is considered to be fixed in this analysis, and surface velocity is constant the equation simplifies to:

\[ \frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial y} \right) = u_a \frac{\partial}{\partial x} \left( \frac{\rho h}{2} \right) + v_a \frac{\partial}{\partial y} \left( \frac{\rho h}{2} \right) \]

\[ + \rho w_a - \rho u_a \frac{\partial h}{\partial x} - \rho v_a \frac{\partial h}{\partial y} + h \frac{\partial \rho}{\partial t} \] \tag{5.6}
Fluid expansion with respect to time is neglected, as it is assumed to provide only a small contribution to pressure build-up in the areas of interest. However, to capture the motion of the piston surface accurately, the $h_K$ term must be used in the translational squeeze term, as described in section 2.1.2. Also similarly to the standard piston cylinder model, the squeeze term $w_a$ is composed of rigid piston motion and transient squeeze calculated in the standard model.

\[
\frac{\partial}{\partial \hat{x}} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \hat{y}} \right) = \frac{u_a}{\hat{x}} \frac{\partial}{\partial \hat{x}} \left( \frac{\rho h}{2} \right) + \frac{v_a}{\hat{y}} \frac{\partial}{\partial \hat{y}} \left( \frac{\rho h}{2} \right) + \rho \frac{\partial h}{\partial t} - \rho \frac{u_a}{\hat{x}} \frac{\partial h_K}{\partial \hat{x}} - \rho \frac{v_a}{\hat{y}} \frac{\partial h_K}{\partial \hat{y}} \tag{5.7}
\]

### 5.1.6 Result Compilation

The results of each patch can be displayed individually as shown in Figure 5.6 which shows fluid film thickness contours along with pressure build-up throughout the analysis area. In the case shown, the patch is capable of supporting the required load while maintaining significantly higher fluid film thicknesses compared to the standard piston cylinder model.

After all the patches have been individually analyzed, the collective results are compiled by a post-processing MATLAB script. The script loads the outputs from every analysis and reads both the required and calculated load support. These values can then be graphically displayed as shown in Figure 5.7. Each circle represents the analysis of a single patch of collapsed film. Circle size corresponds to the load support required of each patch. Horizontal position indicates the shaft angle at which each patch occurred. Vertical position indicates the percentage of required load which is supported using the HD model.

Figure 5.7 shows that the majority of patches analyzed achieve 100% of the required load support. However, there remain many patches which are not capable of supporting any significant load. One possible reason for this is the decoupled nature of the HD code. Load support calculated on the highly refined mesh of the HD
Figure 5.6. Sample *HD* analysis output for single analysis region. Color indicates fluid pressure in *bar*, contours indicate film thickness in *µm*.

model is not communicated to the piston cylinder model’s force balance calculations or its pressure deformation calculations. Improved results can be expected if the load support from the refined patches is considered within the piston cylinder model.
Figure 5.7. Sample *HD* analysis cumulative output. Circle size indicates required load support. Tribo Test Rig, 500rpm, 120bar.
6. INTEGRATING HD PRESSURE CALCULATIONS WITH THE PISTON CYLINDER MODEL

A first step to remedy the shortcomings of the HD model presented in the previous chapter is to integrate its adaptive fluid domain calculations directly into the piston cylinder model presented in Chapter 3. Numerical errors in the solution of Reynolds equation are likely in the piston cylinder model when the film thickness changes dramatically in the space of one grid cell. Therefore refining only the piston cylinder model’s pressure calculations is likely to improve the accuracy of the simulation even with standard definition surface deformation calculations. An important advance in this research is the development of a robust physics based force balance solver.

