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Uncertainty Analysis of the Experimental Results of Air Conditioner Performance due to the Uncertainty of the Equation of State

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Uncertainty Quantification of Experimental Air Conditioner Performance due to the Uncertainty of the Equation of State^a

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ABSTRACT

In previous literature, the uncertainty analyses of experimental performance metrics of air conditioners usually ignored the uncertainty due to the equation of state (EoS) of the refrigerants. One possible reason was that the uncertainty reported in the EoS literature was much smaller than the one of the system performance measurement. However, with the advancement of measurement technologies, the impact of measurement uncertainty on the air conditioner performance calculation is lowered and becomes on par with that of the EoS. Simultaneously, new research findings give more comprehensive understanding of the EoS uncertainties, such that the uncertainty of EoS reported in previous studies was underestimated under some conditions. To examine if the uncertainty of experimental results of a thermal system are significantly affected by the new findings, an uncertainty analysis is carried out with experimental data of an air conditioner using propane. The results show that the uncertainty of the EoS has a more significant impact on experimental results involving saturation temperature such as subcooling and superheat measurement than the uncertainties of the measurement, while its impact on the uncertainty of the measured heat transfer rate is still not as significant in most cases.

1. INTRODUCTION

Uncertainty analyses of air conditioner performance mainly focus on propagating experimental measurement uncertainties of temperature, pressure and mass flow rate to the output results of the cycle. For example, ASME Performance Test Code 30 (ASME, 2016) only considered measurement uncertainty and changes in the environment as the sources of uncertainties to the test results of heat exchangers. Payne et al. (1999) only quantified uncertainties due to measurement sensors in an air conditioner experiment. Considering measurement uncertainties only may miss other important sources of uncertainties in air conditioner experiments such as the uncertainties of the equation of state (EoS) that is used to estimate thermodynamic properties in the analyses. For example, Cheung et al. (2017) showed that the uncertainty of EoS may contribute to the uncertainty of superheat more than the uncertainty of the pressure transducers. Cheung and Wang (2018) also demonstrated that the uncertainties of heat transfer rate due to

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sensors have the same order of magnitude as the uncertainties propagated from the EoS of the refrigerant used in the air conditioner. These examples show that other uncertainty sources, such as the uncertainty of EoS of refrigerant properties, that may also be important to experimental analyses of the performance of air conditioners.

The aforementioned studies used the uncertainties recorded in the literature of EoS (Lemmon, 2003) while other literature has already used more comprehensive methods to quantify the uncertainties of EoS. For example, Feistel et al. (2016) calculated uncertainties of EoS based on uncertainties of experimental results in the literature by refitting EoS with generalized least-squares method to quantify the uncertainty of EoS of steam. Frutiger et al. (2016) conducted a similar study using the Monte Carlo method to quantify the effects of uncertainties of EoS of various refrigerants on organic Rankine cycle power outputs. Unlike the uncertainties reported in the literature of EoS that were calculated solely based on the EoS accuracy (Lemmon, 2003), these studies calculated the uncertainties of EoS based on statistical methods (JCGM 2008; Coleman and Steele, 2009) that are more appropriate than the accuracy of the models. These methods should be able to account for the effects of uncertainties of EoS to the experimental performance metrics of air conditioners more reasonably than ones in Cheung et al. (2017) and Cheung and Wang (2018).

In this study, the effects of uncertainties of EoS on the uncertainties of the air conditioner performance are studied by using an uncertainty calculation method based on Seber and Wild (1989). The method was used to derive the uncertainty calculation method of Helmholtz-energy-based EoS in Cheung et al. (2018). This study applied the technique to the air conditioner test result in Abdelaziz et al. (2015) as a case study to examine the effect of uncertainty of EoS on the experimental analyses of air conditioner performance.

2. CALCULATION METHOD OF EOS UNCERTAINTY

Cheung et al. (2018) developed a method to calculate the uncertainty of Helmholtz-energy-based EoS (HEoS) based on the uncertainty calculation method of a regression model in Seber and Wild (1989) and demonstrated the method using the EoS of propane in Lemmon et al. (2009). Regression models are mathematical models that estimate a value of a dependent variable based on some independent variables and a set of parameters. These parameters are estimated from a set of training data with observations of dependent variables and independent variables in the system to be modeled. Its mathematical description is shown in Equations (1) and (2).

$$y_{\text{pred}} = f(\vec{x}, \vec{\beta}) \quad (1)$$

$$\vec{\beta} = g(X_{\text{train}}, \vec{y}_{\text{train}}) \quad (2)$$

where y_{pred} is the predicted dependent variable, \vec{x} is a vector of independent variables, $\vec{\beta}$ is a vector of parameters, X_{train} is a matrix of independent variables in the training data and \vec{y}_{train} is a vector of dependent variables in the training data.

The choice of the mathematical form in Equation (1) and the training data used to estimate the parameters in Equation (2) is subject to the discretion of the model developer. The choice is also limited by the availability of resources to obtain the training data. Depending on the criteria of the choices, the results of the estimation of Equation (1) may vary. Seber and Wild (1989) provide a method to calculate the uncertainty of the model prediction by calculating the confidence interval of the estimation of the dependent variable by Equations (3), (4) and (5).

$$\Delta y_{\text{pred}} = \sqrt{\text{diag}(\mathbf{COV}(y_{\text{pred}}))} t(n - m, \gamma_t/2) \quad (3)$$

$$\mathbf{COV}(y_{\text{pred}}) = \vec{j}(\vec{x}, \vec{\beta}) \mathbf{COV}(\vec{\beta}) \vec{j}(\vec{x}, \vec{\beta})^T \quad (4)$$

$$\mathbf{COV}(\vec{\beta}) = \left(\frac{SSE}{n - m}\right)^2 \left(\mathbf{J}(X_{\text{train}}, \vec{\beta}) \mathbf{J}(X_{\text{train}}, \vec{\beta})^T\right)^{-1} \quad (5)$$

where Δy_{pred} is the uncertainty of y_{pred} , $\mathbf{COV}(y_{\text{pred}})$ is the covariance matrix of y_{pred} , $\text{diag}(\mathbf{COV}(y_{\text{pred}}))$ are the diagonal entries of the matrix, $t(n - m, \gamma_t/2)$ is the Student t-statistics with $n - m$ degree of freedom and Type I error γ_t , $\vec{j}(\vec{x}, \vec{\beta})$ is the Jacobian vector of $f(\vec{x}, \vec{\beta})$ with respect to $\vec{\beta}$, $\mathbf{COV}(\vec{\beta})$ is the covariance matrix of $\vec{\beta}$, $\mathbf{J}(\mathbf{X}_{\text{train}}, \vec{\beta})$ is the Jacobian matrix of $\vec{f}(\mathbf{X}_{\text{train}}, \vec{\beta})$, SSE is the sum of square of errors of the regression model, n is the number of training data points and m is the number of coefficients.

To use the technique for the uncertainty calculation of the HEoS of thermodynamic properties of pure substances, the mathematical form of the HEoS has to be understood. The HEoS can be described by a nonlinear equation of dimensionless Helmholtz energy α as a function of temperature T , density ρ and parameters θ_{EoS} as shown in Equation (6).

$$\alpha = f(T, \rho, \theta_{\text{EoS}}) \quad (6)$$

By calculating the dimensionless Helmholtz energy, other thermodynamic properties can be calculated by explicit equations depending on the model parameters, dimensionless Helmholtz energy, its partial derivatives, and temperature and density values (Lemmon et al., 2009). If the temperature or density values are unknown, numerical methods will be used with the equations to calculate the temperature, density and Helmholtz energy values before calculating other thermodynamic properties. To describe the transformation between vapor and liquid due to a change of temperature and density of the pure substance, Maxwell's criteria are used to find the temperature and density that define the transition between vapor, liquid-vapor mixture, and liquid (Bejan, 2006).

The application of the Seber and Wild (1989) method on Equation (6) requires three major assumptions. They are (a) the negligibility of systematic errors, (b) the optimality of θ_{EoS} from the literature as the optimal coefficients of the HEoS and (c) linear error propagation despite the nonlinearity of the HEoS. In addition to these assumptions, multiple features of HEoS also hinder the direct application of the method in Seber and Wild (1989) for the uncertainty of HEoS. These features are:

1. Training data of HEoS contain multiple types of dependent variables such as pressure and specific heat capacity but the uncertainty calculation method in Seber and Wild (1989) is made to be applied to a model using one type of dependent variable only.
2. Seber and Wild (1989) only provides a method to calculate uncertainties of dependent variables, but applications of HEoS may also require the uncertainties of temperature and density values that are independent variables in Equation (6). An uncertainty calculation method of HEoS should also calculate these uncertainties.
3. The differences of values of some properties such as enthalpy and entropy are more important than the magnitude of a single property value, so it is important to account for the correlation between the uncertainties of properties to accurately describe the uncertainties of the differences of these properties.
4. The calculation of uncertainties of properties at saturation depends on the use of Maxwell's criteria which contain a set of implicit equations. A method to propagate the uncertainties of Equation (6) through Maxwell's criteria is needed to calculate the uncertainties of the properties at saturation.
5. The training process involves the differences of Gibbs energy of saturated liquid and vapor at vapor pressure data points instead of the measured and predicted vapor pressure because Gibbs energy of saturated liquid and vapor at the same pressure should be equal according to Maxwell's criteria (Bell et al. 2018).