6.1 Pressure Buildup Calculations

A simplified form of the Reynolds equation for the piston cylinder interface introduced by Shang [41] is implemented:

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \mu} \frac{\partial p}{\partial y} \right) = \frac{u_t \partial (\rho h)}{2} \frac{\partial h}{\partial x} + \frac{v_t \partial (\rho h)}{2} \frac{\partial h}{\partial y} + \rho \frac{\partial h}{\partial t_{total}} \tag{6.1}
\]

Where all changes in fluid film thickness with respect to time are captured in the term:

\[
\frac{\partial h_{total}}{\partial t} = -\frac{h_{current} - h_{previous}}{dt} \tag{6.2}
\]
6.2 Adaptive Mesh Refinement

Pressure calculations are carried out by the adaptive multigrid model presented in Chapter 5. As the goal of this model is to refine calculations in areas where the film thickness is changing quickly, a new grid refinement criterion is needed. Whereas previously the grid was refined based on the film thickness in each cell, it is now desirable to refine based on the film thickness nonuniformity across each cell:

\[ U_h = \frac{h_{\text{max}} - h_{\text{min}}}{h_{\text{max}} + h_{\text{min}}} \] (6.3)

Examples of \( U_h \) calculated at several points in a one dimensional grid are shown in Figure 6.1. Using this equation, the grid is refined for each point in the grid if \( U_h \) calculated between itself and any of its neighbors exceeds a predetermined threshold. In the case of the presented model, the grid is refined for \( U_h > 0.3 \) to a maximum of five levels of refinement.
6.3 A New Force Balance Solver

In the work of Pelosi [15] Newton’s root finding method is used to find a shifting velocity \( \dot{e} \) which balances the forces. Newton’s method assumes the second derivative of the function is continuous. In this case the function whose roots are to be found is the force balance of the piston including the correction forces described in Section 3.5.1. The second derivative of piston force imbalance is discontinuous at the piston position at which contact forces begin to occur. Therefore Newton’s method is not guaranteed to achieve a stable solution to the force balance problem and a more robust method is required.

Because the force balance of the piston is based in physics, knowledge of the physics involved can be used to formulate a new force balance algorithm. The fundamental purpose of the force balance algorithm is to maintain the piston in a state of force equilibrium by changing its shifting motion \( \dot{e} \). From Newton’s laws of motion we know that any object in a state of force imbalance will move in accordance with the imbalanced forces. Accordingly the following approach is devised. First a unit vector \( \textbf{dv} \) is defined based on the direction of force imbalance \( \textbf{dF} \) acting on the piston.

\[
\textbf{dF} = F_f + F_e + F_{cK} \tag{6.4}
\]

\[
\textbf{dv} = - \frac{\textbf{dF}}{|\textbf{dF}|} \tag{6.5}
\]

Then a line search is performed in the direction defined by \( \textbf{dv} \) by varying the change in velocity \( \Delta v \) until a minimum \( |\textbf{dF}| \) is found.

\[
\dot{\text{e}}_{\text{trial}} = \dot{e} + \Delta v \cdot \textbf{dv} \tag{6.6}
\]

Once a minimum is found a new search direction is found using Equation (6.4). This process is iterated until \( |\textbf{dF}| \) falls below a predetermined threshold.
The EHD test rig introduced in Section 2.2 was used by Pelosi [15] to record the temperature and pressure distribution in the piston cylinder lubricating gap. The operating condition is detailed in Table 7.1. These measurements are here compared to the simulation outputs of the high definition piston cylinder model. Cylinder bore temperature and lubricating gap pressures are presented as described in Section 3.2.

Table 7.1. EHD test rig operating condition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump Speed</td>
<td>1000rpm</td>
</tr>
<tr>
<td>Inlet Pressure</td>
<td>25bar</td>
</tr>
<tr>
<td>Outlet Pressure</td>
<td>175bar</td>
</tr>
<tr>
<td>Inlet Temperature</td>
<td>43C</td>
</tr>
<tr>
<td>Outlet Temperature</td>
<td>45C</td>
</tr>
<tr>
<td>Drain Temperature</td>
<td>55C</td>
</tr>
</tbody>
</table>
7.1 Measurement Data Post-Processing

Some care must be taken in comparing measured pressure values to simulated values. The pressure sensors do not directly sense the pressure in the lubricating film. Instead, as described in Section 2.2 they are separated from the lubricating gap by a small capillary hole. Figure 7.1 shows in detail the geometry of the sensor cavity and its connection to the lubricating gap. Because the operating fluid is compressible, a small amount of fluid must flow through the capillary to change the pressure in the sensor volume. When the lubricating film thickness surrounding the capillary is low, the flow required to change the pressure in the sensor volume begins to change the pressure in the lubricating gap. To analyze this effect, the gap surrounding the capillary is modeled as the land of a radial hydrostatic bearing. The mass flow in the surrounding gap can be calculated [39]:

\[ \dot{m}_c = \frac{\rho \pi h^3 (p_g - p_s)}{6 \mu \ln \left( \frac{R_o}{R_i} \right)} \]  