In order to deal with these issues, the method in Cheung et al. (2018) modifies the uncertainty calculation method in Seber and Wild (1989) with the following measures:

1. Normalizing the Jacobian matrix in Equation (5);
2. Using the Kline and McClintock (1953) method and the finite difference method (Nocedal and Wright, 2006) to propagate the uncertainty of EoS of other properties such as pressure and entropy to the temperature and density values;
3. Calculating the covariance of differences of dependent variables in Equation (4) instead of the covariance of a single dependent variable to calculate the uncertainties of differences of properties;

4. Using the Kline and McClintock (1953) method to calculate the uncertainties of saturation densities by propagating the uncertainties from Equation (6) through the equations in Maxwell's criteria for the uncertainties of other saturation properties; and
5. Involving Gibbs energy of vapor pressure data points in the Jacobian vectors and the calculation of *SSE*.

The detailed mathematical description of the modification can be found in Cheung et al. (2018). With the modified method, the uncertainty of HEoS can be calculated, and the effects of uncertainties to the calculation of performance metric of air conditioners can be quantified.

3. DESCRIPTION OF TEST SETUP AND PERFORMANCE ANALYSIS

Abdelaziz et al. (2015) conducted an experiment of the performance of a 5.25 kW split air conditioner using propane in a pair of environmental chambers as shown in Figure 1.

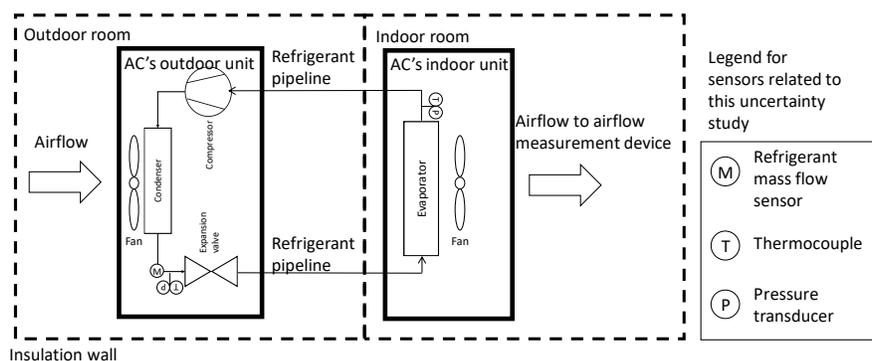


Figure 1: Test setup of a split air conditioner in environmental chambers

To quantify the performance of the air conditioner comprehensively, the study tested its steady-state performance under 6 different conditions defined based on AHRI standard 210/240 (AHRI 2008) as shown in Table 1.

Table 1: Testing conditions of the split air conditioner

| Test condition | AHRI B | AHRI A | T3* | T3 | Hot | Extreme |
|----------------------------------|--------|--------|------|----|-----|---------|
| Outdoor temperature [°C] | 27.8 | 35 | 46 | 46 | 52 | 55 |
| Indoor dry-bulb temperature [°C] | 26.7 | 26.7 | 26.7 | 29 | 29 | 29 |
| Indoor wet-bulb temperature [°C] | 19.4 | 19.4 | 19 | 19 | 19 | 19 |

The performance of the air conditioner was quantified by measuring the temperature, pressure and flows of air as well as refrigerant at multiple locations of the setup. Since this uncertainty study only involved the uncertainty calculation method of refrigerant properties but not the air properties, only the refrigerant-side measurements were investigated. The information of the sensors for the refrigerant-side measurement are listed in Table 2.

Table 2: Uncertainty of sensors on the refrigerant-side of the air conditioner

| Type of sensor | Measurement | Uncertainty |
|-------------------------|-------------------------------|--------------------------|
| T-type thermocouple | Refrigerant temperature | $\pm 0.28^\circ\text{C}$ |
| Pressure transducer | Pressure in refrigerant pipes | $\pm 0.08\%$ of reading |
| Coriolis mass flowmeter | Refrigerant mass flow rate | $\pm 0.1\%$ of reading |

The measurement by sensors in Table 2 collected data for the calculation of the air conditioner performance metrics as shown in Equations (7), (8) and (9).

$$\dot{Q} = \dot{m} \left(h_{\text{evap,out}}(T_{\text{evap,out}}, p_{\text{evap,out}}) - h_{\text{cond,out}}(T_{\text{cond,out}}, p_{\text{cond,out}}) \right) \quad (7)$$

$$SH = T_{\text{evap,out}} - T_{\text{evap,out,sat}}(p_{\text{evap,out}}) \quad (8)$$

$$SC = T_{\text{cond,out,sat}}(p_{\text{cond,out}}) - T_{\text{cond,out}} \quad (9)$$

where \dot{Q} is cooling capacity, \dot{m} is refrigerant mass flow rate, h is enthalpy, p is pressure, “evap,out” refers to a variable at the evaporator outlet, “cond,out” refers to a variable at the condenser outlet, SH is superheat, SC is subcooling and “sat” refers to a variable for a substance at saturation.

Equation (7) calculates the cooling capacity of the air conditioner and quantifies the maximum amount of cooling the air conditioner can deliver under the test condition. Equation (8) calculates its superheat, and an appropriate value around 11.1 °C indicates that the compressor is running appropriately (Dabiri and Rice, 1981). Equation (9) calculates its subcooling, and a value around 8.3 °C indicates that the refrigerant charge level inside an air conditioner is appropriate (AHRI, 2004).

Since the equations depend on measurements of the refrigerant temperature, pressure and mass flow rate, the contribution of the measurement uncertainty to the uncertainty of the performance metrics in Equations (7), (8) and (9) can be calculated by Equations (10), (11) and (12) based on Kline and McClintock (1953).

$$\Delta\dot{Q}_{\text{mea}} = \sqrt{\left(\frac{\partial\dot{Q}}{\partial\dot{m}}\Delta\dot{m}_{\text{mea}}\right)^2 + \left(\frac{\partial\dot{Q}}{\partial h_{\text{evap,out}}}\frac{\partial h_{\text{evap,out}}}{\partial T_{\text{evap,out}}}\Delta T_{\text{evap,out,mea}}\right)^2 + \left(\frac{\partial\dot{Q}}{\partial h_{\text{evap,out}}}\frac{\partial h_{\text{evap,out}}}{\partial p_{\text{evap,out}}}\Delta p_{\text{evap,out,mea}}\right)^2 + \left(\frac{\partial\dot{Q}}{\partial h_{\text{cond,out}}}\frac{\partial h_{\text{cond,out}}}{\partial T_{\text{cond,out}}}\Delta T_{\text{cond,out,mea}}\right)^2 + \left(\frac{\partial\dot{Q}}{\partial h_{\text{cond,out}}}\frac{\partial h_{\text{cond,out}}}{\partial p_{\text{cond,out}}}\Delta p_{\text{cond,out,mea}}\right)^2} \quad (10)$$

$$\Delta SH_{\text{mea}} = \sqrt{(\Delta T_{\text{evap,out,mea}})^2 + \left(\frac{\partial T_{\text{evap,out,sat}}}{\partial p_{\text{evap,out}}}\Delta p_{\text{evap,out,mea}}\right)^2} \quad (11)$$

$$\Delta SC_{\text{mea}} = \sqrt{(\Delta T_{\text{cond,out,mea}})^2 + \left(\frac{\partial T_{\text{cond,out,sat}}}{\partial p_{\text{cond,out}}}\Delta p_{\text{cond,out,mea}}\right)^2} \quad (12)$$

where “mea” refers to a measured variable.

The equations to calculate the performance metrics also depend on the HEoS because enthalpy values and saturation temperature values are calculated from the measurements using the HEoS. The contribution of EoS uncertainties to the uncertainties of performance metrics is quantified by Equations (13), (14) and (15).

$$\Delta\dot{Q}_{\text{EoS}} = \dot{m}\Delta(h_{\text{evap,out}} - h_{\text{cond,out}})_{\text{EoS}} \quad (13)$$

$$\Delta SH_{\text{EoS}} = \Delta T_{\text{evap,out,sat,EoS}} \quad (14)$$

$$\Delta SC_{\text{EoS}} = \Delta T_{\text{cond,out,sat,EoS}} \quad (15)$$

where “EoS” refers to a variable calculated from EoS.

The uncertainties of enthalpy difference and saturation temperature in Equations (13), (14) and (15) are calculated based on the uncertainty calculation method in Section 2.

Other details of instrumentation of sensors, the testing procedure and the measurement data can be found in Abdelaziz et al. (2015).

4. RESULTS AND DISCUSSION

4.1 Cooling capacity

The cooling capacity in each test calculated from Equation (7) and their uncertainties calculated from Equations (10) and (13) are tabulated in Figure 1.