(7.1)

Assuming a constant sensor volume:

\[ \dot{m}_c = \dot{\rho} V_s \]  

(7.2)

And considering the bulk modulus of the working fluid:

\[ \dot{\rho} = \frac{\rho \ddot{p}}{K} \]  

(7.3)

Combining and rearranging yields a differential equation describing the pressure buildup in the sensor volume:

\[ \frac{6V \mu \ln \left( \frac{R_o}{R_i} \right)}{K \pi h^3} \dot{p}_s + p_s = p_g \]  

(7.4)

This equation is used to calculate the pressure within each sensor based on the local gap pressure and film thickness during the simulated operation of the pump.
Figure 7.1. Detail view of pressure sensor relationship to lubricating gap.

Table 7.2. EHD pressure sensor model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity</td>
<td>$140 \cdot 10^{-3} Pa \cdot s$</td>
</tr>
<tr>
<td>Sensor Volume</td>
<td>$50 \cdot 10^{-12} m^3$</td>
</tr>
<tr>
<td>Bulk Modulus</td>
<td>$2.6 \cdot 10^{-12} Pa$</td>
</tr>
<tr>
<td>Density</td>
<td>$900 Kg m^{-3}$</td>
</tr>
<tr>
<td>Inner Radius</td>
<td>$150 \cdot 10^{-6} m$</td>
</tr>
<tr>
<td>Outer Radius</td>
<td>$300 \cdot 10^{-6} m$</td>
</tr>
</tbody>
</table>

Values other than pressures and film thickness are assumed constant with the values given in Table 7.2.
7.2 Simulation Comparison to Measurement

7.2.1 Cylinder Surface Temperature

Figure 7.2 shows the measured and simulated temperature fields of the cylinder surface. Good agreement is achieved between the converged simulation and the steady state measurements both in absolute value and overall pattern.
Figure 7.2. Temperature field of EHD cylinder surface measured (top) and simulated (bottom).
7.2.2 Film Pressure

Figures 7.3 through 7.6 show simulated sensor pressure data for the HD piston cylinder model and the standard piston cylinder model compared to measurement. Good overall agreement is achieved by both models in comparison to the measured data. However, the HD piston cylinder model generates a pressure profile that more consistently matches the measurements compared to the standard piston cylinder model. This is evident primarily in the low pressure area on the left side of the figures.
Figure 7.3. Pressure field of EHD fluid film simulated at 45° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).
Figure 7.4. Pressure field of EHD fluid film simulated at $90^\circ$ with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).
Figure 7.5. Pressure field of EHD fluid film simulated at 135° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).
Figure 7.6. Pressure field of EHD fluid film simulated at 270° with HD piston cylinder model (top) and standard definition model (bottom) compared with measurement (middle).
7.3 Comparison of Standard Piston Cylinder Model to HD Piston Cylinder Model

7.3.1 Load Support

Figure 7.7 shows the correction forces resulting from the HD piston cylinder model and the standard definition version. Here the HD piston cylinder model can be seen to eliminate the correction forces required in the standard definition model. Combined with the pressure field comparisons presented previously, this supports the assumption of full film lubrication in the piston cylinder interface of the EHD test rig.

Figure 7.7. Simulated correction forces for HD piston cylinder model (top) and standard definition model (bottom).
7.3.2 Friction Forces

Predicted axial friction forces for both the HD and standard definition piston cylinder models are shown in Figure 7.8. The standard definition model predicts higher Couette friction while correction forces are present due to the reduced film thickness in the collapsed regions.

Figure 7.8. Simulated axial friction forces for HD piston cylinder model and standard definition model.
7.3.3 Energy Dissipation

Figure 7.9 compares the power loss predictions of the HD piston cylinder model and the standard definition model. Overall predicted power loss from the HD piston cylinder model is reduced by 15% compared to the standard definition model. Two factors contribute to this difference. First, the lower predicted friction discussed previously leads to lower mechanical power loss. Second, the volumetric power loss is reduced over much of the high pressure stroke.