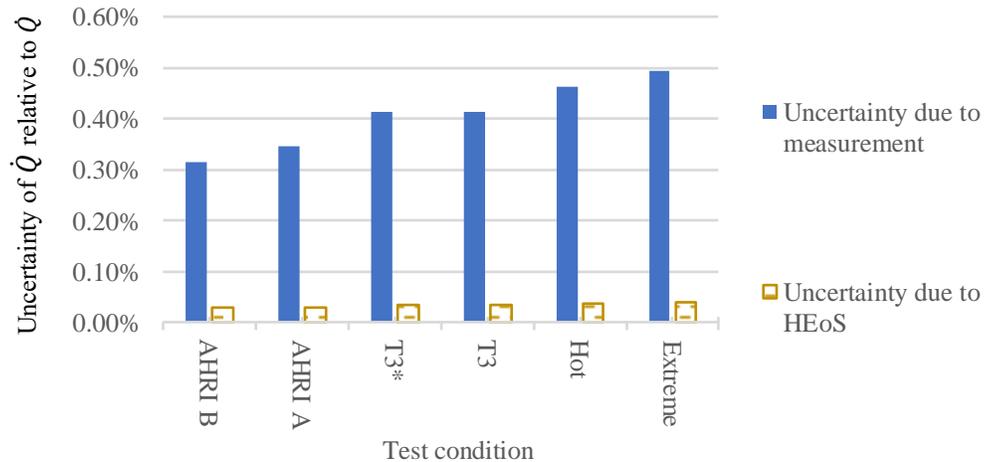


Figure 1: Comparison of heat transfer rate uncertainties due to measurement and due to HEoS

Figure 1 shows that the cooling capacity uncertainties due to HEoS are only approximately 10 % of the uncertainties due to measurement. This shows that the uncertainties of HEoS are not very significant relative to the cooling capacity uncertainties due to measurement. The reason of the small uncertainties due to EoS is the correlation of uncertainties of enthalpy values in Equation (7). The uncertainties of enthalpy values in Equation (7) are found to be highly correlated with each other, and a large part of the uncertainties cancel each other out as their differences are calculated in Equation (7). Hence the uncertainty of the enthalpy difference in Equation (13) and the uncertainties of cooling capacity due to HEoS in Figure 1 become small.

4.2 Superheat and subcooling

The superheat and subcooling of the air conditioner in various tests and their uncertainties are tabulated in Figure 2.

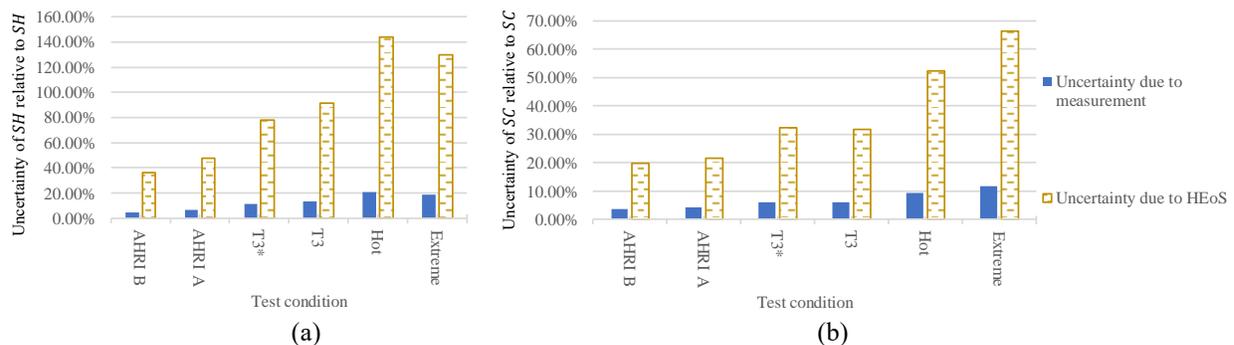


Figure 2: Comparison of (a) SH and (b) SC uncertainty due to measurement and HEoS

Figure 2 shows that the uncertainties of superheat and subcooling due to HEoS are much larger than that of the uncertainties due to measurement. This is caused by Maxwell's criteria which mandates the calculation steps of saturation pressure and the lack of correlation between uncertainties of pressure values. Maxwell's criteria determine the saturation pressure by solving Equations (16) and (17) simultaneously.

$$g(T, \rho_l) - g(T, \rho_v) = 0 \quad (16)$$

$$p(T, \rho_l) - p(T, \rho_v) = 0 \quad (17)$$

where g is Gibbs energy.

The solution yields not only the density values of the saturated liquid and vapor but also the Gibbs energy and pressure at saturation. The uncertainty of saturation temperature can then be calculated by converting the pressure difference uncertainty in Equation (17) to the uncertainty of saturation temperature by the Kline and McClintock (1953) method. The equations show that the uncertainty of saturation temperature is highly dependent on the uncertainty of pressure values calculated at the saturated liquid condition. The high values of the derivative of liquid pressure with respect to density causes high uncertainty of pressure differences in Equation (17). This implies that a small change of measured properties in the liquid region in the training data may lead to a very different liquid pressure value in Equation (17) and hence a very different saturation temperature. As a result, the uncertainty of saturation temperature and the uncertainties of subcooling and superheat due to HEoS in Figure 2 are much larger than that of measurement.

To illustrate that the cause of the large uncertainty is the presence of liquid pressure as a function of density in Maxwell's criteria, the uncertainties of superheat and subcooling due to EoS in Figure 2 are also calculated by imposing the uncertainty calculation method of regression model in Seber and Wild (1989) on the ancillary equation used in Lemmon et al. (2009) as shown in Equation (18).

$$\ln\left(\frac{p_{\text{sat}}}{p_c}\right) = \beta_0 \left(\frac{T_c}{T_{\text{sat}}}\right) \left(1 - \frac{T_{\text{sat}}}{T_c}\right) + \beta_1 \left(\frac{T_c}{T_{\text{sat}}}\right) \left(1 - \frac{T_{\text{sat}}}{T_c}\right)^{1.5} + \beta_2 \left(\frac{T_c}{T_{\text{sat}}}\right) \left(1 - \frac{T_{\text{sat}}}{T_c}\right)^{\beta_3} + \beta_4 \left(\frac{T_c}{T_{\text{sat}}}\right) \left(1 - \frac{T_{\text{sat}}}{T_c}\right)^{\beta_5} + \beta_6 \left(\frac{T_c}{T_{\text{sat}}}\right) \left(1 - \frac{T_{\text{sat}}}{T_c}\right)^{\beta_7} \quad (18)$$

Equation (18) calculates saturation pressure from saturation temperature of propane. It is used to calculate the saturation pressure from temperature, and Equations (3), (4) and (5) can be used to calculate the uncertainty of saturation pressure from Equation (18). The validation results and other details of the ancillary equation such as its theoretical background can be found in Lemmon et al. (2009).

The uncertainty of saturation temperature from the EoS can then be calculated from that of saturation pressure by Equation (19) using by the Clausius-Clapeyron relation (Çengel and Boles 2005).

$$\Delta T_{\text{sat}} = \left. \frac{dT}{dp} \right|_{\text{sat}} \Delta p_{\text{sat}} \quad (19)$$

Using Equation (19) to calculate the saturation temperature can help to analyze the cause of high uncertainty in Figure 2, because it only describes the relationship between saturation temperature and pressure. It does not depend on liquid pressure as a function of density, and the derivatives of liquid pressure with respect to density and Maxwell's criteria cannot affect the uncertainty of Equation (19). If the uncertainty of superheat and subcooling due to the uncertainty of saturation temperature from Equation (19) is small, it shows that the high uncertainty in Figure 2 is a result of the use of Maxwell's criteria in HEoS but not the vapor pressure data.

To calculate the uncertainty of saturation temperature from Equation (19), the covariance matrix of the coefficients in Equation was first calculated using the 1376 phase boundary pressure data points of propane listed in Cheung et al. (2018). The data were also used to calculate the Jacobian matrices and vectors in Equations (4) and (5) to find the uncertainty of the saturation pressure. The uncertainties of saturation temperature, superheat and subcooling can be calculated using Equation (19). The results are tabulated in Figure 3.

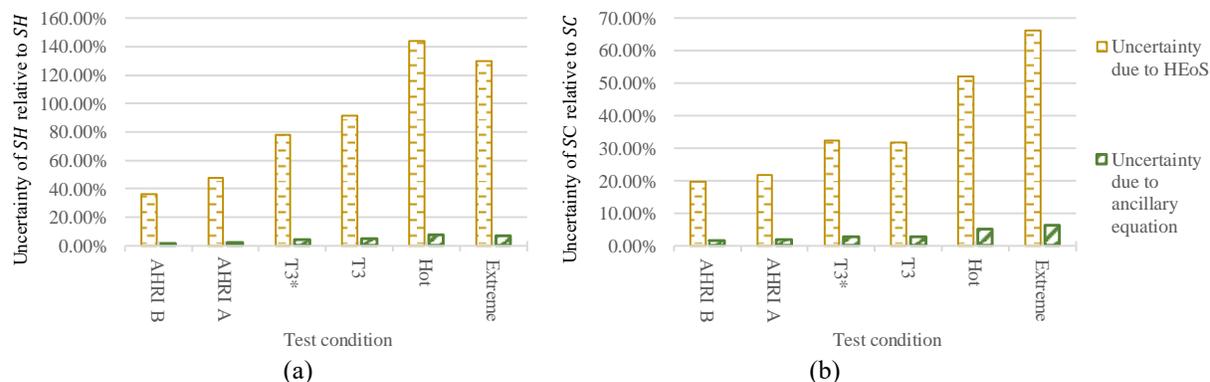


Figure 3: Comparison of (a) SH and (b) SC uncertainties due to HEoS and the ancillary equation