Figure 7.9. Simulated power losses for HD piston cylinder model and standard definition model.
7.3.4 Leakage Flow

The decreased volumetric power loss is the result of decreased leakage flow as shown in Figure 7.10.

![Simulated leakage flow for HD piston cylinder model and standard definition model.](image)
7.3.5 Piston Motion

The decreased leakage flow is in turn caused by a significant difference in the predicted piston position during the respective portion of the shaft revolution. This $18\mu m$ shift in piston position begins during the predicted contact in the standard definition model and persists until the end of the revolution.

Figure 7.11. Simulated leakage flow for HD piston cylinder model and standard definition model.
7.3.6 Film Thickness

Figure 7.12 shows the film thickness predicted by the HD piston cylinder model and the standard definition model at 90° shaft angle. The primary leakage flow path can be seen following the path of high film thickness from the left side of the plot to the right. This flow path experiences significantly lower film thicknesses in the HD piston cylinder model’s prediction, leading to the lower predicted leakages. Figure 7.13 shows the difference between the film thickness predictions of the HD piston cylinder model and the standard definition model. The impact of the 18µm difference in piston position is visible in the different film thicknesses. Note however that the difference of film thickness in the critical areas of low film thickness are minimal.
Figure 7.12. Simulated film thickness at 90° for HD piston cylinder model (top) and standard definition model (bottom).
Figure 7.13. Absolute difference in simulated film thickness between HD piston cylinder model and standard definition model.
8. CONTRIBUTIONS

A novel piston cylinder interface model is introduced that advances the understanding of the tribological effects in play at the piston cylinder interface. Model advances resulting from this work are:

- Physical effects have been added to the piston cylinder model Reynolds Equation:
  - Elastohydrodynamic (EHD) squeeze is added.
  - Accurate surface velocities are considered with respect to translational squeeze.
  - Compressible flow is considered.
- Two-dimensional surface wear profiles are defined and implemented.
- Measured surface profiles are implemented for all simulated operating conditions.
- Methods for preventing fluid film collapse and ensuring force balance are introduced.
- Finite Element Method (FEM) solver implemented for solid body thermal analysis.

The improved standard piston cylinder model is compared against measurements made using an improved Tribo test rig. The reconfigured Tribo test rig exhibits an improved signal to noise ratio and a more stable and controllable inlet fluid temperature. Comparison between simulation results and measured friction profiles shows
good agreement across the limited range of operating conditions available for com-
parison.

A novel high definition lubrication model is introduced and developed. The high
definition model refines areas of collapsed fluid film predicted by the standard piston
cylinder model to investigate the impact of a finer grid spacing on improving load
support. Advances resulting from this work are:

- An adaptive multigrid solver is implemented, with refinement based on film
  thickness.
- A linear half-space deformation model is implemented for high-resolution defor-
mation calculations.
- A new fluid model extending to $3000\, bar$ is developed for the fluid used in
  measurement.

Results of the high definition lubrication model suggest that sufficient load support
can be calculated in order to prevent fluid film collapse, leading to the integration
of the high definition pressure calculations with the standard piston cylinder model.
This high definition piston cylinder model is used to simulate the operating condition
of EHD Test Rig measurements made by Pelosi [15]. Advances resulting from this
work are:

- Development of a physics based method for calculating the motion of the piston
  resulting from force balance.
- Improved understanding and modeling of the impact of film thickness on mea-
sured pressure fields in the EHD test rig.

Results comparison with measurement shows a good match with both temperature
and pressure fields, and indicates full film lubrication prevails in the lubricating film
between piston and cylinder throughout steady state operation.
LIST OF REFERENCES
LIST OF REFERENCES


VITA
VITA

Daniel Mizell was born in 1989 in Southeast Michigan. He earned his B.S. in Mechanical Engineering in 2011 at Michigan Technological University in Houghton, MI. In August 2011, Dan came to Purdue to join the team at the Maha Fluid Power Research Center. There, he earned his M.S. in Mechanical Engineering in 2014 while working toward his Direct PhD. Dan’s research activities focus on advanced modeling of the piston cylinder interface of swash plate type axial piston machines.

Dan met his wife Ashby while studying at Michigan Technological University, and they were married in 2011. Their son Troy was born in 2013.