The uncertainties of superheat and subcooling values due to ancillary equation are much smaller than the uncertainties due to HEoS in Figure 3. This shows that the cause of the high uncertainty of HEoS in Figure 2 is the use of Maxwell's criteria in HEoS. If Maxwell's criteria are not used to calculate the saturation temperature, the saturation temperature values from an HEoS will not be influenced by the high sensitivity of the pressure values of liquid with respect to density, and the uncertainty of saturation temperature can be lowered significantly.

Although the uncertainties due to the ancillary equation in Figure 3 are smaller than the uncertainties due to HEoS, the values of uncertainties in Figure 3 are still approximately 42 % of the uncertainties of superheat and subcooling due to measurement in Figure 2. This shows that the uncertainties of EoS are significant to the uncertainties of superheat and subcooling values evaluated from experiments of air conditioners.

6. CONCLUSIONS

To conclude, the effects of uncertainty of Helmholtz-energy-based equation of state (HEoS) on the uncertainty of air conditioner performance metrics from laboratory experiments are evaluated. The study was conducted by applying the uncertainty calculation method of HEoS of propane properties on the uncertainty calculation of the performance metrics of an air conditioner tested in a laboratory. While the results show that the uncertainty of equation of state (EoS) is negligible in the calculation of the air conditioner's cooling capacity, it is significant to the calculation of the superheat and subcooling from the experimental results of the air conditioner. If the saturation temperature is calculated based on Maxwell's criteria, the uncertainty of the superheat and subcooling values are dominated by the uncertainty of EoS due to the high sensitivity of liquid pressure with density. However, if the saturation temperature is calculated from an auxiliary polynomial, the uncertainty of superheat and subcooling values due to HEoS will only be around 42 % of that due to measurement.

NOMENCLATURE

| | | |
|----------------|---------------------------------|--------|
| $COV(x)$ | covariance of variable x | (-) |
| $COV(\vec{x})$ | covariance matrix of \vec{x} | (-) |
| $diag(X)$ | diagonal elements of matrix X | (-) |
| f | function | (-) |
| g | Gibbs energy | (J/kg) |
| h | enthalpy | (J/kg) |
| \vec{j} | Jacobian vector | (-) |
| J | Jacobian matrix | (-) |
| m | number of parameters | (-) |
| \dot{m} | mass flow rate | (kg/s) |
| n | number of data points | (-) |
| p | pressure | (Pa) |
| \dot{Q} | cooling capacity | (W) |
| SC | subcooling | (°C) |

| | | |
|------------------|-----------------------------------|----------------------|
| SH | superheat | (°C) |
| SSE | sum of square error | (-) |
| t | Student t value | (-) |
| T | temperature | (°C) |
| \vec{x} | independent variable vector | (-) |
| X | independent variable matrix | (-) |
| y | dependent variable | (-) |
| \vec{y} | dependent variable vector | (-) |
| Greek | | |
| α | dimensionless Helmholtz energy | (-) |
| β | regression model parameter | (-) |
| $\vec{\beta}$ | regression model parameter vector | (-) |
| γ_t | p-value for Student t statistics | (-) |
| Δx | uncertainty of variable x | (-) |
| ρ | density | (kg/m ³) |
| Subscript | | |
| c | critical | |
| cond | condenser | |
| evap | evaporator | |
| EoS | equation of state | |
| mea | measurement | |
| out | outlet | |
| pred | predicted | |
| sat | saturation | |

REFERENCES

- Abdelaziz, O., Shrestha, S. S., Munk, J. D., Linkous, R. L., Goetzler, W., Guernsey, M., & Kassuga, T. (2015). Alternative Refrigerant Evaluation for High-Ambient-Temperature Environments: R-22 and R-410a Alternatives for Mini-Split Air Conditioners. ORNL/TM-2015/536. Oak Ridge National Laboratory (ORNL). Building Technologies Research and Integration Center (BTRIC). Retrieved from <https://www.osti.gov/scitech/biblio/1223676/>
- AHRI. (2004). *ANSI/AHRI Standard 540: 2004 Standard For Performance Rating Of Positive Displacement Refrigerant Compressors And Compressor Units*. Arlington, VA: Air-Conditioning, Heating and Refrigeration Institute.
- AHRI. (2008). *ANSI/AHRI Standard 210/240: 2008 Standard for Performance Rating of Unitary Air-Conditioning & Air-Source Heat Pump Equipment*. Arlington, VA: Air-Conditioning, Heating and Refrigeration Institute.
- ASME. (2016). *ASME PTC 30-1991 (RA 2016) Air-Cooled Heat Exchanger*. New York: American Society of Mechanical Engineers.
- Bejan, A. (2006). *Advanced Engineering Thermodynamics* (3 edition). Hoboken, N.J: Wiley.
- Bell, I. H., Satyro, M., & Lemmon, E. W. (2018). Consistent Two Parameters for More than 2500 Pure Fluids from Critically Evaluated Experimental Data. *Journal of Chemical & Engineering Data*. In Press. <https://dx.doi.org/10.1021/acs.jced.7b00967>
- Çengel, Y. A., & Boles, M. A. (2005). *Thermodynamics: An Engineering Approach (5th edition)*. Boston: McGraw-Hill Science/Engineering/Math.
- Cheung, H., Omer, S., & Bach, C. K. (2017). A Method to Calculate Uncertainty of Empirical Compressor Maps with the Consideration of Extrapolation Effect and Choice of Training Data. *Science and Technology for the Built Environment*. <https://doi.org/10.1080/23744731.2017.1372805>

Cheung, H., & Wang, S. (2018). Impact of Dynamics on The Accuracies of Different Experimental Data Processing Methods for Steady-state Heat Transfer Rate Measurement. *Journal of Thermal Science and Engineering Applications*, 10. <https://doi.org/10.1115/1.4037543>

Cheung, H., Frutiger, J., Bell, I. H., Abildskov, J., Sin, G., & Wang, S. (2018). A comprehensive uncertainty analysis for reference equations of state. *Journal of Chemical & Engineering Data*. Submitted

Coleman, H. W., & Steele, W. G. (2009). *Experimentation, Validation, and Uncertainty Analysis for Engineers (3rd edition)*. Hoboken, N.J: Wiley.

Dabiri, A. E., & Rice, C. K. (1981). A Compressor Simulation Model with Corrections for the Level of Suction Gas Superheat. In *ASHRAE Transactions* (Vol. 87, p. 771). American Society of Heating, Refrigerating and Air-conditioning Engineers, Inc. Retrieved from <http://www.osti.gov/scitech/biblio/829414>

Feistel, R., Lovell-Smith, J. W., Saunders, P., & Seitz, S. (2016). Uncertainty of empirical correlation equations. *Metrologia*, 53(4), 1079. <https://doi.org/10.1088/0026-1394/53/4/1079>

Frutiger, J., Andreasen, J., Liu, W., Spliethoff, H., Haglind, F., Abildskov, J., & Sin, G. (2016). Working fluid selection for organic Rankine cycles – Impact of uncertainty of fluid properties. *Energy*, 109, 987–997. <https://doi.org/10.1016/j.energy.2016.05.010>

JCGM. (2008). *JCGM 100:2008 Evaluation of measurement data — Guide to the expression of uncertainty in measurement*. Joint Committee for Guides in Metrology.

Kline, S. J., & McClintock, F. A. (1953). Describing uncertainties in single-sample experiments. *Mechanical Engineering*, 75(1), 3–8.

Lemmon, E. W. (2003). Pseudo-Pure Fluid Equations of State for the Refrigerant Blends R-410A, R-404A, R-507A, and R-407C. *International Journal of Thermophysics*, 24(4), 991–1006. <https://doi.org/10.1023/A:1025048800563>

Lemmon, E. W., McLinden, M. O., & Wagner, W. (2009). Thermodynamic Properties of Propane. III. A Reference Equation of State for Temperatures from the Melting Line to 650 K and Pressures up to 1000 MPa. *Journal of Chemical & Engineering Data*, 54(12), 3141–3180. <https://doi.org/10.1021/jc900217v>

Nocedal, J., & Wright, S. (2006). *Numerical Optimization*. Springer Science & Business Media.

Payne, W. V., Domanski, P. A., & Muller, J. (1999). *A Study to Water-to-Water Heat Pump Using Hydrocarbon and Hydrofluorocarbon Zeotropic Mixtures* (NISTIR 6330). National Institute of Standards and Technology.

Seber, G., & Wild, C. J. (1989). *Nonlinear regression*. Hoboken, NJ, USA: John Wiley & Sons, Inc.